## Comparatives and quantifiers

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## 1 Introduction

A traditional issue in the analysis of comparatives is whether or not degrees are essential. In the first part of this paper I discuss the traditional analyses that account for comparatives with (Seuren, von Stechow) and without (Klein) degrees, and remind the reader that these are very similar to each other. A more recent issue is how to account for quantifiers in the than-clause. The traditional analyses account well for Negative Polarity Items in comparative clauses, but have problems with conjunctive quantifiers. The strength of the proposals of Larson (1988) and Schwarzchild \& Wilkinson (2002), on the other hand, goes exactly in the opposite direction. I will discuss two types of strategies so as to account for both types of quantifiers: (i) one based on the traditional analysis, but by making use of more coarse-grained models or of intervals, (ii) one where comparatives are taken to be ambiguous between the traditional reading and the Larson-reading, and where the actual reading is selected with the help of the strongest meaning hypothesis.

## 2 The traditional analyses of comparatives

There exist two major types of approaches to the analysis of gradable adjectives: comparison class approaches and degree-based approaches. In this section I sketch the traditional approaches along these lines, and show how close they are to each other.

Intuitively, John can be counted as tall when we compare him with other men, but not tall when we compare him with (other) basketball players. Thus, whether someone of 1.80 meters is tall or not is context dependent. Wheeler (1972) and Klein (1980) propose that every adjective should be interpreted with respect to a comparison class. A comparison class is just a set of objects/individuals and is contextually given. In particular if the adjective stands alone, we might assume that the contextually given comparison class helps to determine what counts as being tall. Klein (1980) assumed that with respect to a given comparison class, some elements of this set are considered to be definitely tall, some definitely not tall, and the others are borderline cases. The truth of the positive sentence (1)

## (1) John is tall.

[^0]depends on the contextually given comparison class: (1) is true in context (or comparison class) $c$ iff John is counted as tall in this class. The proposition expressed by a comparative like (2) is context independent.
(2) John is taller than Mary.
and the sentence is true iff there is a comparison class according to which John counts as tall, while Mary does not: $\exists c[T(j, c) \wedge \neg T(m, c)] .{ }^{1}$

According to the degree-based approaches (e.g. Seuren, 1973; Cresswell, 1976; Bierwisch, 1984; von Stechow, 1984, Kennedy, 1999, 2007), relative adjectives are analyzed as relations between individuals and degrees, where these degrees are associated with the dimension referred to by the adjective. Individuals can posses a property to a certain measurable degree, and the truth conditions of comparative and absolutive sentences are stated in terms of degrees. According to the most straightforward degreebased approach, the absolutive (1) is true iff the degree to which John is tall is (significantly) greater than a (contextually given) standard of length, while the comparative (2) is true iff the degree to which John is tall is greater than the degree to which Mary is tall. But this straightforward degree-based approach has a problem with examples where the scope of the comparative contains a disjunction, an indefinite ('any'), or existential modal:
(3) a. John is taller than Mary or Sue.
b. John is taller than anyone else.
c. John is taller than allowed.

It is not easy to see how the above degree-based approach can account for the intuition that from (3-b), for instance, we infer that John is taller than everybody else. To account for this, and the other examples above, Von Stechow (1984) introduced a maximality operator. Example (3-a) is predicted to be true iff the degree to which John is tall is higher than the maximal degree to which Mary or Sue is tall.

$$
\begin{equation*}
\max \{d \in D: T(j, d)\}>\max \left\{d^{\prime} \in D \mid T\left(m, d^{\prime}\right) \vee T\left(s, d^{\prime}\right)\right\} . \tag{4}
\end{equation*}
$$

Such an analysis predicts correctly for examples (3-a), (3-b), and (3-c).
According to Seuren (1973), (2) 'John is taller than Mary' is true iff there is a degree $d$ of tallness that John has but Mary does not: $\exists d[\operatorname{Tall}(j, d) \wedge \neg \operatorname{Tall}(m, d)]$. In this for-

[^1](i) a. John is as tall as Mary.
b. In every context where Mary is tall, John is tall as well.

Klein (1980) notes that on this analysis, the negation of (i-a), i.e. (ii-a), is correctly predicted to be equivalent with (ii-b):
(ii) a. John is not as tall as Mary.
b. Mary is taller than John.

Standard pragmatics can explain why in the context of question How tall is John?, (i-a) would come to mean that John and Mary are equally tall.
malization, $T(j, d)$ means that John's degree of tallness includes at least $d .^{2}$ This analysis easily accounts for the intuition concerning (3-a), (3-b), and (3-c), by representing them by ( $5-\mathrm{a}$ ), ( $5-\mathrm{b}$ ), and ( $5-\mathrm{c}$ ) respectively (treating 'any' as an existential quantifier):
a. $\quad \exists d[T(j, d) \wedge \neg(T(m, d) \vee T(s, d))]$.
b. $\exists d[T(j, d) \wedge \neg \exists x[x \neq j \wedge T(x, d)]]$.
c. $\exists d[T(j, d) \wedge \neg \diamond T(j, d)]$.

It is obvious that von Stechow's analysis is very close to Seuren's analysis if the formula ' $T(j, d$ )' means that John's degree of tallness includes at least $d$. On this assumption, Seuren's analysis of John is taller than Mary is true iff $\{d \in D: T(j, d)\} \supset\{d \in D$ : $T(m, d)\}$. Now assume that the sentence is true on von Stechow's analysis: $\operatorname{Max}\{d \in D$ : $T(j, d)\}>\operatorname{Max}\{d \in D: T(m, d)\}$. Because of the 'at least' reading of tallness, it follows that $\forall d \in\{d \in D: T(m, d)\}: d \in\{d \in D: T(j, d)\}$, i.e., $\{d \in D: T(m, d)\} \subseteq\{d \in D: T(j, d)\}$. Because $\operatorname{Max}\{d \in D: T(j, d)\}>\operatorname{Max}\{d \in D: T(m, d)\}$, it is immediate that $\operatorname{Max}\{d \in D$ : $T(j, d)\} \notin\{d \in D: T(m, d)\}$. Thus we can conclude $\{d \in D: T(m, d)\} \subset\{d \in D: T(j, d)\}$, which is Seuren's analysis.

Seuren's analysis - and thus von Stechow's analysis - is obviously close to Klein's comparison-class account. And indeed, also Klein has no problem with examples like $(3-a),(3-b)$, and $(3-c) .{ }^{3}$ This is obvious for (3-a) and (3-b). To see why the comparisonclass approach accounts successfully for (3-c), represented by $\exists c[T(j, c) \wedge \neg \diamond T(j, c)]$, notice that this sentence is predicted to be true iff there is a context in $\{c \in C \mid T(j, c)\}$ that is not an element of $\{c \in C \mid \diamond T(j, c)\}$. Suppose that we have five individuals, John, Mary, Sue, Bill, and Lucy, such that Bill $>$ Mary $>$ Sue $>$ Lucy. To be allowed (to become an astronaut, for instance), one has to be taller than Lucy, but one may not be taller than Bill. In that case, the set of contexts (containing 2 individuals) where John's tallness is allowed, $\{c \in C \mid \diamond T(j, c)\}$, is $\{\{j, m\},\{j, s\},\{j, l\}\}$. But this means that (3-c) is

[^2]predicted to be true according to the comparison-class account iff John is taller than Bill, i.e. taller than the tallest individual that is allowed. Similarly, the set of contexts denoted by $\{c \in C \mid \square T(j, c)\}$ in this example would be $\{\{j, l\}\}$, and it is predicted that 'John is taller than required' is true iff John is taller (or equally tall) than the smallest individual that is allowed. These predictions are the same as those made by the degree-based approach.

One of the obvious requirements for any theory of comparatives is that they should account for the converse relation that holds between the comparatives of antonyms: (2) 'John is taller than Mary' is true iff 'Mary is shorter than John' is true. Seuren's degree-based approach seems to have no problem with this. The straightforward proposal is simply to define 'short' as 'not tall'. 'Mary is shorter than John' is then true iff $\exists d[\neg T(m, d) \wedge \neg \neg T(j, d)]$, meaning that $\exists d[T(j, d) \wedge \neg T(m, d)]$ and thus that (2) is true. Notice, moreover, that on the assumption that degrees have an 'at least' reading, one immediately predicts what Kennedy (1999) calls cases of 'cross polar anomaly. ${ }^{4}$ That is, it is correctly predicted that 'John is taller than Mary is short' and 'Mary is shorter than John is tall' are inappropriate. To see this, notice that $T(j, d)$ is true iff John is as least as tall as $\mathbf{d}$ : $j \geq_{T} \mathbf{d}$, where $\mathbf{d}$ is the degree (or an individual in the equivalence class) 'corresponding' with $d$. Assuming that $x \geq_{T} y$ iff $y \geq_{S} x$, it follows that $S(m, d)$ is true iff $m \geq_{S} \mathbf{d}$ iff $m \leq_{T} \mathbf{d}$. But this means that $\neg S(m, d)$ is true iff $m>_{T} \mathbf{d}$. From this it follows that it is predicted that both 'John is taller than Mary is short' $(\exists d[T(j, d) \wedge \neg S(m, d)])$ and 'Mary is shorter than John is tall' $(\exists d[S(m, d) \wedge \neg T(j, d)])$ denote the (almost) trivial proposition.

