A modal analysis of presupposition and modal subordination

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Abstract

In this paper I will give a modal two-dimensional analysis of presupposition and modal subordination. I will think of presupposition as a non-veridical propositional attitude. This allows me to evaluate what is presupposed and what is asserted at different dimensions without getting into the binding problem. What is presupposed will be represented by an accessibility relation between possible worlds. The major part of the paper consists of a proposal to account for the dependence of the interpretation of modal expressions, i.e. modal subordination, in terms of an accessibility relation as well. Moreover, I show how such an analysis can be extended from the propositional to the predicate logical level.

1 Introduction

Consider the following examples:

(1) a. A thief *might* break into the house. He *might* take the silver.

    b. It is *possible* that John used to smoke and *possible* that he just *stopped* doing so.

    c. It is *possible* that Mary will come and it is *possible* that Sue will come *too*.

The sequence (1a) was discussed by Roberts (1989) as a problematic example for standard discourse representation theory and dynamic semantics (Kamp, Heim, Groenendijk & Stokhof). The sequence (1b) was already given by Gazdar (1979) as a counterexample to the satisfaction theory of presupposition defended in the seventies by, among others, Karttunen and Stalnaker. This satisfaction theory predicted that if A presupposes P, the sentence *It is possible that A* presupposes P as well. The sequence (1b) was taken to be a counterexample to this theory, because it is typically used in a situation in which it is not presupposed that John used to smoke. The closely related (1c) is even a more serious problem for proponents of the satisfaction theory who claim that a trigger like *too* does not entail what it presupposes, in this case that (it is possible that) Mary will come. The reason is that according to such a *two-dimensional* analysis of presuppositions it is predicted that (1c) can be true and appropriate without there being a possible world in which both Mary and Sue are coming, i.e. the *binding problem*.

In this paper I will formulate a *two-dimensional* theory of presupposition satisfaction in which the binding problem does not arise. I will do this by taking seriously the proposal of Stalnaker that presupposition should be thought of as a propositional attitude and represented by an *accessibility relation* between possible worlds. In the most substantial part of the paper, I will show how such a *modal* analysis can help to account for the phenomenon that the appropriate interpretation of one embedded sentence can depend on that of another: a phenomenon normally discussed under the heading of 'modal subordination'. This modal analysis of modal subordination will be rather different from the more *representational* analyses proposed by, among others, Roberts (1989) and Geurts (1995).

This paper will be organized as follows. First, I will briefly motivate and formalize a two-dimensional analysis of presupposition satisfaction. In section 3, I will discuss the phenomenon of modal subordination and propose a modal analysis in terms of a changing accessibility relation. This analysis will be developed further in the remaining sections to account for disjunctions, conditionals, belief and desire attributions, and the subjunctive mood. Until then I limit myself to the propositional level. In the final substantial section of this paper I will briefly indicate how the analysis can be extended such that anaphoric dependencies across modals can be taken care of as well. I will end with some conclusions.

2 Presupposition

2.1 The representation of presuppositions

The notion of presupposition plays a crucial role in dynamic semantics. A context is supposed to represent what is presupposed. Stalnaker (1974, 1998, 2002) has always argued that presupposition should be thought of as a propositional attitude and thus represented in a similar way: by means of an *accessibility relation*. But what do agents presuppose? The standard answer is: what is common ground between the participants of the conversation. According to discourse representation theory, what is common ground is what is explicitly represented in a discourse representation structure, a DRS. This DRS, in turn, represents what has been *explicitly* agreed upon by the conversational participants. This suggests that presupposition should by default be fully *introspective*: what is presupposed is also presupposed to be presupposed, and what is not presupposed is also presupposed not to be presupposed.[1] I will represent what is presupposed by a *primitive* accessibility relation R, i.e., one that cannot be reduced to what the participants of the conversation know or believe.[2] Although the presuppositional accessibility relation should be fully introspective, the relation should not be based on the assumption that what is presupposed also has to be true: discourse can be based on an assumption that later turns out to be false. So, presupposition should be represented by an accessibility relation that need *not* be *reflexive*.

The non-veridicality of what is presupposed suggests that we should treat the valuation of truth separately from context change – distinguish content from force. In this section I will show how we can systematically account for presupposition satisfaction without giving up the possibility of determining the content of a sentence separately from the way it changes the context. For context change, I will rely mainly on work in dynamic epistemic semantics, where updates are defined in terms of *eliminating arrows* instead of eliminating worlds.[3]

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[1] See Fernando (1995) for an analysis of context where full introspection is assumed as well.

[2] Stalnaker (2002) suggests what is presupposed by an agent is what she believes is commonly believed by the discourse participants. This has as a result, however, that the attitude of presupposition does not obey negative introspection, because more things can be taken to be commonly believed than what is explicitly agreed upon.

[3] Updating through the elimination of arrows instead of worlds has been used, among others, by Landman (1986a) and Veltman (1996). Its limitations for multi-agent settings are discussed in Gerbrandy (1999).

2.2 Formalization

When a speaker presupposes something, he presupposes it in a world or a possibility. A possibility will be represented by a *pointed model*, $\langle R, w \rangle$,[4] where w is a distinguished world representing the actual world and should be thought of as a valuation function from atomic propositions to truth values and where R is the presuppositional accessibility relation that is (by default) *serial*, *transitive* and *Euclidean*.[5] I will take $R(v)$ to be the worlds accessible from v: $\{u \in W : vRu\}$. As a result, it will be the case that what is presupposed is introspective: $\forall v, w$: if $v \in R(w)$, then $R(v) = R(w)$, although it need not be veridical, i.e., it might be that $w \notin R(w)$. To determine in possibility $\langle R, w \rangle$ whether P is presupposed, we have to check what is presupposed in this possibility, $R(w)$. The two-dimensional (or four-valued) analysis of presupposition that was popular in the seventies treats the logic of truth and that of presupposition at separate dimensions. This is appealing because sometimes a sentence can – at least according to Karttunen & Peters (1979, for instance – be true, although its presupposition is false. They claim that a sentence like *Even John was there*, for example, can be true without it actually being unlikely that John was there.[6]

Standard dynamic semantics treats conjunction in an *asymmetric* way: the second conjunct should be interpreted with respect to the initial context updated with the first conjunct. This is a desirable feature of a framework to account for the asymmetric behavior of presuppositions in conjunctive sentences. In this section I will combine the desirable features of both the two-dimensional and the dynamic analysis of presuppositions. Thinking of presupposition as a non-veridical propositional attitude, we can account for the dynamic aspects of presupposition satisfaction without giving up the idea behind a two-dimensional analysis of presupposition satisfaction. That is, we will predict that conjunction behaves *asymmetrically* with respect to presupposition satisfaction because what is presupposed

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[4]If we think of a world as representing everything that is the case, including some modal facts, a pointed model should be thought of as such a world.

[5]A relation R is *serial* if $\forall x : \exists y : xRy$; *transitive* if $\forall x, y, z : (xRy \ \& \ yRz) \rightarrow xRz$; and *Euclidean* if $\forall x, y, z : (xRy \ \& \ xRz) \rightarrow yRz$.

