Definition 1 (val(\(\varphi\))). Let \(\varphi\) be a propositional formula in CNF. Then val(\(\varphi\)) is the maximum ratio of clauses of \(\varphi\) that can be satisfied simultaneously by any truth assignment.

Thus, if \(\varphi\) is satisfiable, then val(\(\varphi\)) = 1, and if \(\varphi\) is not satisfiable, then val(\(\varphi\)) < 1.

Definition 2 (PCP verifier). Let \(L \subseteq \{0, 1\}^*\) be a language and let \(q, r : \mathbb{N} \rightarrow \mathbb{N}\) be functions. We say that \(L\) has an \((r(n), q(n))\)-PCP verifier if there is a polynomial-time probabilistic algorithm \(V\) that satisfies:

- **(Efficiency)** When given as input a string \(x \in \{0, 1\}^n\) and when given random access to a string \(\pi \in \{0, 1\}^*\) of length at most \(q(n)2^r(n)\) (which we call the proof), \(V\) uses at most \(r(n)\) random coin flips and makes at most \(q(n)\) nonadaptive queries to locations of \(\pi\).
  - Random access: \(V\) can query an oracle that gives the \(i\)-th bit of \(\pi\).
  - Nonadaptive queries: the queries do not depend on the answers for previous queries.
- **(Completeness)** If \(x \in L\), then there exists a proof \(\pi \in \{0, 1\}^*\) of length at most \(q(n)2^r(n)\) such that \(\Pr[V^* (x) = 1] = 1\). This string is called the correct proof for \(x\).
- **(Soundness)** If \(x \notin L\), then for every proof \(\pi \in \{0, 1\}^*\) of length at most \(q(n)2^r(n)\), it holds that \(\Pr[V^* (x) = 1] \leq 1/2\).

Definition 3 (PCP(\(r(n), q(n))\)). Let \(q, r : \mathbb{N} \rightarrow \mathbb{N}\) be functions. The class PCP(\(r(n), q(n))\) consists of all languages \(L \subseteq \{0, 1\}^*\) for which there exist constants \(c, d > 0\) such that \(L\) has a \((c \cdot r(n), d \cdot q(n))\)-PCP verifier.