It seems that analyzing 'short' as 'not tall' within a degree-based analysis gives rise to the wrong prediction that 'John is tall and short' and 'John is neither tall nor short' are equivalent. This problem does not exist for Klein's comparison class-based analysis: if there is any comparison class in which John but not Mary counts as tall, this is also the case in the comparison class containing just John and Mary. But this means that in this context Mary is short, while John is not. From this we can conclude that we can account for the intuition that (2) John is taller than Mary and Mary is shorter than John have the same truth conditions without assuming that we should analyze short as meaning not tall. But in fact, also the degree-based account need not generate this problem. ${ }^{5}$ Recall that according to degree-based approaches, the positive use of adjectives is treated in a somewhat different way from adjectives occurring in comparatives. For positive uses, an additional POS-operator is assumed, and ‘John is tall', for instance, is represented as $\exists d[\operatorname{POS}(T)(j, d)]$, where $\operatorname{POS}(T)(j, d)$ is true iff John has the degree of tallness $d$ and $d$ is higher than the contextually given standard of tallness. The sentence 'John is short' is then analyzed as $\exists d[\operatorname{POS}(\neg T)(j, d)]$, where $\operatorname{POS}(\neg T)(j, d)$ is true iff John has degree $d$ of $\neg T$ and $d$ is higher than the contextually given standard of not-tallness. If we then assume that the orderings of tallness and not-tallness are duals of each other, and that the standards of tallness and not-tallness need not be the same, also a degree-based approach does not predict that 'John is tall and short' and 'John is neither tall nor short' are equivalent.

There has been a lot of discussion about the pro's and con's of the comparison class-

[^3]based analysis versus the degree-based analysis. According to wide-spread opinion, the comparison class analysis is conceptually more appealing because it assumes that the positive use of the adjective is basic, and it better reflects our basic ability to draw comparisons. ${ }^{6}$ On the other hand, the degree-based analysis can account for more examples. In particular examples where we explicitly talk about degrees.

Von Stechow (1984) and Kenedy (1999) argue that even if we don't explicitly talk about degrees, we are still required to have degrees at our disposal to account for socalled subdeletion examples like (6-a) and (6-b) that involve two different types of adjectives:
(6) a. This table is longer than that table is wide.
b. This table is longer than it is wide.

But, actually, Klein (1980) himself already suggested an analysis of subdeletion comparatives. His final analysis is somewhat more complicated than I suggested until now: rather than quantifying over comparison classes, he existentially quantifies over (the meanings of) modifiers of adjectives, like very and fairly. One motivation for quantifying over such modifiers is to be able to account for subdeletion comparatives like (7-a), ${ }^{7}$ which are interpreted as something like (7-b) as suggested earlier by McConnellGinet (1973).
a. John is more happy than Mary is sad.
b. $\quad \exists \mathbf{f} \in\{$ very, fairly, quite,..$\}[\mathbf{f}($ Tall $)(j) \wedge \neg \mathbf{f}($ Sad $)(m)]$.

Klein (1980) accounts for modifiers of adjectives in terms of comparison classes and shows that existentially quantifying over comparison classes is only a special case of quantifying over these modifiers. To illustrate this, suppose we have a set of 4 individuals: $I=\{w, x, y, z\}$. One comparison-class, call it $c_{0}$, is $I$. Suppose now that $P\left(c_{0}\right)=\{w, x\}$, and (thus) $\bar{P}\left(c_{0}\right)=\{y, z\}$. We can now think of $P\left(c_{0}\right)$ and $\bar{P}\left(c_{0}\right)$ as new comparison classes, i.e., $P\left(c_{0}\right)=c_{1}$, and $\bar{P}\left(c_{0}\right)=c_{2}$. Let us now assume that $P\left(c_{1}\right)=\{w\}$ and $P\left(c_{2}\right)=\{y\}$. If so, this generates the following ordering via Klein's definition of the comparative we used before: $w>_{P} x>_{P} y>_{P} z$. Let us now assume that $\mathbf{f}$ is an expression of type $\langle\langle\langle e, t\rangle,\langle e, t\rangle\rangle,\langle\langle e, t\rangle,\langle e, t\rangle\rangle\rangle$, i.e., as a modifier of adjectives. According to Klein's (1980) final analysis, he represents comparatives of the form ' $x$ is $P$-er than $y$ ' as follows:

$$
\begin{equation*}
\left.\exists \mathbf{f}\left[\mathbf{f}(P)\left(c_{0}\right)(x) \wedge \neg \mathbf{f}(P)\left(c_{0}\right)(y)\right)\right] \tag{8}
\end{equation*}
$$

To continue our illustration, we can define the following set of modifier functions on domain $I$ in terms of the behavior of $P$ with respect to different comparison classes: $\mathbf{f}_{\mathbf{1}}(P)\left(c_{o}\right)=P(c), \mathbf{f}_{2}(P)\left(c_{o}\right)=P\left(P\left(c_{0}\right)\right), \mathbf{f}_{\mathbf{3}}(P)\left(c_{o}\right)=P\left(\bar{P}\left(c_{0}\right)\right)$, and $\mathbf{f}_{\mathbf{4}}(P)\left(c_{o}\right)=c_{0}$. Take $\mathbf{F}$ to

[^4]be $\left\{\mathbf{f}_{\mathbf{1}}, \mathbf{f}_{2}, \mathbf{f}_{3}, \mathbf{f}_{4}\right\}$. Obviously, this new analysis of the comparative gives rise to the same order: $w>_{P} x>_{P} y>_{P} z$. Moreover, any of those comparatives can only be true according to the new analysis, if it is true according to the old analysis: The statement ' $w>_{P} x$ ' is true, for instance, because of function $\mathbf{f}_{2}$. But $\mathbf{f}_{2}(P)\left(c_{0}\right)(w) \wedge \neg \mathbf{f}_{2}(P)\left(c_{0}\right)(x)$ holds iff $P\left(c_{1}\right)(w) \wedge \neg(P)\left(c_{1}\right)(x)$, which demonstrates (for this special case) that the old analysis is indeed a special case of the new analysis.

Klein $(1980,1991)$ suggests, furthermore, that measure phrases can be thought of as modifiers of adjectives, which means that Seuren's (1973) analysis is a special case of what Klein (1980) proposed. To do so, we have to assume an 'at least' meaning of adjective modifiers, i.e, that if $x \in \mathbf{f}(P)\left(c_{o}\right)$ and $y \geq_{P} x$, then it has to be that $y \in \mathbf{f}(P)\left(c_{0}\right)$, if $x, y \in c_{0}$. The function $\mathbf{f}_{3}$ defined above does not satisfy this constraint, but we can define a similar function that does so: $\mathbf{f}_{\mathbf{3}}^{\prime}(P)\left(c_{0}\right)={ }_{\text {def }} P\left(c_{0}\right) \cup P\left(\bar{P}\left(c_{0}\right)\right)$. Something like this can be done in general. If we do so, it holds for every $\mathbf{f} \in \mathbf{F}$ that $\mathbf{f}(P)\left(c_{0}\right)$ denotes the set of all individuals at least as $P$ as a particular individual, which might, but need not, have property $P$ (w.r.t. comparison class $c_{0}$ ). Indeed, by the construction of set $\mathbf{F}$ it doesn't follow that for all $\mathbf{f} \mathbf{f}(P)\left(c_{0}\right) \subseteq P\left(c_{0}\right)$. But this means that if we limit ourselves to one adjective and its antonym, we can think of the fs, intuitively, as degrees. In that case we might as well forget about the comparison class and reduce (8) to $\exists \mathbf{f}[\mathbf{f}(P)(x) \wedge$ $\neg \mathbf{f}(P)(y)]$, which indeed is (very close to) Seuren's degree-based analysis.