[6]A presuppositional analysis can be 'two-dimensional' in two different ways. One is through the assumption that the presupposition and the assertion of the sentence can and must be determined relatively independently of each other. The analysis of what Karttunen & Peters (1979) call 'conventional implicatures' and the presuppositional analyses of Herzberger (1973), Gazdar (1979), Soames (1979), and Van der Sandt (1988) are all two-dimensional in this sense.

But there is also a less radical way in which an analysis of presuppositions might be called 'two-dimensional'. On this reading it means only that at least some sentences in which a presupposition trigger occurs can be true even though the triggered presupposition is actually false. I believe that Stalnaker's analysis of presuppositions, just as my own analysis, should be thought of as two-dimensional in this more liberal sense.

when the second conjunct is interpreted is predicted to be what is presupposed when the first conjunct is interpreted updated with the first conjunct itself. Still, '*and*' will be treated truth-conditionally in a *symmetric* way. The reason is that truth and presupposition satisfaction are defined *separately* from the update function (although they will be defined simultaneously). Making use of Beaver's (1995) presupposition operator, I will represent an atomic sentence A that presupposes P as follows: $\partial P \wedge A$. For the time being, I will concentrate only on the truth-conditional connectives. I will assume that a sentence has two values: (i) a sentence is true or false, i.e. 1 or 0; (ii) a sentence has no presupposition failure or it has one, i.e. + or -. The combined *truth* and *presupposition satisfaction conditions* of sentences are given below (where '·' is a placeholder):[7]

- $[[A]]^{R,w} = \langle 1/0, + \rangle$, iff $w(A) = 1/0$, if A is atomic (then always defined)

- $[[\neg A]]^{R,w} = \langle 1/0, + \rangle$ iff $[[A]]^{R,w} = \langle 0/1, + \rangle$, $\langle \cdot, - \rangle$ otherwise

- $[[A \wedge B]]^{R,w} = \langle 1, + \rangle$ iff $[[A]]^{R,w} = \langle 1, + \rangle$ and $[[B]]^{Upd(A,R),w} = \langle 1, + \rangle$
  $= \langle \cdot, - \rangle$ iff $[[A]]^{R,w} = \langle \cdot, - \rangle$ or $[[B]]^{Upd(A,R),w} = \langle \cdot, - \rangle$
  $= \langle 0, + \rangle$ otherwise

- $[[\partial A]]^{R,w} = \langle 1, + \rangle$ iff $\forall v \in R(w) : [[A]]^{R,v} = \langle 1, + \rangle$
  $= \langle \cdot, - \rangle$ otherwise

Observe again that the presupposition value of a conjunction is determined in a *symmetric* way. That is, if either A or B is false or has a presupposition failure, the conjunction $A \wedge B$ will be false or have a presupposition failure as well. However, to determine the presupposition value of a conjunction of the form $A \wedge B$ in possibility $\langle R, w \rangle$, we look at the presupposition value of B in possibility $\langle Upd(A, R), w \rangle$ – the update function is being relevant here. This is the point at which we take over the insights of dynamic semantics. The update $Upd(A, R)$ is defined as follows:

- $Upd(A, R)$ $=$ $\{\langle u, v \rangle \in R |\ [[A]]^{R,v} = \langle 1, + \rangle\}$.

Notice that this update function is *eliminative*, but instead of eliminating worlds in $R(w)$ it eliminate tuples, or *arrows*, in R. It eliminates all arrows in R that point to an non-A-world. This has the effect that after the update of R with A, not only all worlds v

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accessible from w verify A, but all worlds u accessible from v make A true as well. Thus, after the update with A it is not only presupposed that A, but it is also presupposed to be presupposed that A. Moreover, on the assumption that R is fully introspective, $Upd(A, R)$ will be fully introspective as well. Also after the update, everything that is not presupposed is also presupposed to be not presupposed.

Our analysis is very similar to standard dynamic semantics. If we would say that $[[\Diamond A]]^{R,w} = \langle 1, \cdot \rangle$ iff $\exists v \in R(w) : [[A]]^{R,v} = \langle 1, \cdot \rangle$ and assume that possibility statements don't have any dynamic effect,[8] we predict just like Veltman (1996) an asymmetry between $\Diamond A \wedge \neg A$ and $\neg A \wedge \Diamond A$; the former is okay, the latter is not. However, this contrast in acceptability is explained in a somewhat different way: Veltman's explanation appeals to acceptability of update, while we explain the contrast in terms of truth. We predict that the former sequence can be true, but the latter cannot.

If we assume that sentence A presupposes P iff $\forall \langle R, w \rangle$: if $[[A]]^{R,w} = \langle \cdot, + \rangle$, then $\forall v \in R(w) : [[P]]^{R,v} = \langle 1, + \rangle$, the above implementation gives rise to the same presuppositional predictions as the standard implementation of the satisfaction account. In particular, on the assumption that *John stopped smoking* gives rise to the presupposition that John used to smoke, this implementation predicts that sentences like *John didn't stop smoking* and *John stopped smoking and Mary is sick* will also gives rise to this presupposition, but *John used to smoke and he stopped doing so* will never give rise to presupposition failure.

Although the predictions of the above implementation of the satisfaction approach are similar to the predictions of the standard approach, there are still some important differences. First, by treating presupposition as a propositional attitude, we can evaluate in a *distributive* way whether a presupposition associated with a sentence is satisfied by what the speaker presupposes. This is possible, of course, because we have represented in a single possibility all the information that is normally represented only in a whole context/information state. Second, and related, we can now account for the dominant view in the seventies that presupposition satisfaction and truth should be evaluated at *different dimensions*.

According to Karttunen & Peters (1979) and others a sentence like *Even Bill likes Mary* presupposes something that it does not entail. Thus, the sentence can be true without it actually being unlikely that Bill likes Mary, because what is presupposed need not be true.[9] Notice that we can now account for this intuition without assuming with Karttunen & Peters (1979) that we should thus *represent* presuppositions separately from

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[8]Though we will give a somewhat different analysis of possibility statements later.

[9]Soames (1989) observed already that this is problematic for the standard way of accounting for presupposition satisfaction in dynamic semantics.

assertions. On the other hand, we can also account for the intuition that a factive verb both presupposes and entails that its complement is true.[10] To analyze *Sam realizes that P* we add the following construction to the language: if P is a sentence, $Real(s, P)$ is a sentence too. To interpret the formula, we add a reflexive accessibility relation to the model, K_s, modeling what Sam realizes.[11] The formula is then interpreted as follows:

- $[[Real(s, P)]]^{R,w} = \langle 1, + \rangle$ iff $\forall v \in K_s(w) : [[P]]^{R,v} = \langle 1, + \rangle$
  $= \langle 0, + \rangle$ iff $\exists v \in K_s(w) : [[P]]^{R,v} = \langle 0, + \rangle, \quad \langle \cdot, - \rangle$ otherwise

Notice that because K_s is reflexive, according to this analysis the formula entails, but does not presuppose, that P. To account for the presupposition, we represent the sentence *Sam realizes that P* by the following formula $\partial P \wedge Real(s, P)$, which both presupposes and entails that P. If we now represent *Sam does not realize that P* by $\neg(\partial P \wedge Real(s, P))$, this sentence presupposes that P, but can still be true in case P is false (in case $w \notin R(w)$).

