Computational Complexity

Handout – Lecture 11

Definition 1 (δ_q)**.** For $q \geq 3$, let δ_q be the infimum of the set of constants c for which there exists an algorithm solving qSAT in time $O(2^{cn}) \cdot m^{O(1)}$, where n is the number of variables in the qSAT input and m the number of clauses.

Definition 2 (ETH)**.** The *Exponential-Time Hypothesis (ETH)* is the unproven conjecture that $\delta_3 > 0$.

The ETH implies that there is no $2^{o(n)}$-time algorithm algorithm that solves 3SAT, and therefore also that P $\neq$ NP.

Definition 3 (SETH)**.** The *Strong Exponential-Time Hypothesis (ETH)* is the unproven conjecture that:

$$\lim_{q \to \infty} \delta_q = 1.$$

Definition 4 (Sparsification Lemma)**.** For all $\epsilon > 0$, there is a constant $\kappa(\epsilon)$ such that every 3CNF formula φ with n variables and m clauses can be expressed as:

$$\varphi \equiv \bigvee_{i=1}^{t} \psi_i,$$

where $t \leq 2^{\epsilon n}$ and each ψ_i is a 3CNF formula on the same variables as φ and with $\kappa(\epsilon) \cdot n$ clauses. Moreover, this disjuction $\bigvee_{i=1}^{t} \psi_i$ can be computed in time $2^{\epsilon n} \cdot m^{O(1)}$.