One might think that once we have these 'degrees', we can immediately account for (6-a) and (6-b). But there is still a problem: there need not be any relation between any f applied to $L$ (ong), and the same $\mathbf{f}$ applied to $W$ (ide). As noted by Kennedy (1999), as a result it is not clear how Klein (1980) could make a distinction between the appropriateness of (6-a) versus the inappropriateness of 'John is longer than Mary is clever' on at least one of its readings. What this points to is that we have to assume that on the normal reading, the modifier functions have to have more structure, and be partial. ${ }^{8}$ The functions that take 'Long' as argument should take 'Wide' as argument as well, but not 'clever' or 'ugly'. The obvious intuition for this is that in contrast to cleverness, length and width are commensurable, i.e., have the same dimension. Notice, however, that according to natural language, commensurabiltiy is a flexible term. Not only do we have examples like John is more happy than Mary is sad, even examples like Little John is more centimeters tall than Big Pete is meters wide, and John is more centimeters tall than it is ${ }^{\circ} C$ warm in Amsterdam don't seem to be totally out. In the last two comparatives, we only compare numbers, not degrees of any more concrete type. Perhaps this is all we ever do, and the reason why sentences like John is taller than it is warm in Amsterdam are inappropriate is that it is unclear what the units and zero-points of measurements are to measure tallness and warmth. For standard comparatives like John is taller than Mary the unit and zero-point are irrelevant, and standard subdeletion cases like This table is longer than it is wide are so natural because it is very natural to assume that the units and zero-points of tallness and width are the same.

How does this relate with the analysis of ( $7-\mathrm{a}$ ) as suggested by McConnell-Ginet (1973) and taken over by Klein (1980)? I believe that these cases are good in case happiness and sadness have obvious zero-points and clear units. Notice first that in case

[^5](7-a) is true, we intuitively infer that John is happy and that Mary is sad. ${ }^{9}$ This suggests that 'normal' happiness and 'normal' sadness are now the contextually salient zero-points. In terms of Klein's semantics this means that with respect to the contextually salient comparison class $c$, John is an element of $\operatorname{Happy}(c)$ and Mary an element of $\operatorname{Sad}(c)$. As for units, let us assume that we take with McConnell-Ginet and Klein a function that models the meaning of an adjective modifier, like very. It sounds in accordance with our intuition to say that John is more happy than Mary is sad is true iff John is, for instance, very very happy, but Mary is only very sad. More in general, we can say that the sentence is true iff there is an intersective function $\mathbf{f}$ that can be applied more times to happiness such that John is an element of it, than it can be applied to sadness such that Mary is an element of it.

## 3 Quantifiers in than-clauses

### 3.1 The problem

Although Klein's account of comparatives is in some respect crucially different from the standard degree-based approaches, we have seen that they have a lot in common. Many examples treated appropriately in one theory are treated appropriately in the other theory as well. Unfortunately, all these traditional approaches have some problems as well.

The approaches discussed in section 2 all have problems with conjunctive quantifiers in the than-clause. As noted already by Von Stechow (1984), these approaches give rise to the wrong predictions for sentences of the following form:
(9) a. John is taller than everybody else is.
b. John is taller than Mary and Sue.

Intuitively, (9-b) is true iff John is taller than Mary and John is taller than Sue. But on the analyses discussed above, this doesn't come out:

$$
\begin{align*}
\text { a. } & \exists c[T(j, c) \wedge \neg(T(m, c) \wedge T(s, c))] .  \tag{10}\\
& \equiv \exists c[T(j, c) \wedge(\neg T(m, c) \vee \neg T(s, c))] .
\end{align*}
$$

[^6](i) $\quad \exists \mathbf{f} \in \mathbf{F}^{*}\left[\mathbf{f}(P)\left(c_{0}\right)(x) \wedge\left((N E G(\mathbf{f})(P))\left(c_{0}\right)(y)\right]\right.$

## EVALUATIVE

In this formula, we use the same typing as before, but we assume that for each adjective $P$ and modifier $\mathbf{f} \in \mathbf{F}^{*}:(\mathbf{f}(P))(c) \subseteq P(c)$. We assume that NEG is a function from adjective modifiers to objects of the same type (is of a type too long to give here) with meaning: $N E G=\lambda \mathbf{f} \cdot \lambda P \cdot \lambda c .(P(c)-(\mathbf{f}(P))(c))$. Recall that from (i) and our assumptions it follows immediately that both $x$ and $y$ have property $P$ with respect to comparison class $c_{0}$, which accounts for the intuition that from ( $7-\mathrm{a}$ ) we entail that John is happy and Mary is sad, just as desired.
b. $\quad \exists d[T(j, d) \wedge \neg(T(m, d) \wedge T(s, d))]$.
(Seuren) $\equiv \exists d[T(j, d) \wedge(\neg T(m, d) \vee \neg T(s, d))]$.
c. $\max (T(j))>\max \{d \in D: T(m, d) \wedge T(s, d)\} \quad$ (von Stechow) $\equiv \max (T(j))>\max (T(m)) \vee \max (T(j))>\max (T(s))$.

For suppose that John is taller than Sue, but that Mary is taller than John. In that case, (9-b) is predicted to be true on the comparison-class approach because there is a context, $\{j, s\}$, where John is tall but not Sue, while ( $9-b$ ) is predicted to be true on the degree-approaches because there is a degree of tallness that John has, but not Sue. Of course, this prediction is false. Obviously, (9-a) gives rise to the same problem.

A second well-known problem involves existential quantification. The traditional approaches can account for the fact that (3-b) is interpreted in the intuitively correct way.
(3-b) John is taller than anybody else.
But all these approaches have a problem with examples where the NPI anybody is replaced by a standard existential quantifier like somebody, as in (11). ${ }^{10}$
(11) John is taller than somebody is.

An obvious suggestion to account for this kind of example is to assume that the universal effect is not observed because the domain of quantification of the existential quantifier is strongly restricted. Alternatively, one can argue that somebody in (11) has a referential instead of a quantification reading. Arguably, however, neither of these suggestions solves all of our problems, because there exist similar examples that certainly don't involve referential uses of the indefinite and where domain selection seems out of the question: ${ }^{11}$
(12) John is taller than at least one woman is.

Moreover, as noted by Schwarzchild \& Wilkinson (2002), other examples like (13) involving existential quantification over worlds with might are predicted falsely as well.
(13) Today it is warmer than it might be tomorrow.

Von Stechow (1984) suggested the straightforward solution to solve such problems by assuming that quantifiers in than-clauses might undergo quantifier raising: The universal quantifier and the conjunction in (9-a) and (9-b), and the existential quantifiers in (11), (12), and (13) can then simply take scope over the subject term.

Larson (1988) posed a number of problems for von Stechow's straightforward scoping solution to the problems discussed above. A first argument is that on such a move the parallelism between well-known constraints imposed on Wh-movement and quantifier raising has to be given up. The following example shows that Wh -words may not be moved from inside a clause under than, which suggests - by paralellism - that the

[^7]same holds for quantifiers in such clauses:
(14) ${ }^{*}[\text { Which bird }]_{i}$ are you taller than $t_{i}$ was?

A second problem is that the following sentences are on the standard scopal analysis predicted to have a reading that does not exist:
(15) Someone is taller than everybody else is.
(11) John is taller than somebody else.
(9-a) John is taller than everybody else.
A standard quantifier scope analysis predicts that (15) has a reading where everyone takes scope over someone. Larson claims this sentence does not have such a reading. The standard scopal analyses also predicts that (11) has a reading saying that for everybody, John is taller than that person, and that (9-a) can mean that there is somebody that is shorter than John. Again, Larson rightly claims that these sentences don't have these readings. Similar problems arise with modals used in the comparative clause.

### 3.2 Larson's scopal account

Larson (1988) concludes from the arguments given above that the quantifier in the than-clause is not allowed to quantify over the whole sentence (as standard scopal accounts would predict). He proposes, instead, that the quantifier takes obligatory scope over the negation (and only the negation). For generality - but still in accordance with Larson's proposal - I will work with degree functions that 'correspond' with degrees. Larson's proposal then comes down to assuming that 'is taller than' has the following meaning: $\lambda x_{e} \cdot \lambda Q_{\langle\langle e, t\rangle, t\rangle} . \exists \mathbf{d}[\mathbf{d}(T)(x) \wedge Q(\lambda y . \neg \mathbf{d}(T)(y))] .^{12}$ Thus, (15), (11), and (9-a) will be represented as ( $16-\mathrm{a}$ ), ( $16-\mathrm{b}$ ), and ( $16-\mathrm{c}$ ), respectively:
a. $\exists x \exists \mathbf{d}[\mathbf{d}(T)(x) \wedge \forall y[x \neq y \rightarrow \neg \mathbf{d}(T)(y)]$.
b. $\quad \exists \mathbf{d}[\mathbf{d}(T)(j) \wedge \exists y[j \neq y \wedge \neg \mathbf{d}(T)(y)]$.
c. $\exists \mathbf{d}[\mathbf{d}(T)(j) \wedge \forall y[j \neq y \rightarrow \neg \mathbf{d}(T)(y)]$.