3   Modal subordination

3.1   Previous analyses

According to standard dynamic semantics (Veltman 1996), the embedded sentence of 'possibly A' should be interpreted with respect to the same context as the whole sentence. This gives rise to the prediction that 'possibly A' triggers the same presupposition as A itself. However, if it has already been established that it is possible that John used to smoke, i.e. after (2a) has been asserted, (2b) need not presuppose that John used to smoke.

(2)   a. It is *possible* that John used to smoke,

     b. and it is *possible* that he just *stopped* doing so.

The phenomenon that a modal expression depends for its interpretation on another modal, as illustrated by (1a) and (2a)-(2b), is known as 'modal subordination'. Consider Roberts's (1a) from the introduction again:

(1a)   A thief *might* break into the house. He *might* take the silver.

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[10]Throughout the paper I will assume the same for an aspectual verb like *stop*.

[11]Our simple update function has limitations here: if we would attribute to Sam attitudes about what the discourse participants presuppose, things go wrong. I will ignore such attributions in this paper. See, among others, Gerbrandy (1999) for an analysis in which this problem is overcome.

In both (1a) as in (2a)-(2b), we intuitively get the correct reading if we assume that the modal in the first clause takes scope over the whole sequence. Apart for reasons of compositionality, however, Roberts (1989) showed already that such an analysis would be on the wrong track. It would not be able to account for a slightly different sequence like (3a), where we have a necessity instead of a possibility operator in the second sentence.

(3) a. A thief *might* break into the house. He *would* take the silver.

  b. *If* a thief broke into the house, he would take the silver.

Intuitively, the second sentence of (3a) means something like (3b). To account for this, Roberts takes up Kratzer's (1981) idea that the domain of a modal is context dependent, and extends it by proposing that the actual selection goes via 'accommodation' of the material that has been mentioned explicitly in an earlier modal statement. For (1a), (2a)-(2b), and (3a), for instance, this means that the embedded clauses of their first sentences will be accommodated to function as the antecedents of the modals *might*, *possible*, and *would*, respectively. In this way she makes correct predictions for all of (1a), (2b) and (3a).

Although Roberts's analysis reflects what intuitively goes on in the examples illustrated above, the exact mechanism that she uses has been rightly criticized by Kibble (1994), Geurts (1995), and others. Not only is her use of accommodation rather *ad hoc* and non-compositional, it also seems to be much too powerful a device, even with the constraints on accommodation that she proposes.[12]

Kibble (1994) and Geurts (1995) propose that instead of selecting the domain of a modal by means of (antecedent) accommodation, we should assume that the domain is picked up *anaphorically*.[13] Moreover, they suggest that modal statements also make such domains, or subordinated contexts, anaphorically available for later modals: they introduce propositional discourse variables to the discourse and these are mapped by assignment functions to the set of possibilities that verify their embedded clauses. Thus, they can account for (3a) by making sure that the first sentence introduces a propositional discourse referent that is assigned to a set of possibilities that verify the embedded clause. The second

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[12]Roberts's (1989) constraints are the following (i) modal subordination 'requires non-factual mood' (p. 701); (ii) 'it must be plausible that the modally subordinate utterance has a hypothetical common ground suggested by the immediately preceding context' (p. 701); and (iii) modal subordination may not make antecedents available to anaphoric expressions that have no explicit representation in the given DRS (p. 705). For a critical discussion of these constraints, see Kibble (1994) and Geurts (1995).

[13]Geurts (1995) claims that a modal *presupposes* its domain and assumes an exclusively *anaphoric* account of presupposition (satisfaction) for these cases. Given the important role that (non-global) accommodation plays in Geurts's (1995) analysis of presuppositions, this restriction is somewhat surprising.

sentence of (3a) is then analyzed as true in case all those possibilities verify the embedded clause of this latter sentence.

Although these anaphoric analyses of modal subordination are more constrained and appealing than Roberts's, they are not unproblematic. For one thing, they are still too unrestricted: modal statements just introduce and are allowed to pick up propositional discourse markers. This makes it possible that a clause embedded under one kind of modality can figure as the antecedent of a modal expression of a completely different kind. A second problem is that Geurts' analysis doesn't allow for enough interaction between main and subordinated context, while Kibble demands too strong such an interaction. Consider first an example (discussed in Kibble, 1994) that is problematic for Geurts (1995). He falsely predict that (4c) is an appropriate continuation of (4a)-(4b):[14]

(4) a. John might be at home reading a book$_x$

    b. Actually, he's still at the office.

    c. *It$_x$'ll be War and Peace.

The reason is that in Geurts' account of modal subordination, non-modal sentences that talk about the actual world such as (4b) are not taken to eliminate any of the elements in the set of world-assignment pairs introduced by the embedded sentence of (4a). Thus, Geurts' analysis of modal subordination does not allow for enough interaction between the main context and the subordinated ones.[15]

Kibble (1994) allows for such interaction, but his analysis cannot account for interpretational dependence of clauses in counterfactual or subjunctive mood. This is so because in order to interpret a clause (with free variables) embedded under a modal operator, each possibility in the introduced subordinated context has to be compatible in his formalism with the 'main' context. Geurts's (1995) analysis doesn't have this limitation, because each possibility of the 'main' context carries the information contained in the subordinated contexts. However, by making use of standard set theory, Geurts does not allow for the situation that if $\langle w, f \rangle$ is a possibility that satisfies the main context (DRS) and also assigns a set of possibilities to newly introduced propositional discourse marker p, that there is a world v such that $\langle v, f \rangle \in f(p)$. A somewhat *ad hoc* analysis is given to assure

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[14]Kaufmann (1997) discusses a similar example involving a conditional:
(i) If John bought a book, he'll be home reading it by now.
(ii) John works at a gas station.
(iii) *It'll be a murder mystery.

[15]As shown in Frank (1997), however, this problem can be overcome.

that this won't happen. In particular, a somewhat arbitrary distinction is made between embedded and unembedded information: although a propositional discourse marker can be mapped to what is presupposed to be possible, it is not allowed there to be a propositional discourse marker that captures what is presupposed in the discourse as a whole. Not only do I believe that this is undesirable for conceptual reasons, it also has an unfortunate empirical consequence. It is predicted that modal statements cannot anaphorically pick up what is presupposed in the entire discourse. Because a modal can, intuitively, use this kind of information as its domain of quantification, a somewhat artificial distinction has to be made between anaphoric and non-anaphoric dependent modals.

One way to overcome this problem is to introduce a distinguished propositional discourse marker that represents with respect to each world assignment pair what is presupposed in that possibility. A straightforward implementation of this idea, however, requires the use of non-wellfounded set theory.[16] In this paper, instead, I will propose a more traditional way to account for presuppositions and modal expressions: in terms of accessibility relations.

