Computational Complexity

Exercise Session 1

Definition 1. Let f, g be functions $\mathbb{N} \to \mathbb{R}$.

$$f \in O(g) \quad \text{means} \quad (\exists c, n_0 \in \mathbb{N})(\forall n \geq n_0)(|f(n)| \leq c|g(n)|) \tag{$\leq$}$$

$$f \in o(g) \quad \text{means} \quad (\forall \varepsilon > 0)(\exists n_0 \in \mathbb{N})(\forall n \geq n_0)(|f(n)| < \varepsilon|g(n)|) \tag{$<$}$$

$$\text{or equivalently} \quad \lim_{n\to\infty} \frac{|f(n)|}{|g(n)|} = 0$$

$$f \in \Omega(g) \quad \text{means} \quad g \in O(f) \tag{$\geq$}$$

$$f \in \omega(g) \quad \text{means} \quad g \in o(f) \tag{$>$}$$

$$f \in \Theta(g) \quad \text{means} \quad f \in O(g) \cap \Omega(g) \tag{$=$}$$

Exercise 1.

1. Let $f(n) = pn^3 + qn^2 + rn + s$ for some $p, q, r, s \in \mathbb{R}$. Show that $f(n) \in O(n^3)$ and $f(n) \in o(n^4)$.
2. Show that $\sin(n) \in O(1)$ and $\sin(n) \notin o(1)$.
3. Show that $n^{\log n} \in O(2^n)$.

Exercise 2 (Exercise 1.2 from the book (Arora & Barak, 2009))**.** Complete the proof of Claim 1.5 in the book by writing down explicitly the description of the machine $\tilde{M}$.