These representations give rise to the correct predictions. In fact, Larson's proposal also makes the correct predictions for the following examples (discussed, for example, by Scharzchild \& Wilkinson (2002)):
(13) Today it is warmer than it might be tomorrow.
(17) John is taller than he ought to be.

The only readings available for these sentences are the ones where the modals take scope over the negation.

[^8]Notice also that Larson (1988) predicts well for some examples discussed by Schwarzchild \& Wilkinson (2002) that are taken to be problematic for scopal accounts. First, consider example (18-a) which Larson (1988) would represent as (18-b):
a. It is colder in Paris today than it usually is in Amsterdam.
b. $\quad \exists \mathbf{d}\left[\mathbf{d}(C)\left(p_{0}\right) \wedge \operatorname{Most}_{a}(\operatorname{Day}(a), \neg \mathbf{d}(C)(a))\right]$.

Larson (1988) correctly predicts that this sentence can be true even if there is no single temperature that characterizes Amsterdam most of the time.

Now consider an example of Schwarzchild and Wilkinson (2002) that involves another scope taking element that lies between the quantifier and the comparative over which it will take scope:

John is older than Bill thinks most of his students are.
Suppose that Bill believes that most of his students are between 20 and 24 years old, that John is 25 years old, but also that Bill has no particular belief of the age of any specific student. In this scenario, (19) is intuitively true, although (20) is not:
(20) For most of Bill's students $x$ : John is older than Bill believes $x$ is.

But this is predicted, because on Larson's account, there exists a wide scope reading that is intuitively the correct one:

$$
\begin{equation*}
\exists \mathbf{d}\left[\mathbf{d}(O)(j) \wedge \operatorname{Bel}^{\left.\left(b, \operatorname{Most}_{x}(\operatorname{Stud}-o f(x, b), \neg \mathbf{d}(O)(x))\right)\right] . ~}\right. \tag{21}
\end{equation*}
$$

Second, let's look at some examples not involving upward-monotonic quantifiers. First, a non-monotonic quantifier:
(22) a. John is taller than exactly 3 of the others.
b. $\quad \exists \mathbf{d}[\mathbf{d}(T)(j) \wedge|\{x \in I: x \neq j \wedge \neg \mathbf{d}(T)(x)\}| \neq 3]$.

Unfortunately, (22-b) doesn't really represent the meaning of (22-a). It might be, for instance, that there are only three others that are less than 1.70 meters, although there are 10 other that are less than John's actual length, 1.90 meters. To account for this I propose to follow Jon Gajewski's (ms) ${ }^{13}$ suggestion to look in this case at the most informative degree that John has (i.e. the maximal one), instead of just an arbitrary one. Other examples involving monotone decreasing quantifiers could be analyzed similarly
(23) John is taller than at most 5 of the others.

On Larson's analysis the predicted reading of (23) is given by (24),

$$
\begin{equation*}
\exists \mathbf{d}[\mathbf{d}(T)(j) \wedge|\{x \in I: x \neq j \wedge \neg \mathbf{d}(T)(x)\}| \leq 5] . \tag{24}
\end{equation*}
$$

[^9]which is not correct for the same reason. The suggestion made by Gajewski (ms) would be to use the trick here as well.

I conclude that Larson's account makes pretty good predictions. These predictions are in fact very similar to the predictions made by the interval-based approach developed by Schwarzchild \& Wilkinson (2002) - an approach also motivated by the problems observed by Larson (1988) for the traditional approaches. Both type of approaches give - intuitively (though not technically) speaking - quantified phrases in the than-clause obligatory 'wide scope' over the than-clause. However, as noted by Larson (1988) himself already, such an 'obligatory wide scope'-analysis predicts wrongly for sentences analyzed correctly by the standard approaches:
(3-a) John is taller than Mary or Sue.
(3-b) John is taller than anyone else.
(25) John is taller than allowed/required.

Examples (3-b) and (25) only have readings where the quantifier or modal has narrow scope with respect to the negation, while the most natural reading (although perhaps not the only one) of ( $3-\mathrm{a}$ ) is the one where negation scopes over the disjunction.

One strategy to solve this problem would be to still adopt Larson's proposal (or the one of Schwarzchild \& Wilkinson (2002)), and try to 'explain away' the mispredictions of the Larsonian analysis. One might do so by suggesting that the meaning of any in (3-b), for instance, should not be represented in terms of an existential, but rather in terms of a universal quantifier. And perhaps 'or' has a special 'conjunctive NPI-reading' as well. Finally, we might follow Schwarzchild (ms), who suggests that allowed and required have special scopal properties. He proposes that in contrast to might and should, allowed and required are scope splitters, that take scope over the (in our terms) negation. Although I am not sure how to make sense of this idea compositionally within a Larsonian approach, the result would be as desired: $\exists \mathbf{d}[\mathbf{d}(T)(j) \wedge \neg \diamond / \square \mathbf{d}(T)(j))]$.

Another strategy would be to stay close to the Seuren/Klein/von Stechow account, and to 'explain away' the problems of the traditional approach. The main challenge here is to be able to account for, for instance, universal quantifiers in the than-clause.

According to a yet different strategy, one can propose that comparatives are ambiguous between the reading proposed by the traditional analyses and the Larsonreading. This proposal gives rise to a new task: explain why (15), (11), and (9-a) could only be interpreted as ( $16-\mathrm{a}$ ), ( $16-\mathrm{b}$ ), and ( $16-\mathrm{c}$ ), while ( $3-\mathrm{b}$ ) and ( $3-\mathrm{c}$ ) should be interpreted as originally proposed by Seuren (1973), Klein (1980), and von Stechow (1984).

In the rest of this paper I would like to sketch ways in which the second and third strategies might be worked out. I will spend most time on describing two versions of the second strategy, but I take the third strategy to be a viable option as well.

## 4 Modifying Klein

The strategy to solve the problem how to account for quantifiers in than-clauses I will discuss in this section is the one where we stay close to the Seuren/Klein/von Stechow account, and to 'explain away' the problems of the traditional approach. To meet the
challenge how to account for conjunctive than-clauses, I will first suggest to make use of a notion of 'fine-grainedness', and then reformulate the analysis by making use of intervals.

### 4.1 Fine-grainedness

One idea to account for comparatives with conjunctive 'than'-clauses is to allow for several standards of precision, and analyze such comparatives with respect to a standard of precision such that it blurs any differences between individuals that 'witness' the comparative clause.

According to the degree-based approach towards comparatives, we start with an ordering relation between degrees, and derive from that an ordering between individuals. According to the comparison class approach, instead, we start with the meaning of predicate $P$ with respect to a set of comparison classes and derive from that the meaning of the relation ' $>_{P}$ '. Let us assume that both approaches give rise to the same ordering between individuals. In terms of such an ordering relation we can define a new relation ' $\approx_{P}$ ' as follows: $x \approx_{P} y$ iff ${ }_{\text {def }} x \ngtr_{P} y$ and $y \ngtr P_{P} x$. If the ordering relation ' $>_{P}$ ' is a so-called weak order, ${ }^{14}$ the new relation ' $\approx{ }_{P}$ ' is an equivalence relation. For different weak ordering relations ' $>_{P}$ ', however, ' $\approx{ }_{P}$ ' might come out very differently.