In contrast to Roberts' analysis of modal subordination in terms of accommodation, the above described anaphoric analyses keep the subordinated contexts made use of in the interpretation of previous sentences 'in memory' by adding propositional discourse markers to the discourse. Another way to store previously used subordinated contexts was proposed by Kaufmann (1997, 2000). Instead of representing a context just by a set of possibilities that verify everything established until now, he represents it by a *stack* of such sets (see Zeevat (1992) as well), where a set 'below' the top-level represents a subordinated context. I have no principled objection to such an analysis. Still, I would like to see a less 'representational' approach towards modal subordination where what is presupposed at a particular moment in time can simply be represented by a single set of possibilities. But now the challenge is how to account for the introduction of subordinated contexts without giving up the assumptions that we represent what is presupposed at a particular moment in time in terms of a *single* accessibility relation.

3.2 A modal analysis

The basic idea is technically very simple: possibility statements introduce an *ordering* on the worlds. Because we assume that what is presupposed is a propositional attitude and should be represented by an accessibility relation, we can implement this idea in an

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[16]See, however, Fernando (1996) and Frank (1997) for less straightforward implementations of this idea within standard set theory.

appealing way. Following Veltman's (1996) analysis of *normally*, I will assume that the dynamic effect of a possibility statement is that the worlds that make the embedded clause true are the most preferred worlds by eliminating arrows from A-worlds to $\neg A$-worlds.[17]

- $Upd(\Diamond A, R) = \{\langle u, v \rangle \in R|$ if $[[A]]^{R,u} = \langle 1, + \rangle$, then $[[A]]^{R,v} = \langle 1, + \rangle\}$

According to the update function, possibility statements *disconnect* A-worlds from $\neg A$-worlds, although A-worlds can still be seen from $\neg A$-worlds and from the actual world w. Suppose that before the update $R = \{\langle w, v \rangle, \langle w, u \rangle, \langle v, v \rangle, \langle v, u \rangle, \langle u, u \rangle, \langle u, v \rangle\}$, where v is an A-world and w and u are $\neg A$-worlds. Then R is introspective: $R(w) = R(v) = R(u) = \{v, u\}$. After the update with $\Diamond A$, however, the new accessibility relation $Upd(\Diamond A, R)$ won't be introspective anymore: the tuple $\langle v, u \rangle$ will be eliminated, which means that $Upd(\Diamond A, R)(w) \neq Upd(\Diamond A, R)(v) = \{v\} \neq Upd(\Diamond A, R)(u)$.[18] Thus, if R was Euclidean before the update with $\Diamond A$, it won't be Euclidean anymore afterwards.

Possibility statements will be interpreted as follows:

- $[[\Diamond A]]^{R,w} = \langle 1, + \rangle$ iff $\exists v \in R(w) : [[A]]^{R,v} = \langle 1, + \rangle$
          $= \langle 0, + \rangle$ iff $\exists v \in R(w) : [[A]]^{R,v} = \langle \cdot, + \rangle$ and
                        $\forall v \in R(w) :$ if $[[A]]^{R,v} = \langle \cdot, + \rangle$, then $[[A]]^{R,v} = \langle 0, + \rangle$,
          $= \langle \cdot, - \rangle$ otherwise

According to this rule it holds that if A presupposes P, $\Diamond A$ can be used appropriately only if it is assumed to be possible that P is presupposed. Because out of context (or so we assumed) it holds that $\forall v \in R(w) : R(v) = R(w)$, under normal circumstances $\Diamond A$ presupposes the same as A itself. However, the rule can also account for the sequence (2a)-(2b), where the presupposition of the embedded clause of (2b) is not a presupposition

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[17]That the proposed analysis is technically similar to Veltman's (1996) analysis of *normally* doesn't mean that the ordering relation represents the same kind of information.

[18]This update rule is defined on the assumption that either $w \notin R(w)$ or w is not an A-world, because otherwise we would falsely predict that after the use of the possibility statement only other A-worlds would be accessible from w. In general we cannot make this assumption, of course. Fortunately, there are several ways to solve this problem. One way is to assume that w is the distinguished actual world, and that we change the update rule for possibility statements as follows: $Upd(\Diamond A, R) = \{\langle u, v \rangle \in R|$ (if $[[A]]^{R,u} = \langle 1, + \rangle$, then $[[A]]^{R,v} = \langle 1, + \rangle)$ or $u = w\}$. In this paper I leave the update rule unchanged, and adopt a more tricky solution. Assume that if $w \in R(w)$ and w is an A-world, we don't go to the new pointed model $\langle R', w \rangle$, but rather to the pointed model $\langle R', w^* \rangle$ with a new world w^*. This new world is exactly like w of the original pointed model, except that $w^* \notin R(w^*)$ although $w \in R(w^*)$. Because our technical problem has a simple solution, I will ignore this complication in the main text. (Thanks to Frank Veltman and Henk Zeevat for discussion on this point).

of its embedding sentence as a whole. The reason is that after the interpretation/update of (2a) there is a world v consistent with what is presupposed in the actual world w in which John used to smoke and in which it is presupposed that John used to smoke. Thus, because from such a world v only worlds in which John used to smoke are accessible, the embedded sentence of (2b) can be interpreted appropriately as well.

The concrete accessibility relation R discussed above illustrates what it means that after the update of R with $\Diamond A$, the A-worlds are the *preferred* ones: although in each $v \in R(w)$ it was the case that both $\Diamond A$ and $\Diamond \neg A$ were true, this is only the case for $\Diamond A$ for all $v \in Upd(\Diamond A, R)(w)$.

In the introduction we noted that Karttunen & Peters' (1979) two-dimensional analysis gives rise to the *binding problem*: if it is assumed that *Sue will come <u>too</u>* presupposes, but does not entail, that somebody different from Sue will come, it falsely predicts that (1c) can be true although there is no possible world in which anyone besides Sue will come.

(1c) It is *possible* that Mary will come and it is *possible* the Sue will come *too*

Our analysis does not give rise to this false prediction. The reason is that, although we have defined update separately from truth, the definitions of truth and appropriateness are closely related: for a possibility statement to be true and appropriate, there has to be an accessible world in which the embedded sentence is *both* true and appropriate. Thus, the logics of truth and appropriateness (or presupposition satisfaction) are not as independent of each other as proposed in Herzberger (1973) and Karttunen & Peters (1979).

Notice that if we take $\Box A$ to be an abbreviation of $\neg \Diamond \neg A$, we predict that A has to be interpreted only in possibilities that satisfy the presupposition of A: $\Box A$ has value $\langle 1, + \rangle$ in $\langle R, w \rangle$ iff $\exists v \in R(w) : [[A]]^{R,v} = \langle \cdot, + \rangle$ and $\forall v \in R(w) :$ if $[[A]]^{R,v} = \langle \cdot, + \rangle,$ then $[[A]]^{R,v} = \langle 1, + \rangle$. But this means that $\Box(\partial P \wedge A)$ can only be true in $\langle R, w \rangle$ iff either P itself is presupposed and A is true in all accessible worlds, or it is presupposed that P is possible and A is true in all accessible P-worlds.[19]

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[19]Notice that this analysis predicts that if A doesn't give rise to a presupposition, the sentence *Would A* can only be true if A is true in *all* accessible worlds. But the following sequence suggests that this is a wrong prediction: *It might be rainy next week. The strawberries would rot in the garden.* Here we seem to have modal subordination even though the embedded clause of the second sentence doesn't trigger a presupposition. One way in which this problem might be solved is to assume that the embedded clauses of *would*-sentences should be interpreted only with respect to accessible worlds v such that $\forall u \in R(v) :$ $R(u) = R(v)$. In the above example, the embedded clause would now only be interpreted in worlds where it rains next week.

