Exercise 1 (4pt). Consider the following polynomial-time reduction \( f \) from 3SAT to 3SAT. Let \( \varphi \) be a 3CNF formula with clauses \( c_1, \ldots, c_m \) and containing the variables \( x_1, \ldots, x_n \). Then \( f(\varphi) \) is defined as the following 3CNF formula:

\[
f(\varphi) = \varphi \land \left( \bigwedge_{j=1}^{m} y_j \right) \land \left( \bigwedge_{j=1}^{m} \bigwedge_{j' = 1}^{m} (y_j \lor y_{j'}) \right),
\]

where each of the variables \( y_j \) is a fresh variable that does not occur in \( \varphi \). (Note: here we define a 3CNF formula as a CNF formula where each clause contains at most 3 literals.)

Let \( F \) be the following set of 3CNF formulas:

\[
F = \{ f(\varphi) \mid \varphi \text{ is a 3CNF formula} \},
\]

and let \( \text{FUNNY-3SAT} \) be the following decision problem:

\[
\text{FUNNY-3SAT} = F \cap 3\text{SAT}.
\]

(a) Show that \( \text{FUNNY-3SAT} \) is solvable in time \( 2^{O(\sqrt{|x|})} \), where \( |x| \) denotes the input size.

(b) Show that \( \text{FUNNY-3SAT} \) is not solvable in time \( 2^{o(\sqrt{|x|})} \), where \( |x| \) denotes the input size, assuming that the ETH is true.

Exercise 2 (1pt). Give an example of a decision problem that is not solvable in polynomial time (assuming \( P \neq \text{NP} \)), yet that is solvable in time \( 2^{o(|x|)} \), where \( |x| \) denotes the input size.

Exercise 3 (2pt). Show that in interactive proof systems we gain nothing by allowing the prover to make use of randomness. That is, show that if we have a probabilistic prover \( P \) that convinces a verifier \( V \) to accept with probability \( p \), where the probability is taken over the random coins of both \( P \) and \( V \), then we have a deterministic prover \( P \) that convinces \( V \) to accept with probability \( \geq p \), where the probability is now taken only over the random bits of \( V \).

Exercise 4 (3pt). Show that \( \text{IP} \subseteq \text{PSPACE} \).