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ACCOUNTING NOISE AND THE PRICING OF CoCos

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Contingent Convertible bonds (CoCos) convert into equity or are written down in times of distress. Existing pricing models assume conversion triggers based on market prices assuming that markets observe all relevant information. We incorporate that markets receive information through noisy accounting reports only, distinguish between market and accounting values and incorporate that coupon payments are subject to a Maximum

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Distributable Amount limit. We examine the impact of CoCo design and accounting noise on prices. Most importantly, we discuss the capital structure decision, explain why nondilutive CoCos tend to be chosen and how these increase the bank's risk-taking incentives.

Keywords: Contingent capital pricing; accounting noise; CoCo triggers; CoCo design; risk taking incentives; investment incentives.

1. Introduction

Contingent Capital instruments or Contingent Convertible bonds (CoCos) are debt instruments designed to convert into equity or to be written down in times of distress. Their use has exploded since the Great Financial Crisis eroded the capital base of banks across the world and regulators responded by raising capital requirements while allowing banks to meet those requirements partially by issuing CoCos.

In this paper, we develop a valuation model that takes into account their particular contingent properties and explicitly incorporates the fact that markets get only imperfect information about the underlying firm dynamics through noisy accounting reports. This allows us to distinguish the accounting triggers that are used exclusively in practice from the market value-based triggers always assumed in the academic literature. And we address another issue that has received insufficient attention in the literature: why do banks choose to issue CoCos to begin with, and what sort of CoCos do they choose? The answers to these capital structure questions suggest that the objective of higher capital requirements may in fact be undermined by allowing them to be met by CoCos with the structure left free for the issuing bank to choose.

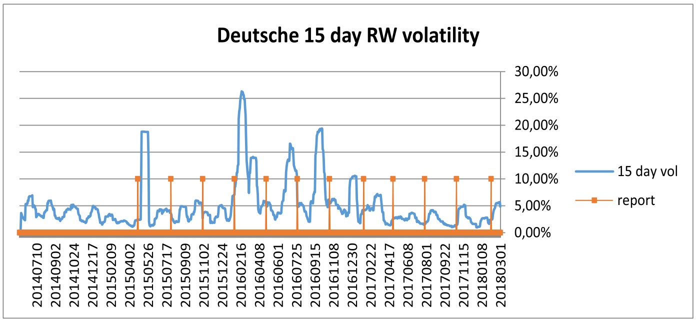
Regulation has a major impact on the design of CoCos. In particular, the eligibility criteria for CoCos to count as (Additional Tier 1 or AT1) capital restrict the way they can be structured. For example, basing conversion triggers on market prices actually makes a CoCo ineligible as regulatory capital under European law[a] under the framework implementing Basel III capital requirements in the European Union. Accordingly, all CoCos issued so far have triggers for conversion based on accounting ratios falling below a particular ratio (the trigger ratio). The distinction between book and market valuations matters since they differ widely and often systematically (Fama & French 1995). Our first contribution is that we introduce plausible informational frictions as a reason why market price-based ratios and accounting ratios can (and generally will) diverge; this allows for a meaningful analysis of accounting ratio-based CoCo triggers.

Another consequence of regulation is that CoCos, to be eligible for AT1 "additional going concern capital" status, will need to be fully loss absorbent on a going

[a]see Capital Requirements Regulation 575/2013/EU (2013, art. 54), henceforth referred to as CRR.

concern basis. This requires that instruments are subordinated, have fully discretionary noncumulative dividends or coupons and have neither a maturity date nor an incentive to redeem; only the issuing bank can call them, typically after a five-year period. And the issuing bank needs to have the contractual right to suspend coupon payment with an explicit contractual provision that such a suspension cannot constitute an event of default (ECB 2010, p. 126). This has consequences for the way Leland-style (Leland 1994) default triggers are introduced; in particular the default trigger cannot depend on whether the CoCo has converted or not.

Corcuera *et al.* (2013) argued early on that there is another problem with most proposed pricing models: they do not address the impact of skew and fat tails on CoCo valuation. Corcuera *et al.* (2013) therefore adopted a smile conform asset valuation process: the Heston mean reversion stochastic volatility model, like de Spiegeleer *et al.* (2017). Chen *et al.* (2013) took an alternative approach by introducing a jump process.[b] Figure 1 gives an example of the volatility of one particular CoCo price which reveals another unusual pattern.



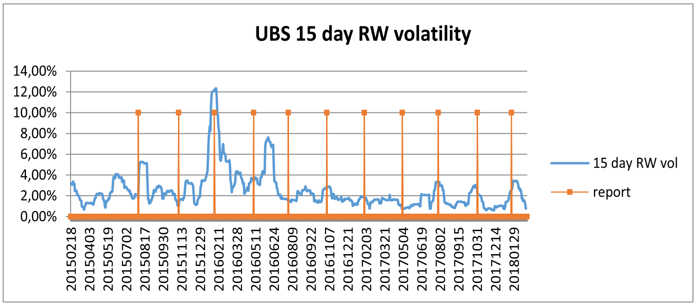


Fig. 1. The volatility of CoCo prices (15-day Rolling Window), for CoCos issued by Deutsche Bank (upper figure, plotted for all days between 2014-07-10 and 2018-03-01) and UBS (lower figure, plotted for all days between 2015-02-18 and 2018-01-29). Observe how the time series of CoCo price volatility displays an irregular pattern and how the CoCo price volatility tends to spike around the release of accounting reports (labeled by the orange markers).