4 Extending the analysis

4.1 Modal Splitting and Questions

Now consider (5).

(5) Either John *stopped* smoking, or he just *started* doing so.

Landman (1985) proposed to account for such examples by assuming that the disjuncts should be interpreted with respect to two mutually exclusive *subordinated contexts*, possibly created by an earlier use of a disjunctive sentence that has *split* the context. Our analysis of possibility statements suggests a straightforward analysis (where $\mathcal{A}$ is a set of formulas and R^A is $\{\langle u, v \rangle \in R : [[A]]^{R,v} = \langle \cdot, + \rangle\}$):

- $[\bigvee \mathcal{A}]]^{R,w} = \langle \cdot, - \rangle$ iff $\exists A_i : [[\Diamond A_i]]^{R,w} = \langle \cdot, - \rangle$
  $= \langle 1, + \rangle$ iff $[[\bigvee \mathcal{A}]]^{R,w} \neq \langle \cdot, - \rangle$ and $\exists A_i : [[A_i]]^{R^{A_i},w} = \langle 1, + \rangle$
  $= \langle 0, + \rangle$ otherwise

- $Upd(\bigvee \mathcal{A}, R) = \bigcap_j(Upd(\Diamond A_j, \bigcup_i Upd(A_i, R)))$

To illustrate the update function, suppose that before the update $R = \{\langle w, v \rangle, \langle w, u \rangle, \langle w, x \rangle, \langle v, v \rangle, \langle v, u \rangle, \langle v, x \rangle, \langle u, u \rangle, \langle u, v \rangle, \langle u, x \rangle, \langle x, x \rangle, \langle x, v \rangle, \langle x, u \rangle\}$, where v is an A-world, u a B-world, and x neither makes A nor B true. To calculate $Upd(A \vee B, R)$, we first calculate $Upd(A, R) \cup Upd(B, R)$. It is easy to see that $Upd(A, R) \cup Upd(B, R) = R^* = \{\langle w, v \rangle, \langle w, u \rangle, \langle v, v \rangle, \langle v, u \rangle, \langle u, u \rangle, \langle u, v \rangle\}$. Now we determine $Upd(\Diamond A, R^*) \cap Upd(\Diamond B, R^*)$. This new accessibility relation, R^{**}, is $\{\langle w, v \rangle, \langle w, u \rangle, \langle v, v \rangle, \langle u, u \rangle\}$, a relation that is not Euclidean. Making use of the last step, I propose that $A \vee B$ requires both $\Diamond A$ and $\Diamond B$ to be appropriate. As for the case of possibility statements, this means that normally all the presuppositions of the disjuncts are presuppositions of the whole disjunction as well. However, when the context is split, this doesn't have to be the case. This accounts for the problematic (5), if we assume that the context was split (perhaps after accommodation of the disjunctive presupposition) between, on the one hand, worlds where John smoked before, and, on the other, worlds where he did not.

Questions give rise to modal subordination too:[20]

(6) Did John used to smoke? and did he *stop* smoking?

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[20]For a different analysis of modal subordination with questions, see van Rooy (1998).

Although it is standardly assumed that a polar question gives rise to a partition, with respect to modal subordination there seems to be a difference between the positive and the negative answer: only the positive answer can be picked up to figure as the domain of a later modal. To account for this, we can simply assume that the update of R with the yes/no question A? is the same as the update of R with $\Diamond A$: $Upd(A?, R) = Upd(\Diamond A, R)$. And this gives rise to the following correct predictions: after question (7a), both (7b) and (7c) are appropriate and do not give rise to presuppositional readings:

(7) a. Did anyone solve the problem?

    b. It is possible that *it was* John *who* solved the problem.

    c. Either *it was* John *who* solved the problem, or the problem was too difficult.

The reason is that after the update with (7a), the most preferred worlds are ones where somebody solved the problem. As a result, if the question was non-redundant, the presupposition of the embedded sentence of (7b) is satisfied in at least one accessible world. Thus, the sequence (7a)-(7b), represented abstractly as $A? \wedge \Diamond(\partial A \wedge B)$, is correctly predicted to be appropriate, without giving rise to the presupposition that A. A similar consideration shows that the same holds for the sequence (7a)-(7c).

4.2 Indicative conditionals

Conditionals show modal subordination behavior as well. Example (8a) shows that the antecedent of a conditional might depend on an earlier epistemic modal; (8b) shows that the interpretation of an epistemic modal may depend on a conditional sentence used earlier, while (8c) shows that the interpretation of one conditional can depend on the interpretation of another.

(8) a. I might have been wrong. If I *realize* that I was wrong, I will tell everybody.

    b. If John feels bad, he will start smoking. His girlfriend might make him to *stop*.

    c. If Mary comes, we'll have a quorum. If Susan comes *too*, we'll have a majority.

I will follow Stalnaker (1976) in assuming that not only subjunctive, but also indicative conditionals should be analyzed in terms of selection functions/similarity relations. I will assume that the selected worlds are such that they satisfy the presupposition of the

antecedent. Out of context, the same will be presupposed in the selected worlds, or possibilities, as in the actual possibility. However, the selected possibilities might also depend on an earlier introduced subordinated context. To account for this, I make use of a similarity relation between worlds, $u <_w v$, meaning that u is at least as close to w as v is. To make the selected antecedent-worlds explicitly depend on what is presupposed, I will define a new relation $<_{R,w}$ between worlds that is dependent both on $<_w$ and on what is presupposed in $\langle R, w \rangle$ (where $v \approx_w u$ iff neither $v <_w u$ nor $u <_w v$):

- $v <_{R,w} u$ iff   (i) $v <_w u$, or
  (ii) $v \approx_w u$ and $R(u) \subset R(v) \subseteq R(w)$, or
  (iii) $v \approx_w u$ and $R(w) \subseteq R(v) \subset R(u)$.