Let us now look at a set of weak ordering relations ' $>_{p}$ ', represented by a set of models $\mathscr{M}$. Let us say that in all models $M, M^{\prime}$ of $\mathscr{M}$, the set of individuals, $I$ is the same: $I_{M}=I_{M^{\prime}}$, but that the relations ' $>_{P}$ ' and ' $\approx{ }_{P}$ ' might differ. Now we can define a refinement relation between models $M$ and $M^{\prime}$ as follows: we say that model $M^{\prime}$ is a refinement of model $M$ with respect to predicate $P$ only if $\exists x, y \in I, M \vDash x \approx_{P} y$, but $M^{\prime} \not \vDash x \approx_{P} y$. So, $M^{\prime}$ is more fine-grained than $M$ with respect to predicate $P$ iff there is at least one pair of individuals equally $P$ in $M$ that is not equally $P$ in $M^{\prime}$. There is a natural constraint on the ordering between models: if Mary is taller than Sue, but smaller than John in fine-grained model $M^{\prime}$, it cannot be the case that John is counted as equally tall as Sue in the more course-grained model $M$, but still taller than Mary. Formally: $M^{\prime}$ is a refinement of $M$ w.r.t. $P, M^{\prime} \geq_{P} M$, only if $\forall x, y, z \in I$ : if $M^{\prime} \mid=x \geq_{P} y \wedge y \geq_{P} z$ and $M \vDash x \approx_{P} z$, then $M \vDash x \approx_{P} y \wedge y \approx_{P} z$. This follows if we define refinements w.r.t. predicate $P$ as follows: $M^{\prime}$ is a refinement of $M$ with respect to $P$, $M \leq_{P} M^{\prime}$, iff $V_{M}\left(>_{P}\right) \subseteq V_{M^{\prime}}\left(>_{P}\right)$. It follows that if $M \leq_{P} M^{\prime}$, then $V_{M}\left(\approx_{P}\right) \supseteq V_{M^{\prime}}\left(\approx_{P}\right) .{ }^{15}$

Now we say that (9-b) John is taller than Mary and Sue is true in $M \in \mathscr{M}$ iff there is a model $M^{\prime}$ at least as coarse-grained as $M$ where Mary and Sue are (considered to be) equally tall, but where John is taller than any of them, i.e., $\exists M^{\prime} \leq_{T} M$ and $M^{\prime} \vDash m \approx_{T}$ $s$ and $\exists x\left[M^{\prime} \mid=T(j, x) \wedge \neg(T(m, x) \wedge T(s, x))\right]$, with $x$ either a degree or a comparison class. From this we can conclude that in $M$ it cannot be that John is either shorter than Mary or shorter than Sue. By our new suggested truth conditions for comparatives this means that both John is taller than Mary and John is taller than Sue are predicted to be true in $M$, just as desired. The reasoning goes as follows: Because $M^{\prime} \vDash m \approx_{T} s$,

[^10]it follows by our constraint on models that $\forall x \in I$ : if $M \vDash m \geq_{T} x \geq_{T} s$, then $M^{\prime} \vDash$ $m \approx_{T} x \approx_{T} s$. Now suppose that in $M$ John is counted as being taller than Sue, but not as being taller than Mary. It follows by our above reasoning that in the more coarsegrained model $M^{\prime}$, John must be counted as being equally tall as Mary and Sue. But that means that $\exists x\left[M^{\prime} \vDash T(j, x) \wedge \neg(T(m, x) \wedge T(s, x))\right]$ is false, which is in contradiction with what we assumed.

It is obvious that this solution extends to the following examples that are considered to be problematic for simple degree and comparison-class approaches to comparatives as well:
a. John is taller than the girls are.
b. John is taller than (a) $\operatorname{dog}(\mathrm{s})$ is (are). (generic reading).

One might suspect that any solution that solves (9-b), (9-a), and the above examples gives rise to problems for sentences like (3-a).
(3-a) John is taller than Mary or Sue.
But this is not the case, because the than-clause of this comparative sentence doesn't require us to consider coarser-grained models, and the analysis for (3-a) remains thus the same as we assumed before. To make this more formal, say that $M \in \mathscr{M}$ is an appropriate model to analyze a comparative with quantifier $Q$ denoted by the noun phrase in the than-clause if there is an element $X$ in $Q$ such that $\forall x, y \in X: x \approx_{T}^{M} y$. The idea is now to analyze the sentence with respect to the most fine-grained appropriate model where the following condition holds: $\forall X \in \operatorname{Min}(Q): \forall x, y \in X: x \approx_{T}^{M} y .{ }^{16}$ Because in contrast to conjunctive noun phrases, the minimal elements of the quantifiers denoted by disjunctive noun phrases are singleton sets, ${ }^{17}$, (3-a) can be interpreted with respect to the most fine-grained model in $\mathscr{M}$.

The traditional analyses predicted that the than-clause was a Downward Entailing context, and thus correctly predicts that it allows for negative polarity items like any and ever (Ladusaw, 1979).
(3-b) John is taller than anybody else.
John is stronger now then he was ever before.
Can our new proposal still account for this? Well, in a sense our new proposal still predicts that the than-clause is a DE context: from (3-a) we can still conclude that John is taller than Mary, and from (3-b) we can still conclude that everybody else is smaller than John. On the other hand, we have seen that from (9-b) John is taller than Mary and Sue we can conclude that John is taller than Mary, and we cannot conclude that John is taller than Mary, Sue, and Lucy. For this example, the than-clause behaves like an Upward Entailing context! So, it seems that we cannot say that the than-clause is always a DE context, or always a U(pwards) E(ntailing) context: it depends on the example (and the fine-grainedness of the model) we have to consider. But if our analysis sometimes predicts the than-clause to behave downward entailing, and at other times

[^11]upward entailing, it seems impossible to come up with a correct logic for comparatives, which would be a surprisingly negative result.

Fortunately, we can claim that the than-clause of a comparative is always downward entailing, but only when the standard of precision is such that the difference in $P$-ness between the individuals that 'witness' the comparative clause is blurred. So, the standard cases of NPIs can be accounted for without a problem. But what about our reasoning from (9-b) John is taller than Mary and Sue to John is taller than Mary? Well, we have just seen that ( $9-b$ ) is analyzed in a model $M$ where Mary is (considered to be) equally tall as Sue. But in such a model, the sentence Mary is tall is true iff Mary is tall and Sue is tall is true (in contexts that contain at least Mary and Sue). But that means that in $M$ the conditional If Mary is tall, Mary and Sue are tall is true. And this is enough to show that the inference from John is taller than Mary and Sue to John is taller than Mary is not in conflict with the than-clause of the comparative to be Downward Entailing. ${ }^{18}$

The analysis proposed in this section gives rise to some desirable empirical predictions. But at least to some, ${ }^{19}$ the analysis is already problematic from a conceptual point of view. The reason is that my proposed 'granularity'-ordering between models doesn't capture the intuition we have about granularity refinements. If each model that I use wants to capture the idea that it represents the tallness relation at a particular level of granularity, it should be the case, intuitively, that all degrees, or equivalence classes, of tallness in coarser-grained models represent the same number of degrees of tallness in finer-grained models. But that idea is not captured at all in this analysis. In fact, it should not be captured, if it wants to predict that the sentence John is taller than Mary and Sue is true in case John is, for instance, 2 cm taller than Mary, but 40 cm taller than Sue. I have to admit that I am not too worried by this complaint: all that I need is the refinement relation between models that I mentioned, and I only used the term 'granularity' for lack of a better name. On the other hand, it is perhaps useful to reformulate the main idea of the proposed analysis in such a way that don't give rise to such misleading interpretations. I will do so in the next section, making use of intervals.

### 4.2 An interval-based reformulation

Since Schwarzchild \& Wilkinson (2002) it is widely assumed that to account for comparatives, we need to make use of intervals. A comparative like John is taller than Mary and Sue is predicted to be true iff there is an interval of tallness that John's tallness is on and an interval of tallness that Mary's and Sue's tallness are on such that any point

[^12](i) a. John is 2 cm taller than the others are.
b. John is taller than Bill expected most students would be.
c. John is taller than exactly 3 others are.

I believe that these examples can be accounted for in terms of the analysis proposed in this section in rather straightforward ways, but won't bother you with it, because I believe there are some serious problems with the proposal made in this section.
${ }^{19}$ The first person who objected to this proposal on this ground was Remko Scha, during a Lego-talk in spring 2007 at the University of Amsterdam.
in the first interval is higher than any point in the second interval. I already claimed in section 3.2 that this type of analysis is in fact very similar to Larson's analysis, and thus very different from the traditional analyses due to Seuren, Klein, and von Stechow. In this section, however, I will make use of intervals to reformulate the main idea presented in the previous section, which is much more in the spirit of the traditional analyses. One can guess immediately that the interval-based analysis I will present in this section will be very different from the one of Schwarzchild \& Wilkinson (2002). In fact, it turns out that the resulting analysis will be very close to a recent one due to Beck (manuscript).

