Computational Complexity

Homework Sheet 4

Hand in before March 6, 17:00
By email to `J.Czajkowski@uva.nl`

Exercise 1 (2pt)**.** Prove that coNP $\subseteq$ PSPACE.

Exercise 2 (2pt)**.** Prove that NL $\subseteq$ P.

- *Hint:* consider the configuration graph of the nondeterministic Turing machine.

Exercise 3 (3pt)**.** In this exercise, all graphs are directed graphs. We define the following decision problem:

$$A = \{ \langle G, s, t \rangle \mid \text{The graph } G \text{ contains a vertex } v \text{ such that } v \text{ is not reachable from } s \text{, and } t \text{ is reachable from } v. \}$$

Show that this problem is in NL.

Exercise 4 (3pt)**.** Define the complexity class DP as follows:

$$\text{DP} = \{ A \cap B \mid A \in \text{NP}, B \in \text{coNP} \}.$$

Let $G = (V, E)$ be an undirected graph. A subset $C \subseteq V$ of vertices is called a *clique* of G if every $v_1, v_2 \in C$ with $v_1 \neq v_2$ are connected by an edge in E. Consider the problem Exact-Clique:

$$\textsc{Exact-Clique} = \{ \langle G, k \rangle \mid G \text{ is an undirected graph that has a clique of size } k \text{ but has no clique of size } k+1 \}.$$

(a) Prove that if DP $\subseteq$ NP, then the Polynomial Hierarchy collapses.

(b) Prove that Exact-Clique is DP-complete.

- *Hint:* You may use the following. Let L be a problem in NP. Then there exists a polynomial-time reduction R from L to Clique such that for every $x \in \{0,1\}^*$ the instance $\langle G, k \rangle = R(x)$ has the following additional property: G has no clique of size $k+1$.