Computational Complexity

Exercise Session 4

Exercise 1. Prove that $\mathrm{L} \subseteq \mathrm{P}$.

Definition 1. We define DP to be the following complexity class:

$$\mathrm{DP} = \{\ A \cap B \mid A \in \mathrm{NP}, B \in \mathrm{coNP}\ \}.$$

Definition 2. Let $G = (V, E)$ be an undirected graph. A subset $C \subseteq V$ of vertices is called a *clique* of G if every $v_1, v_2 \in C$ with $v_1 \neq v_2$ are connected by an edge in E. The problem Exact-Clique is defined as follows:

$$\textsc{Exact-Clique} = \{\ \langle G, k\rangle \mid \begin{array}{l} G \text{ is an undirected graph that has a clique of size } k \\ \text{but has no clique of size } k+1\ \}. \end{array}$$

Exercise 2.

(a) Explain the difference between DP and the class $\mathrm{NP} \cap \mathrm{coNP}$.

(b) Prove that $\textsc{Exact-Clique} \in \mathrm{DP}$.

(c) Prove that $\mathrm{NP} \cup \mathrm{coNP} \subseteq \mathrm{DP}$.

(d) Prove that $\mathrm{P} = \mathrm{DP}$ if and only if $\mathrm{P} = \mathrm{NP}$.