In linguistics it is standard to think of intervals as convex sets of (time)points, with the 'later than' and 'part-of' relations defined in terms of the later-than-relation between points. What I will do here, instead, is to follow the philosophical tradition and start with the primitive notion of an interval, and put some constraints on the 'later than' relation between them. I will say that an Interval order is a structure $\langle I,>\rangle$, with ' $>$ ' a binary relation on $I$ that is irreflexive, and satisfies the so-called 'Interval Order' condition, (IO): $\forall x, y, v, w:(x>y \wedge v>w) \rightarrow(x>w \vee v>y)$. One can easily show that in case $\langle I\rangle$,$\rangle is an interval order, ' >$ ' is also transitive. ${ }^{20}$ From this fact it follows immediately that an interval order is stronger than a strict partial order, but weaker than a weak order: ${ }^{21}$ every interval order is a strict partial order, but not every strict partial order is an interval order, and every weak order is an interval order, but not every interval order is a weak order. Let us now define the indifference relation, ' $\sim$ ', as follows: $x \sim y$ iff $_{\text {def }}$ neither $x>y$ nor $y>x$. It is easy to see that if $\langle I,>\rangle$ is a weak order, ' $\sim$ ' is reflexive, symmetric, and transitive, and thus an equivalence relation. If $\langle I,>\rangle$ is an interval order, however, $I$ is still reflexive and symmetric, but need not be transitive anymore. In terms of relations ' $>$ ' and ' $\sim$ ' we can define two new relations ' $>$ *' and ' $>$ ' as follows: $x>^{*} y$ iff $_{\text {def }} \exists z[x \sim z>y]$, and $x>_{*} y$ iff $_{\text {def }} \exists z[x>z \sim y]$. If ' $>$ ' is an interval-order (or stronger), one can show that ' $>$ *' and ' $>_{*}$ ' are weak orders. ${ }^{22}$

Both weak orders and interval orders are used a lot in semantics, and also for the analysis of comparatives. Lewis (1973), for instance, uses weak orders in his analysis of counterfactuals, and any standard degree-based analysis of comparatives is based on the assumption that relations like taller than are weak orders (between individuals). Interva-based semantics is standardly based on (something like) what I defined above to be an interval order (see especially Thomason (1984), who uses interval orders as I defined them above). The elements of $I$ are assumed to be intervals, and the relations

[^13]' $>$ ' and ' $\sim$ ' are interpreted as 'completely before' (or 'completely after') and 'overlap'. The relations ' $>^{* \prime}$ and ' $>_{*}$ ' now mean 'ends later' and 'ends before', respectively. To assure that we should think of the elements of $I$ really as intervals, define the relation '巨' as follows: $x \sqsubseteq y \operatorname{iff}_{\text {def }} \forall z[y>z \rightarrow x>z] \wedge \forall z[z>y \rightarrow z>x]$. It is easy to prove that ' $\sqsubset$ ' is a partial order, but it also satisfies the following convexity condition, $\forall x, y, z[x>$ $y>z \rightarrow \forall u[x \sqsubseteq u \wedge z \sqsubseteq u \rightarrow y \sqsubseteq u]\rfloor,{ }^{23}$ a condition that is typical for intervals.

Making use of interval orders we will say that $x$ is $P$-er than $y$ iff $x>_{P}^{*} y$, i.e. $\exists z[x \sim$ $\left.z \wedge z>_{P} y\right]$. However, this only makes sense if all real individuals 'start' at the same point. In order to capture that intuition, we make use of the relation ' $={ }_{*}$ '. If ' $\gg_{*}$ ' means 'ends before', ' $=$ *' means 'ends simultaneously', and if ' $>_{*}$ ' means 'has a smaller lowest-point', ' $={ }_{*}$ ' means 'have an equal lowest-point'. The relation is defined as follows: $x=* y$ iff $_{\text {def }} x \ngtr_{*} y$, and $y \not_{*} x$, and is an equivalence relation. We assume that all real individuals (John, Mary, Sue, etc.) - though not all elements in the domain $I$ have the same 'lowest' point, and are thus ' $=$ *' 'related to one another. Combining our analysis of the ' $P$-er than'-relation between real individuals with the above assumption concerning ' $={ }_{*}$ ' just means that $x$ is $P$-er than $y$ is true iff the interval associated with $P$-ness of $x$ is larger than the interval associated with $P$-ness of $y$, just as desired. The intervals in $I$ that are not used to represent (the $P$-ness of) 'real' individuals are just there to determine the ' $>_{P}^{*}$ '-relation for 'real' individuals in terms of the relation ' $>{ }_{P}$ '. ${ }^{24}$

To prepare the way to account for more complex comparatives, we will first reformulate the analysis in a Seuren/Klein-like way as follows: for real individuals $x$ and $y$, we say that $x$ is $P$-er than $y$ iff $\exists z[x \sim z \wedge \neg(y \sim z)]$. Given our assumption on how to represent 'real' individuals, this is equivalent to the analysis above. To account for negative polarity items in the than-clause, we will say that $x$ is $P$-er than $Q$ iff $\exists z[x \sim z \wedge \neg \exists\{y\} \in \operatorname{Min}(Q)(y \sim z)]$, where $Q$ is a quantifier over real individuals, and $\operatorname{Min}(Q)$ the set of its minimal elements. ${ }^{25}$ It immediately follows from this analysis that from John is taller than Mary or Sue, or John is taller than any girl, we conclude that John is taller than Mary, and that John is taller than Sue. ${ }^{26}$ The analysis given sofar is indeed very similar to the analyses proposed by Seuren and Klein, but is obviously wrong in general. This is so in particular because conjunctive quantifiers occurring in the than-clause don't have singleton sets as elements. To account for them, our final analysis will be a modification of the analysis above as follows (where $\operatorname{Max}_{P}(Y)=\left\{y \in Y: \forall z \in Y: y \geq_{P}^{*} z\right\}$, and where $\downarrow Z$ is an arbitrary element of $Z$ ) :

$$
x \text { is } P \text {-er than } Q \quad \text { iff } \quad \exists z\left[x \sim z \wedge \neg \exists Y \in \operatorname{Min}(Q)\left(\downarrow \operatorname{Max}_{P}(Y) \sim z\right)\right]
$$

Notice, first, that this analysis gives rise to the same truth conditions as what I discussed above for comparatives like John is taller than Mary or Sue or John is taller than anybody else. Things are different for a conjunctive quantifier like Mary and Sue, however. The reason is that such a quantifier has only one minimal element. It follows that our analysis correctly predicts that from John is taller than Mary and Sue we conclude

[^14]that John is taller than Mary and John is taller than Sue. Something similar holds for other examples like John is taller than everybody else. In fact, this analysis is really very similar to the analysis I presented in the previous section: ${ }^{27}$ it correctly predicts the conjunctive reading for both disjunctive and conjunctive quantifiers, but still takes the than-clause to be, in a sense, a downward entailing environment. As a result, and perhaps more clearly now, the analysis is again very close to Beck's (manuscript) recent minimax-proposal. ${ }^{28}$ Unfortunately, it also has similar problems.

### 4.3 Problems for Modified Klein

In the previous section I proposed to stick with the traditional Seuren/Klein/von Stechow proposal and tried to 'explain away' some of the unwelcome predictions by making use either of coarser grained models or of intervals. One problem of the original analysis that cannot be explained away in this manner is that it still predicts that the existential quantifier 'somebody' in (11) 'John is taller than somebody else' receives a universal interpretation. I suggested in section 3.1 that this problem might be solved by domain restriction, or by assuming that 'somebody' has a referential reading. Unfortunately, we have seen already that there exist similar examples where this strategy seems less natural:
(11) John is taller than at least somebody else.
(13) Today it is warmer than it might be tomorrow.

A second problem is that it is not very clear how to account for comparative clauses involving downward-entailing quantifiers like ??:
?? John is taller than at most 5 of the others are.
One could suggest that because the downward entailing quantifier at most 5 occurs in a downward entailing position, one should re-interpret it as its complement at least 6 . Although this suggestion predicts remarkably well, it is hard to give any motivation for this type of move.

A final problem is that in case we would like to take degrees seriously, we should be able to account for the following example: ${ }^{29}$
(28) John is an even centimeter taller than Mary and Sue.

Intuitively, this sentence can be true if John is 2 centimeters taller than Mary, and 10 centimeters taller than Sue. It is not clear at all how to account for this intuition on the proposals discussed here.

Obviously, however, the problems discussed in this section can all be accounted for if we adopt Larson's (1988) analysis. Perhaps, then, we should analyze some examples

[^15]as proposed by Larson after all. This is a suggestion we will discuss in the final main part of this paper.

## 5 Resolving ambiguity by strength

In this section I will discuss the proposal that quantifiers (including modals and connectives) in the than-clause can be interpreted in two ways: either as originally proposed by Seuren/Klein, or as proposed by Larson. ${ }^{30}$ This proposal gives rise to a new task: how can we explain that most, if not all, comparative sentences only give rise to one interpretation?

