Computational Complexity

Exercise Session 6

Exercise 1. CLIQUE is the problem of deciding, given a graph $G = (V, E)$ and a natural number $k \in \mathbb{N}$, whether there exists a set $C \subseteq V$ such that $|C| = k$ and for all $c_1, c_2 \in C$ with $c_1 \neq c_2$ it holds that $\{c_1, c_2\} \in E$.

For every $\rho < 1$, an algorithm A is called a ρ-approximation algorithm for MAX-CLIQUE if for every graph $G = (V, E)$, $A(G)$ outputs a clique $C \subseteq V$ of G of size at least $\rho \cdot \mu_G$, where μ_G is the maximum size of any clique of G.

Show that for every $\rho < 1$, there is no polynomial-time ρ-approximation algorithm for MAX-CLIQUE, unless P = NP.

Exercise 2. Consider the following problem:

$$\text{SQROOT-CLIQUE} = \{\ G\ :\ \langle G, \sqrt{m} \rangle \in \text{CLIQUE}, G \text{ has } m \text{ vertices}\ \}.$$

Show that SQROOT-CLIQUE is solvable in time $2^{o(m)}$.