Computational Complexity

Homework Sheet 5

Hand in before March 14, 23:59

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Exercise 1 (2pt)**.** Show that RP $\subseteq$ NP.

Exercise 2 (4pt)**.** Show that ZPP = RP $\cap$ coRP.

- *Hint:* use Markov's inequality for showing that ZPP $\subseteq$ RP $\cap$ coRP. If X is a non-negative random variable and $a > 0$, then:
$$\mathbb{P}(X \geq a) \leq \frac{\mathbb{E}(X)}{a}.$$

Exercise 3 (4pt)**.** The problem MAX2SAT consists of all tuples $\langle \varphi, k \rangle$ where φ is a 2CNF formula and $k \in \mathbb{N}$ such that there exists a truth assignment $\alpha : \mathrm{var}(\varphi) \to \{0, 1\}$ such that α satisfies at least k clauses of φ. (Note: here we define a 2CNF formula as a CNF formula where each clause contains **at most** 2 literals. Note also: φ might contain several copies of the same clause.)

For every $\rho \geq 1$, an algorithm A is called a ρ-approximation algorithm for MAX2SAT if for every 2CNF formula φ with m clauses, $A(\varphi)$ outputs a truth assignment satisfying at least $\rho \cdot \mu_\varphi$ of φ's clauses, where μ_φ is the maximum number of clauses of φ satisfied by any truth assignment.

Consider the following polynomial-time reduction f from 3SAT to MAX2SAT:

> Let $\varphi = c_1 \wedge \cdots \wedge c_m$ be a 3CNF formula with clauses $c_1, \ldots, c_m$ and containing the propositional variables $p_1, \ldots, p_u$. Then $f(\varphi) = \langle \psi, k \rangle$ is defined as follows.
>
> - The formula ψ will contain the propositional variables $p_1, \ldots, p_u$, as well as the new variables $q_1, \ldots, q_u$.
> - For each clause $c_j = l_{j,1} \vee l_{j,2} \vee l_{j,3}$ of φ, we add the following 10 clauses to ψ:
> $$\begin{array}{l}(l_{j,1}), (l_{j,2}), (l_{j,3}), (q_j), \\ (\neg l_{j,1} \vee \neg l_{j,2}), (\neg l_{j,1} \vee \neg l_{j,3}), (\neg l_{j,2} \vee \neg l_{j,3}), \\ (l_{j,1} \vee \neg q_j), (l_{j,2} \vee \neg q_j), (l_{j,3} \vee \neg q_j).\end{array}$$
>
>   That is ψ consists of the conjunction of the $10m$ resulting clauses.
> - We let $k = 7m$.

(a) Show that this reduction is correct.

(b) Show that if there is a polynomial-time ρ-approximation algorithm for MAX2SAT for each $\rho < 1$, then P = NP.

- *Hint:* use the function f described above.

(c) Give a polynomial-time $\frac{1}{2}$-approximation algorithm for MAX2SAT.