Computational Complexity

Exercise Session 3

Exercise 1. Is there an oracle such that, relative to this oracle, ...? If so, then give such an oracle and prove that it works. If not, prove why not.

(a) $\text{DTIME}(n^2) = \text{DTIME}(n^3)$

(b) $\text{DTIME}(n^2) \neq \text{DTIME}(n^3)$

(c) $\text{P} = \text{coNP}$

(d) $\text{P} \neq \text{coNP}$

Exercise 2. Show that if $\text{NTIME}(n) \subseteq \text{DTIME}(n)$, then $\text{P} = \text{NP}$.

- $\text{NTIME}(n)$ can be characterized as the set of all decision problems that can be verified in linear time with a linear-size certificate. That is, $A \in \text{NTIME}(n)$ if and only if there is a linear-time Turing machine $\mathbb{M}$ and a constant c such that for all $x \in \{0,1\}^*$ it holds that $x \in A$ if and only if there exists some $u \in \{0,1\}^{c \cdot |x|}$ such that $\mathbb{M}(x, u) = 1$.
- *Hint:* Use a padding argument.

Definition 1. Let $A \subseteq \{0,1\}^*$ be a language. When a Turing machine $\mathbb{M}$ has access to an A-oracle, we write $\mathbb{M}^A$. We say that A is *auto-reducible* if there is a polynomial-time Turing machine $\mathbb{M}^A$ with oracle access to A such that for all $x \in \{0,1\}^*$:

$$x \in A \text{ if and only if } \mathbb{M}^A(x) = 1,$$

with the special requirement that on input x the Turing machine $\mathbb{M}^A$ is not allowed to query the oracle A for x.

Exercise 3. Prove that 3SAT is auto-reducible.