

Computational Complexity

Homework Sheet 3

Hand in before February 28, 23:59

Preferably by email to J.M.Czajkowski@cwi.nl

Exercise 1 (2pt). Is there an oracle such that, relative to this oracle, ...? If so, then give such an oracle and prove that it works. If not, prove why not.

(a) $P = EXP$

(b) $coNP \subseteq P$ and $NP \not\subseteq P$

(c) $DTIME(n) = DTIME(n^2)$

(d) $NP = coNP \neq EXP$

For example, in (a) you have to either (i) show that there exists an oracle A such that $P^A = EXP^A$ or (ii) show that such an oracle does not exist. In (b), you have to either (i) show that there exists an oracle A such that $coNP^A \subseteq P^A$ and $NP^A \not\subseteq P^A$ or (ii) show that such an oracle does not exist.

Exercise 2 (2pt). Prove that $coNP \subseteq PSPACE$.

Exercise 3 (3pt). Prove that $NL \subseteq P$.

- *Hint:* consider the configuration graph of the nondeterministic Turing machine.

Exercise 4 (3pt). In this exercise, all graphs are directed graphs. We define the following decision problem:

$$A = \{ \langle G, s, t \rangle \mid \text{The graph } G \text{ contains a vertex } v \text{ that is} \\ \text{not reachable from } s, \text{ such that } t \text{ is reachable from } v. \}$$

Show that this problem is contained in NL.