Thus, v is closer to w than u with respect to R iff either v is closer to w than u, or they are equally close, but fewer additional presuppositions (in addition to those made in w) are made in v than in u, or fewer presuppositions (among those made in w) are suspended in v than in u. The set of closest A-worlds to $\langle R, w \rangle$ is the following set:

- $f_{\langle R,w \rangle}(A) = \{v \in W |\; [[A]]^{R,v} = \langle 1, + \rangle\; \& \; \neg \exists u \in W : [[A]]^{R,u} = \langle 1, + \rangle\; \& \; u <_{\langle R,w \rangle} v\}$

A conditional sentence of the form *if A then B* is then counted as true in $\langle R, w \rangle$ iff B is verified in all pointed models consisting of a closest A-world to $\langle R, w \rangle$ and accessibility relation $Upd(A, R)$:

- $[[A > B]]^{R,w} = \langle 1, + \rangle$ iff $f_{\langle R,w \rangle}(A) \subseteq \{v \in W |\; [[B]]^{Upd(A,R),v} = \langle 1, + \rangle\}$

Following Stalnaker's (1976) suggestion that the antecedent of an indicative conditional selects, if possible, worlds compatible with what is presupposed, I will assume that the use of an indicative conditional demands there to be an accessible world in which its antecedent is true and appropriate. That is, if $A > B$ represents an indicative conditional, it can only be appropriate in $\langle R, w \rangle$ iff $[[\Diamond A]]^{R,w} = \langle 1, + \rangle$. Notice that this enables us already to account for sequence (8a). This is because after the update of R with possibility statement $\Diamond A?$, the appropriateness condition that an indicative conditional of the form $(\partial A \wedge B) > C$ comes with, i.e., that A is presupposed in at least one world in $Upd(\Diamond A, R)(w)$, is guaranteed.

To account for sequences (8b) and (8c), we have to make sure that conditional sentences themselves make subordinated contexts accessible. In these subordinated contexts, both antecedent and consequent should be true and presupposed. This suggests that we should define the update rule for (indicative) conditionals as follows:

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- $Upd(A > B, R) = \{\langle u, v \rangle \in R : [[A > B]]^{R,v} = \langle 1, + \rangle$ and
  if $[[A \wedge B]]^{R,u} = \langle 1, + \rangle$, then $[[A \wedge B]]^{R,v} = \langle 1, + \rangle\}$

With the help of this update rule we can account for (8b) and (8c) as well. The reason is that according to this rule, all accessible worlds make the conditional true, and worlds in which both the antecedent and consequent are true only 'see' other such worlds. But this means that a later modal or conditional statement that presupposes what is entailed by the antecedent and/or consequent, as do the second sentences of (8b) and (8c), can now also be interpreted appropriately.

4.3 Belief and Desire

According to Karttunen (1974), Stalnaker (1988), Heim (1992) and Zeevat (1992), a belief attribution as (9a) presupposes (9b):[21]

(9) a. John believes that Mary *stopped* smoking.

b. John believes that Mary used to smoke.

How can we account for this in our two-dimensional approach? We have assumed that what is presupposed can be represented by a primitive accessibility relation in the model. Suppose that our model contains the accessibility relation B_j which represents what John believes as well. If $R(w)$ represents what is presupposed in w, then $R_j(w) = \bigcup\{B_j(v) : v \in R(w)\}$ represents what is presupposed in w about what John believes. This suggests to define the following indexed accessibility relation: $R_j^w = \{\langle u, v \rangle \in W^2 | \exists x \in R(w) : v \in B_j(x)\}$. This definition has the effect that for all worlds v, $R_j^w(v) = R_j(w)$. Notice that R_j^w is introspective: $\forall v \forall u \in R_j^w(v) : R_j^w(u) = R_j^w(v)$. Using this indexed accessibility relation, we can adopt the following combined truth and appropriateness conditions:

- $[[Bel(j, A)]]^{R,w} = \langle 1, + \rangle$ iff $\forall v \in B_j(w) : [[A]]^{R_j^w, v} = \langle 1, + \rangle$
  $= \langle 0, + \rangle$ iff $\exists v \in B_j(w) : [[A]]^{R_j^w, v} = \langle 0, + \rangle$
  $= \langle \cdot, - \rangle$ otherwise

This immediately accounts for the non-presuppositional reading of the sequence (9b)-(9a). In particular – and similarly as for Stalnaker (1988) –, we don't have to introduce a new update function for belief attributions: any update of R automatically updates R_j^w as well.

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[21]For disagreement, see Geurts (1998).

According to Heim (1992), desire attributions should be interpreted with respect to what is (presupposed to be) believed as well. The following discourse seems perfectly acceptable:

(10) a. John believes that Mary used to smoke,

     b. but he hopes that she *stopped* doing so.

But as noted by Asher (1987), Heim (1992), and later by Geurts (1998), desire attributions can be conditionally dependent on other desire attributions as well:

(11) a. John wants Mary to come,

     b. and he wants Bill to *know* that Mary will come.

Intuitively, this suggests that desire attributions can introduce subordinated contexts and can be interpreted with respect to such contexts as well. How can we account for this? Just as before we took R to be a primitive accessibility relation that cannot be reduced to what the participants of the conversation know or believe, I think that now we have to assume the existence of a primitive accessibility relation R_j that represents what has been explicitly established about what John believes. Thus, in the beginning of the conversation $R_j = W \times W$ and $Upd(Bel(j, A), R_j) = \{\langle u, v \rangle \in R_j : [[A]]^{R_j,v} = \langle 1, + \rangle\}$.

I will assume for simplicity the following analysis of desire attributions: $Des(j, A)$ is true in w iff all the A-worlds in $B_j(w)$ are preferred to the $\neg A$-worlds in $B_j(w)$.[22] Taking presupposition satisfaction into account as well, we define the interpretation rule as follows (where $X > Y$ iff $\forall x \in X : \forall y \in Y : x > y$):

- $[[Des(j, A)]]^{R,w} = \langle \cdot, - \rangle$ iff $[[\Diamond A]]^{R_j,w} = \langle \cdot, - \rangle$
  $\qquad\qquad\qquad\quad = \langle 1, + \rangle$ iff $[[\Diamond A]]^{R_j,w} = \langle 1, + \rangle$ and
  $\qquad\qquad\qquad\quad \{v \in B_j(w) : [[A]]^{R_j,v} = \langle 1, + \rangle\} > \{u \in B_j(w) : [[A]]^{R_j,u} = \langle 0, + \rangle\}$
  $\qquad\qquad\qquad\quad = \langle 0, + \rangle$ otherwise

- $Upd(Des(j, A), R) \quad = \quad \{\langle u, v \rangle \in R | \; [[Des(j, A)]]^{R,v} = \langle 1, + \rangle\}$

- $Upd(Des(j, A), R_j) \quad = \quad \{\langle u, v \rangle \in R_j | \text{ if } [[A]]^{R_j,u} = \langle 1, + \rangle, \text{ then } [[A]]^{R_j,v} = \langle 1, + \rangle\}$

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[22] For a discussion of some alternative analyses of desire attributions, see Van Rooy (1999).

The truth condition basically says that the embedded sentence should be interpreted with respect to all worlds in $B_j(w)$ in which this sentence can be interpreted appropriately. It also demands that if A presupposes P, the desire attribution is predicted to be appropriate if either P is presupposed to be believed by the agent, or P is presupposed to be desired. The update rules are similar to what we have discussed above: the first one just demands that the desire attribution has to be true, while the second turns the A-worlds in $R_j(w)$ into the preferred ones.

4.4   Subjunctive mood and negative sentences

In section 3 we saw already that certain examples of modal subordination involving subjunctives are unproblematic for our account. Our analysis of possibility operators and their duals in section 3.2 immediately gives the desired reading for a sequence such as (3a):

(3a) A thief *might* break into the house. He *would* take the silver.