It is easy to explain why (9-a) and (9-b)
(9-a) John is taller than everybody else is.
(9-b) John is taller than Mary and Sue.
are predicted to give rise to the wide scope reading of the universal quantifier and conjunction with respect to negation: scoping them over the negation gives rise to a stronger reading. This suggests that we should select always the strongest reading of the two, in accordance with the strongest meaning hypothesis of Dalrymple et al. (1998) for reciprocals. Making use of this hypothesis, it is clear why (3-a), (3-b), and (3-c)
(3-a) John is taller than Mary or Sue.
(3-b) John is taller than anyone else.
(3-c) John is taller than allowed.
are now predicted to give rise to the reading proposed by Seuren and Klein: small scope of disjunction or existential quantifier with respect to negation gives rise to a stronger meaning than wide scope. What about sentences with a monotone decreasing quantifier like (29-a) and (29-b), and with a non-monotonic quantifier like (29-c)?
(29) a. John is taller than nobody else.
b. John is taller than at most 3 others.
c. John is taller than exactly 3 others.

Notice first that the Seuren/Klein-reading of (29-a) and (29-b), i.e., (30-a) and (30-b), are trivial (because 'tall' is monotone decreasing in degrees, and everybody shares the same 'minimal' degrees):

$$
\begin{align*}
\text { a. } & \exists \mathbf{d}[\mathbf{d}(T)(j) \wedge \neg \neg \exists x[x \neq j \wedge \mathbf{d}(T)(x)]]  \tag{30}\\
& \equiv \exists \mathbf{d}[\mathbf{d}(T)(j) \wedge \exists x[x \neq j \wedge \mathbf{d}(T)(x)]] \\
\text { b. } & \exists \mathbf{d}[\mathbf{d}(T)(j) \wedge|\{x \in I: x \neq j \wedge \mathbf{d}(T)(x)\}| \not \equiv 3
\end{align*}
$$

This suggests that for pragmtatic reasons (29-a) and (29-b), if they have a reading at all, it is going to be the Larson-reading. However, notice that for the same reason, the Larson-reading of (29-a), i.e. (31), is equally trivial as (30-a), meaning that (29-a) is

[^16]inappropriate on both readings, and thus inappropriate. This seems in accordance with intuition. ${ }^{31}$
\[

$$
\begin{align*}
& \exists \mathbf{d}[\mathbf{d}(T)(j) \wedge \neg \exists x[x \neq j \wedge \neg \mathbf{d}(T)(x)]]  \tag{31}\\
& \equiv \exists \mathbf{d}[\mathbf{d}(T)(j) \wedge \forall x[x \neq j \rightarrow \mathbf{d}(T)(x)]]
\end{align*}
$$
\]

What about (29-b) and (29-c)? We have seen already in section 3.2 that with some extra machinery, Larson (1988) could account for the desired readings. Thus, they are predicted to have the Larson-readings only. ${ }^{32}$

The problematic examples include now at least the following ones:
(13) Today it is warmer than it might be tomorrow.

John is taller than (at least) somebody else.
John is taller than required.
The problem with (13) and (11) is that according to the strongest meaning hypothesis, the Klein-reading of these examples is preferred, although the other reading is the only one that seems to exist. Although these examples were problematic for the original analysis of Seuren and Klein, and for our modification of it as well, now we have a little bit more freedom to account for them. Before, we had to explain the intuitive 'wide scope' reading by still adopting a 'small scope' analysis. Now we can explain the 'wide scope' reading simply by giving independent motivation for why the stronger 'small scope' reading does not exist. I believe that such an independent motivation can be given for (13) and for (11). As for (13), it is not unreasonable (though somewhat stipulative) to assume that epistemic 'might' takes obligatory wide-scope. But this means that the Klein-reading is ruled out. A similar story can be told for (11). It has been argued that '(at least) somebody' is a Positive Polarity Item. As such, this item is not allowed to stand in the scope of negation. This has the desired result that the Kleinreading is ruled out, and that only the weaker Larson-reading is left.

The problem for (32) is perhaps more serious. ${ }^{33}$ The problem now is that according to the strongest meaning hypothesis the Larson-reading is predicted, although (32) only seems to have the minimality reading predicted by Seuren and Klein. Recall that the maximality reading as predicted by Larson seems correct for other universal modals:
(17) John is taller than he ought to/should be.

This suggests that there is something special going on with 'require'. It is unclear to me exactly why 'require' is so special, but at least two proposals have been made in the literature. First, as discussed in section 3.2, Schwarzchild (ms) proposed that in contrast to ought and should, require and have to are 'scope-splitting' modals that take obliga-

[^17]tory scope over the (in our terms) negation. The resulting prediction is in accordance with our intuitions, but the proposal by itself, of course, is not yet very explanatory. Perhaps the 'scope-splitting'-behavior can be explained by a second suggestion due to Krasikowa (2007), taken over by Beck (manuscript). Krasikova observed that 'required' and 'have to' are so-called sufficiently-modals: modals that go well with 'only' to receive a 'sufficiently'-interpretation.

You only have to/*should walk 500 meters before you are at the central station.

On the basis of this observation she suggests that 'required' and 'have to' should thus be given a scalar meaning: If (33) without 'only' is true, it means that walking 500 meters is the minimum amount of meters you have to walk before your are at the central station, although by walking more meters, you might arrive there as well. This, in turn, suggests that 'required to be tall' should receive a minimum-interpretation as well, a suggestion which would indeed predict correctly.

## 6 Conclusion

The traditional analyses (Seuren, Klein, von Stechow) of comparatives are all much alike, and give rise to very similar predictions concerning quantifiers in than-clauses. It is well-known that they can account for a proper - but still significant - subset of examples involving such quantifiers. Larson (1988) and Schwarzchild \& Wilkinson (2002) account for the complementary subset. In the main part of this paper I discussed two strategies how to solve this problem. According to a first strategy, one stays close to the original Seuren/Klein/von Stechow account and tries to 'explain away' the problems by making use either of coarse-grained models, or of intervals. According to a second strategy, one allows comparative sentences to be ambiguous, but explains away the (non-existing) ambiguity by the strongest meaning hypothesis together with some independent reasons why certain undesired readings do not exist. The second strategy makes perhaps the better predictions. The first strategy seems less ad hoc.

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[^1]:    ${ }^{1}$ Equatives can be analyzed in terms of comparison classes as well. Klein (1980) proposes that (i-a) should be interpreted as (i-b).

[^2]:    ${ }^{2}$ According to the delineation account of adjectives due to Lewis (1970) and Kamp (1975), the worlds, or supervaluations, of a vagueness model $\mathscr{M}$ differ from each other in the cutoff point of vague predicates. The comparative 'John is taller than Mary' is considered to be true in $\mathscr{M}$ iff $\exists w \in W_{\mathcal{M}}: \mathscr{M}, w \vDash$ $T(j) \wedge \neg T(m)$, which means that there is a cutoff point for 'tall' such that John is above it, while Mary is not. In standard modal logic, we don't explicitly quantify over worlds in the object language, but sometimes it is conveniant to do so. In that case, the comparative is true iff $\mathscr{M} \vDash \exists w[T(j, w) \wedge \neg T(m, w)]$. A world in a vagueness frame has a cutoff point for each vague predicate, and we might identify the cutoff point for 'tall' in $w$ by $w_{T}$. The easiest way to think of the cutoff point of 'tall' in a world is as a particular number, a degree. But then we can assume that the predicate denotes a relation between individuals and degrees, and the delineation approach just claims that the comparative is true iff $\mathscr{M} \vDash \exists w\left[T\left(j, w_{T}\right) \wedge \neg T\left(m, w_{T}\right)\right]$, meaning that John has a degree of tallness that Mary does not have. This, of course, is exactly Seuren's analysis of comparatives. It should be noted, though, that to account for comparatives in this way, Lewis and Kamp can't allow for all refinements (worlds) of a partial interpretation function being part of their vagueness model. In fact, in contrast to standard supervaluation theory only very few refinements are allowed, and the set of these refinements should come with an independently given ordering as well. Realizing this makes, in my opinion, the delineation account much less attractive than standardly assumed, and by adopting it one can certainly not claim - and this in contrast with the comparison class-account (see van Benthem, 1982) - that one has derived the comparative meaning from the positive use of the adjective, because the comparative meaning was already presupposed.
    ${ }^{3}$ Just like Seuren, also Klein proposes that the than-clause should be represented within the scope of a negation. This use of negation in comparatives goes back to Jespersen (1917), who proposed it to license Negative Polarity Items in these environments.

[^3]:    ${ }^{4}$ It is somewhat surprising to see that this is not the standard explanation of this anomaly, and not even considered (as far as I know) in the literature.
    ${ }^{5}$ Thanks to Chris Kennedy (p.c.) for this.

[^4]:    ${ }^{6}$ This is in accordance of the last sentences of Klein (1991): ‘Presumably the linguistic complexity of comparatives partially reflects the complexity of measurement devices, both conceptual and technological, that the linguistic community has at its disposal. A good theory should be able to show how both kinds of complexity are incrementally built up from our basic ability to draw comparisons.'
    ${ }^{7}$ Notice that this example is exactly parallel to the 'cross polar anomaly' cases like John is taller than Mary is short discussed by Kennedy (1999). Indeed, I believe with Klein that such examples are appropriate, though perhaps only under a non-standard 'evaluative' interpretation. See footnote 9 for more discussion.