But now consider the following example:

(12) I don't smoke. I (also) *would*n't be able to *stop*.

This example is more problematic than (3a) because now there is no modal operator in the first sentence of the sequence that has a non-global effect on the accessibility relation. To account for this example one might propose, again, to accommodate the presupposition locally within the scope of the subjunctive modal. I would like to suggest, however, that, once we represent what is presupposed by an accessibility relation, no need for local presupposition accommodation arises.

The first idea that comes to mind to account for examples like (12) is to propose that *negative* sentences have an effect similar to that of modals. But it is not straightforward to work out this suggestion, because the worlds that verify the negated clause are not accessible anymore after the interpretation of the first sentence. In this section I would like to suggest tentatively two alternative solutions for such examples.

A first proposal would be to assume that the modal in the second sentence is interpreted with respect to an accessibility relation that is determined by taking the complement of the original accessibility relation with respect to which the whole of (12) is interpreted minus the relation resulting from the interpretation of the first sentence of the sequence. Let t be the moment in time at which the second sentence should be interpreted. Then we can define R^{t*} as $\{\langle u, v \rangle \in R^{t-1} | u, v \notin Range(R^t)\}$. Thus, R^{t*} consists of the arrows between worlds in the previous information state, R^{t-1}, that were eliminated by the last assertion. In terms of this accessibility relation, we can analyze the subjunctive mood as follows:

- $[[\text{Would } A]]^{R^t,w} = \langle 1, + \rangle$ iff $\exists v \in R^{t*}(w) : [[A]]^{R^{t*},v} = \langle \cdot, + \rangle$ and
  $\forall v \in R^{t*}(w) :$ if $[[A]]^{R^{t*},v} = \langle \cdot, + \rangle$, then $[[A]]^{R^{t*},v} = \langle 1, + \rangle$

Notice that according to this interpretation rule of a subjunctive modal, the second conjunct of a formula like $\neg A \wedge \text{Would } (\partial A \wedge B)$ is predicted to be true with respect to accessibility relation R if all accessible A-worlds are B-worlds as well. In particular, we can now account for the sequence (12) without making use of local accommodation.

Still, I don't think this proposal is unproblematic. The most problematic aspect, I believe, is the fact that the suggested analysis can't explain the contrast between positive and negative sentences: why can subjunctive modals only 'pick up' *negated* sentences?

As discussed by Horn (1989), among others, there exists a crucial distinction between the contexts in which positive and in which negative sentences can be used appropriately: in contrast to their positive counterparts, negative sentences require a context in which the truth of the positive sentence is expected, or at least very salient. One way of being salient is to be the *topic* of conversation: a question to be addressed. Geurts (1995) suggested tentatively that this might be the reason why sequences as (12) are appropriate. I think that this is indeed a good suggestion. We have seen in section 4.1 what the dynamic effect is of a (positive polar) question: the worlds in which the positive sentence is true come to be preferred to worlds where it is false. A subsequently used modal expression can then be interpreted with respect to these most preferred worlds. This can't be the whole story, of course, because after the first sentence of (12) these most preferred worlds are eliminated. So, if t is the moment in time at which the second sentence should be interpreted, the relevant accessibility relation should not be R^t, but rather R^{t-1}, i.e., the context of interpretation for the first sentence of (12).[23] I think that this suggestion is a natural one, especially given the fact that *would* is a past-tense modal. If this first sentence presupposes that the topic of conversation is whether I smoke, the most preferred worlds in $R^{t-1}(w)$ are all worlds where I smoke, and the second sentence of (12) can be interpreted appropriately.

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[23]So, with Kaufmann (2000) I agree that we need to keep previous contexts 'in memory'. Accepting this – according to one reviewer – we give up the assumption that we can account for modal subordination in terms of a single accessibility relation. But I don't agree, for we still might think of R as a single accessibility relation for each time point. In systems that add to possible worlds a temporal dimension it is very normal to assume perfect memory: at each point in time you recall what you knew, believed, or presupposed before. To implement this assumption, we don't have to add to the model some extra accessibility relations, although the accessibility relation now has to represent more information. I won't go into the details of such a system here.

5 Indefinites and pronouns

Although we have discussed Karttunen & Peters's (1979) *binding problem* already with respect to possibility statements, the most famous problematic example involves indefinites: their false prediction that the individual that satisfies the presupposition of a sentence like *Someone managed to succeed George V on the throne of England* need not be the one who actually succeeded George V. In this section I will show that this problem will not arise in our framework if we extend it to the predicate-logical case. However, the main goal of this section is to indicate how we can account for modal subordination phenomena that involve anaphoric dependencies across the sentential boundary.

To take indefinites and pronouns into account, we have to make our accessibility relation one between more fine-grained possibilities. In contrast to standard dynamic semantics, I assume that pronouns are (normally) used referentially, referring back to the speaker's reference of its antecedent indefinite. Such a speaker's reference of (an occurrence of) an indefinite depends on the *referential intentions* the speaker has. This kind of information should be represented already in a possibility (actual and non-actual). But this means that these possibilities have to contain more information than the world-assignment pairs as standardly assumed in dynamic semantics. A clause with an occurrence of an indefinite is represented by $\exists x_r A$, where r is a reference function. Let us assume that the set of possibilities I is a set of functions from (i) n-ary predicates to their interpretations; (ii) variables to individuals; and (iii) reference functions to individuals. If $\exists x_r A$ is interpreted in possibility i, then $i(r)$ is the speaker's reference of the occurrence of the indefinite in i, and the dynamic effect will be that from now on x will be assigned to $i(r)$ in i, i.e. $i(x) = i(r)$. Let us define $R[^x/_r]$ as $\{\langle i[^x/_{i(r)}], j[^x/_{j(r)}]\rangle : \langle i,j\rangle \in R\}$, and $R[^x/_d]$ as $\{\langle i[^x/_d], j[^x/_d]\rangle : \langle i,j\rangle \in R\}$. Now we define the update of $\langle R,a\rangle$ with $\exists x_r A$ as follows (where a is the actual possibility):

$$Upd(\exists x_r A, \langle R,a\rangle) \quad = \quad \langle\{\langle i,j\rangle \in R[^x/_r] : [[A]]^{R[^x/_{j(r)}],j} = \langle 1,+\rangle\}, a[^x/_{a(r)}]\rangle$$

Thus, in the actual possibility, the speaker's reference of the indefinite is introduced (although this need not be an individual that makes the sentence true). Also in each possibility that is compatible with what is presupposed the speaker's reference of the indefinite in that possibility is introduced, although this introduced individual has to verify the sentence in that possibility.