[^5]:    ${ }^{8}$ Klein (1991) already noted that a similar problem holds for degrees. We cannot assume that degrees are simply real numbers, because in that case it doesn't explain incommensurability.

[^6]:    ${ }^{9}$ As noted by Kennedy (1999), however, there is nothing in Klein's (1980) original analysis that guarantees that this is the case. Sapir (1944) noticed already that for some types of adjectives, $P$, we can conclude from its use in the comparative ' $x$ is $P$-er than $y$ ', that both $x$ and $y$ have property $P$. This is not the case for 'tall' and 'wide', but is so for so-called 'evaluative adjectives' like 'brilliant'. One (nonpresuppositional) proposal to account for evaluative readings of adjectives within Klein's analysis is to assume that comparatives like ' $x$ is $P$-er than $y$ ' can be interpreted as follows with respect to comparison class $c_{0}$ :

[^7]:    ${ }^{10}$ It has been argued that not all non-NPI existential quantifiers give rise to this problem. Beck (manuscript), for instance, notes that Knut is bigger than a black bear pup is intuitively means that Knut is bigger than the largest black bear pup.
    ${ }^{11}$ Thanks to Schwarzchild (p.c.) for this.

[^8]:    ${ }^{12}$ One might propose to generalize this such that it also accounts for subdeletion complements like This table is longer than that table is wide as follows: [[... is ... er than ... is ... ]] $=$ $\lambda P_{1} \lambda P_{2} . \lambda Q . \lambda x . \exists \mathbf{d}\left[\mathbf{d}\left(P_{1}\right)(x) \wedge Q\left(\lambda y .\left(\neg \mathbf{d}\left(P_{2}\right)\right)(y)\right)\right]$ (or - depending on your favorite syntactic analysis with the lambda's in a different order). Kennedy ( $\mathrm{p}, \mathrm{c}$ ) pointed out to me that Larson's analysis can indeed account for such constructions (Larson (1988) himself has claimed that these constructions 'require a rather different treatment', but I don't understand why he thinks so).

[^9]:    ${ }^{13}$ When I wrote the first version of this paper, I was not aware of Jon Gajewski's work on comparatives. He defends an analysis of comparatives very close to Larson's, and compares it with the more recent one of Schwarzchild \& Wilkinson. Gajewski's proposal is very close to a suggestion made by Schwarzchild \& Wilkinson (2002) themselves as well.

[^10]:    ${ }^{14}$ A relation $R$ gives rise to a weak order, if the relation is (i) irreflexive, (ii) transitive, and (iii) negatively transitive, i.e. $\forall x, y, z:(\neg R(x, z) \wedge \neg R(z, y)) \rightarrow \neg R(x, y)$.
    ${ }^{15}$ From $V_{M}\left(>_{P}\right) \subseteq V_{M^{\prime}}\left(>_{P}\right)$ it follows that $\forall x, y \in I:$ if $\langle x, y\rangle \notin V_{M^{\prime}}\left(>_{P}\right)$, then $\langle x, y\rangle \notin V_{M}\left(>_{P}\right)$. Now suppose $M^{\prime} \vDash x \approx_{P} y$. This means that (a) $\langle x, y\rangle \notin V_{M^{\prime}}\left(>_{P}\right)$ and (ii) $\langle y, x\rangle \notin V_{M^{\prime}}\left(>_{P}\right)$. By (i) it now follows that both (a') $\langle x, y\rangle \notin V_{M}\left(>_{P}\right)$ and (b') $\left.\langle y, x\rangle \notin V_{M}( \rangle_{P}\right)$. But that means that $M \mid=x \approx_{P} y$.

[^11]:    ${ }^{16}$ Thanks to Makoto Kanazawa for pointing out a problem in my earlier formalization.
    ${ }^{17}$ Whereas $\operatorname{Min}($ John and Mary $)=\{\{$ John, Mar y\}\}, Min $($ John or Mary $)=\{\{$ John $\},\{$ Mar $y\}\}$.

[^12]:    ${ }^{18}$ There are at least three kinds of examples discussed by Schwarzchild \& Wilkinson (2002) that cannot be accounted for in this way:

[^13]:    ${ }^{20}$ Suppose $x>y$ and $y>z$. By (IO) it follows that either $x>z$ or $y>y$. Because the latter is ruled out by irreflexivity, we conclude that $x>z$.
    ${ }^{21}$ The structure $\left.\langle I\rangle,\right\rangle$ is a strict partial order iff ' $>$ ' is (i) irreflexive and (ii) transitive. $\left.\langle I\rangle,\right\rangle$ is a weak order if ' $>$ ' is (i) irreflexive, (ii) transitive, and (iii) negatively transitive.
    ${ }^{22}$ Proof: Irreflexive: Suppose $x>^{*} x$, then $\exists z[x \sim z>x]$, which is a contradiction.
    Transitivity. Suppose $x>^{*} y$, meaning that $\exists v_{1}\left[x \sim v_{1}>y\right]$, and $y>^{*} z$ meaning that $\exists v_{2}\left[y \sim v_{2}>z\right]$. We have to prove that $x>^{*} z$, i.e., $\exists w[x \sim w>z]$. Because $v_{1}>y$ and $v_{2}>z$ it follows by (IO) that either $\nu_{1}>z$ or $\nu_{2}>y$. But because $y \sim v_{2}$, it has to be the case that $\nu_{1}>z$, which means that $x \sim v_{1}>z$, and thus $\exists w[x \sim w>z]$ and thus $x>^{*} z$.
    Negatively transitive: Suppose $x>^{*} y$ that is, $\exists v[x \sim v>y]$. To show $\exists w_{1}\left[x \sim w_{1}>z\right]$ or $\exists w_{2}\left[z \sim w_{2}>y\right]$. Assume that neither of them is true. Because $\neg \exists v[x \sim v>z]$ and $x \sim v$ it must be that $v \ngtr z$. Because $\neg \exists v[z \sim v>y]$ and $v>z$ it must be that $z \nsucc v$. Because $v \ngtr z$ and $z \nsucc v$ it must be that $z>v$. From $v>y$ and $z>v$ it follows with (IO) that either $v>v$ or $y>z$. Because the former is false, we conclude $y>z$. By transitivity it follows that $x>z$ which contradicts our assumption that neither $x>^{*} z$ nor $z>^{*} y$.

[^14]:    ${ }^{23}$ Proof. Suppose $x>y>z$ and $x \sqsubseteq u$ and $z \sqsubseteq u$. Consider any $v>u$. Because $x \sqsubseteq u$, it follows that $v>x$, and thus $v>y$ (i). Likewise, if $u>v$, then $z>v$, and hence $y>v$ (ii). From (i) and (ii) we conclude $y \sqsubseteq u$ by the above definition.
    ${ }^{24}$ From now on, I will mostly ignore the subscript ' $P$ '.
    ${ }^{25}$ Recall that Min(John and Mary) $=\{\{$ John, Mar $y\}\}$, while Min(John or Mary) $=\{\{$ John $\},\{$ Mar $y\}\}$.
    ${ }^{26}$ Assuming again that any girl should be represented by an existential quantifier.

[^15]:    ${ }^{27}$ There is a formal reason for this similarity, of course. Intuitively, there exists a one-to-one relation between the intervals in the interval-based approach and the set of equivalence classes of 'equally tall' individuals when one looks at all models coarser grained than a finest grained model $M$.
    ${ }^{28}$ Whereas my proposal is based more on the analyses of Seuren and Klein, her analysis is more reminiscent to von Stechow (1984).
    ${ }^{29}$ A similar example is due to Sauerland (p.c.).

[^16]:    ${ }^{30}$ Lerner \& Pinkal (1992) and Heim (2006) proposed solutions very similar to this.

[^17]:    ${ }^{31}$ It seems, however, that on the phrasal reading of comparatives, (29-a) has a reading according to which John is the shortest person. On the other hand, one needs extra (focal) stress on 'nobody' for this reading to come about. Perhaps this non-predicted reading can be explained in terms of this extra required stress. Thanks for Chris Tancredi (p.c.) for bringing up this example.
    ${ }^{32}$ Unfortunately, if we use the extra machinery also for the Klein/Seuren-reading, it is not predicted anymore that (29-b) and (29-c) are trivial. I am not sure what to do with this problem.
    ${ }^{33}$ I should notice, though, that (32) is a problem for the analyses discussed in section 4 as well.