The (rigid) *truth* and *presupposition satisfaction conditions* of the new clauses are given below (where $\vec{x}$ is an n-ary sequence)):

- $[[P\vec{x}]]^{R,i} = \langle 1/(0),+\rangle$ iff $i(\vec{x}) \in i(P)$ (or $i(\vec{x}) \notin i(P)$) and $i(\vec{x})$ is defined

- $[[\exists x_r A]]^{R,i} = \langle 1, + \rangle$ iff $[[A]]^{R[x/i(r)],i[x/i(r)]} = \langle 1, + \rangle$

Notice that the above rules say that $\exists x_r P(x)$ is rigidly true in $\langle R, a \rangle$ if and only if the speaker's referent of the indefinite in a has property P in this world/possibility. Existential sentences, however, don't seem to have such strong truth conditions. As argued for in van Rooy (2001), although speaker's reference is crucial for the analysis of pronouns, it doesn't seem to influence the truth or falsity of the clause in which the indefinite occurs. To account for this, I will follow the same procedure as I proposed in van Rooy (2001), and define the *semantic* notion of truth as an *abstraction* of the more pragmatic notion of rigid truth where speaker's reference is crucial for the interpretation of indefinites. Let us say that $j \approx i$ iff j is a possibility just like i, except that j might assign different individuals to reference functions than i does. Now I define the notion of truth (and presuppositional appropriateness) of sentence A in possibility $\langle R, a \rangle$ in terms of this notion as follows:

- $R, a \models^{+} A$ iff $\exists a' \approx a : [[A]]^{R,a'} = \langle 1, + \rangle$

Now it follows that $R, a \models^{+} \exists x_r Px$ iff $\exists d \in D : [[Px]]^{R[x/d],a[x/d]} = \langle 1, + \rangle$. But this means that the sentence is true in a just in case there is an individual that has property P in this world, just as expected.

In van Rooy (2001), I argued that the notion of speaker's referent is important for at least two reasons. First, to account for the phenomenon of pronominal contradiction: although speaker's reference has no truth conditional effect on the interpretation of indefinites, it does for the interpretation of *pronouns*. Second, to understand what a discourse referent used in dynamic semantics really represents: a discourse referent represents what is presupposed about *the* actual speaker's referent. I will not discuss these arguments any further here. However, in one sense the present implementation of the second intuition is much more appealing than the one I have given in van Rooy (2001): it's accounted for now in terms of standard set theory making use of a standard way to model propositional attitudes, i.e. an accessibility relation.

The *binding problem* of Karttunen & Peters' (1979), involving indefinites, was due to the fact that they *represented* presupposition and assertion separately. Our analysis, instead, only *interprets* them at different dimensions. We represent their problematic sentence abstractly as follows: $\exists x_r [\partial Px \wedge Qx]$. An easy calculation shows that this formula is predicted to be true and appropriate in $\langle R, a \rangle$, $R, a \models^{+} \exists x_r [\partial Px \wedge Qx]$, just in case $\exists d \in D : \forall i \in R(a) : [[Px]]^{R[x/d],i[x/d]} = \langle 1, + \rangle$ & $[[Qx]]^{R[x/d],a[x/d]} = \langle 1, + \rangle$. Thus, it is required that the same individual has to satisfy both the presuppositional part and the assertive part: the binding problem does not occur. This prediction is independent of our assumption that indefinites come with speaker's referents.

How does our analysis of indefinites and pronouns account for anaphoric dependencies across modal statements? Consider the classical sequence of Roberts (1989):

(13) a. A wolf *may* come in. It *would* eat you first.

   b. $\Diamond \exists x_r [Wx \wedge Cx] \wedge \Box Ex$

Notice that $[[\Diamond \exists x_r[Wx \wedge Cx] \wedge \Box Ex]]^{R,a} = \langle 1,+ \rangle$ iff $[[\Diamond \exists x_r[Wx \wedge Cx]]]^{R,a} = \langle 1,+ \rangle$ and $[[\Box Ex]]^{R',a'} = \langle 1,+ \rangle$, where $\langle R', a' \rangle = Upd(\Diamond \exists x_r[Wx \wedge Cx], \langle R,a \rangle)$. The first conjunct is true iff $\exists i \in R(a) : i(r) \in i(W) \cap i(C)$. The update of $\langle R, a \rangle$ with the first conjunct results in $\{\langle j, i \rangle \in R[^x/_r] : i(r) \in i(W) \cap i(C)\} \cup \{\langle j, i \rangle \in R : \neg \exists d \in D : d \in i(W) \cap i(C)\}$.[24] If we then only look at possibilities $i \in R'(a)$ where $i(x)$ is defined, the second conjunct is predicted to be true iff $\forall i \in R(a) : \forall d \in D :$ if $d \in i(W) \cap i(C)$, then $d \in i(E)$. Thus the second conjunct says that in every possibility where there is a wolf who comes in, it eats you first. I believe that this is the correct reading for the second sentence of (13a).

In section 4.1 we followed Landman's *modal splitting* analysis of disjunctive sentences. Intuitively, such an analysis must be able to predict that (14a) really means (14b).

(14) a. Call this number. The phone will be answered by either a doctor or a secretary. The doctor can tell you right away what's a matter with you, or the secretary can make an appointment for you.

   b. Either a doctor will answer and he can tell you what is wrong, or a secretary will answer and she can make an appointment.

Let us assume that $Upd(A \vee B, \langle R, a \rangle) = Upd(A, \langle R, a \rangle) \sqcup Upd(B, \langle R, a \rangle)$, where $\langle R, a \rangle \sqcup \langle R', a' \rangle = \langle R \cup R', a \cup a' \rangle$.[25] If we now represent (14a) as something like $(\exists x_r A \vee \exists y_s B) \wedge (Px \vee Qy)$, this is indeed what we predict. The new possibility will be $\langle Upd(A, R[^x/_r]) \cup Upd(B, R[^y/_s]), a[^x/_{a(r)}, ^y/_{a(s)}]\rangle$. Because of the definedness condition, we correctly predict that the first disjunct of the second disjunction will be regarded as a continuation of the first disjunct of the first disjunction, and similarly for the second disjuncts.

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[24]Based on the assumption that $[[\exists x_r A]]^{R,i} = \langle 0,+ \rangle$ only if $\neg \exists j \approx i : [[A]]^{R[^x/_{j(r)}], j[^x/_{j(r)}]} = \langle 1,+ \rangle$.

[25]Notice that because we think of worlds as a combined function from (i) propositional variables to truth values; (ii) discourse referents to individuals; and (iii) variables to individuals, the union of two worlds is well-defined. Notice also that after the update of a world with new information, this can have an effect only on the individuals it assigns to variables.

6 Conclusion

In this paper I have proposed a modal two-dimensional analysis of presupposition and modal subordination. For the analysis of presupposition I combined the strong points of the standard dynamic analysis and the two-dimensional one: what is presupposed by a sentence follows from the interpretation rules, and presupposition satisfaction is determined (almost) independently from truth. For the analysis of modal subordination I proposed that the embedded clauses of modal statements should be interpreted with respect to possibilities that verify what is presupposed. Roberts (1989, 1996) discusses many more phenomena under the heading of 'modal subordination' than I do in this paper. Some of her additional examples, e.g. bathroom sentences, I would definitely interpret in terms of descriptive pronouns instead of as involving modal subordination (cf. van Rooy, 2001). Other examples, however, in particular those where the interpretation of one quantified phrase depends on that of another, are closer to the ones discussed in this paper. It remains to be seen whether, and if so how, we should extend our analysis to cover these examples as well.

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