[b]Bates (1996) showed that introducing both stochastic volatility and jump processes leads to identification problems.

Figure 1 depicts time series of DB and UBS volatilities, and demonstrates two striking features. First, the time series of actual CoCo price volatility displays an obviously irregular pattern. and second, the volatility series shows spikes that cluster around accounting dates. None of the various smile conform pricing models can reproduce this pattern. We resolve this puzzle by valuing CoCos under the assumption that the only information available is noisy accounting information, which in addition is only received at pre-specified discrete moments in time, the accounting dates. The underlying processes are continuous, but markets only receive noisy information on those underlying fundamental processes at discrete-time instants: accounting reports are only released at the accounting dates, typically at the end of each quarter. Introducing discretely spaced observation moments has an impact similar to introducing jumps, but also explains why they mostly happen at accounting dates. Although the underlying process is continuous, letting it run unobserved for a discrete-time period does allow the price to cover a discrete distance. The accounting noise approach does allow us to account for the observed clustering of volatility spikes around accounting dates. The theoretical results underlying this approach, involving several essential conditional densities and the resulting key pricing formulas, are new and constitute our second contribution to the literature.

We make a third contribution towards a better pricing model for CoCos, by incorporating regulatory restrictions on coupon distribution. The asset pricing literature has so far concentrated on the conversion contingency. But the possibility of a write down or conversion into equity is not the only option-like characteristic embedded in CoCo designs. The coupon payments are contingent too, they can only be paid out if that payment does not exceed the so-called Maximum Distributable Amount (MDA), a trigger that by design and as a regulatory requirement binds much earlier than the conversion trigger (Kiewiet *et al.* 2017). The relevance of this contingency became very clear at the beginning of 2016, when a profit warning of Deutsche Bank ahead of their first quarter accounting report set off an across the board crash in CoCo prices, see again Kiewiet *et al.* (2017).

Finally, our fourth contribution is arguably the most important one: we use our model to answer the question of why firms issue CoCos to begin with, a question that is not addressed in the literature. We have shown that when faced with the requirement by regulators to raise equity ratios in a situation of debt overhang, CoCos are a way for equity holders to resolve the conflict between the requirement to raise equity ratios and the debt overhang induced desire to actually increase leverage, even if the new CoCo debt is fairly priced. We take our cue from the fact that the vast majority of CoCos has been issued in the years following the Great Financial Crisis (GFC) in 2008/2009 when banks were faced with strong pressure by regulators to increase equity ratios. This result naturally leads to two follow-up questions.

First, given that CoCos are the preferred option to raise new capital when required to do so by regulators, does that solve the debt overhang problem that gave rise to the choice for CoCos to begin with? The second question stems from the rationale behind the call of regulators for higher capital ratios: the desire to reduce risk taking incentives after excessive risk taking by banks triggered and amplified the GFC. A logical question then is whether that objective is achieved when banks meet the higher capital requirements by issuing CoCos? We show that the answer to both questions is no when CoCos are NOT sufficiently dilutive. And unfortunately our discussion of the capital structure question also shows that that is what equity holders will choose: when left free to determine the CoCo parameters, they will issue CoCos that are not dilutive at all but benefit shareholders at conversion, thereby increasing rather than reducing both the initial debt overhang and socially undesirable risk taking incentives.

The remainder of this paper is structured as follows. Section 2 briefly describes CoCos in detail, their design features and their regulatory treatment. Section 3 surveys the existing asset pricing literature on CoCos. Section 4 sets up the pricing model, making a distinction between market values and accounting values and incorporating the possibility of early cancellation of coupons triggered by the MDA regulations referred to earlier. We analyze the role of the different accounting triggers and their impact on the pricing of the CoCos under various circumstances. The theoretical results of this section, essential conditional densities and the resulting key pricing formulas, are new. Section 5 uses the model to analyze the sensitivity of CoCo valuation to various design features, changes in the firm's capital structure and external shocks. In Sec. 6, we discuss why banks choose to issue CoCos when faced with the requirement to raise capital ratios, and what sort of CoCos they will choose. Section 7 summarizes and concludes. Appendix A outlines the MCMC algorithms used for evaluating the integrals involved in the final pricing expressions in Sec. 4, while proofs of the technical results are collected in Appendix B.

2. Contingent Convertible Bonds

The design of a CoCo contract is specified by two main characteristics: the trigger event (when does conversion happen?) and the conversion mechanism (what happens at conversion?).

2.1. *The trigger event*

The trigger event specifies at which moment the conversion takes place. We can distinguish three types of trigger events; an accounting trigger, a market trigger and a regulatory trigger.

In case of an accounting trigger, the conversion is triggered by an accounting ratio, e.g. the Common Equity Tier 1 Ratio (defined as the fraction of common equity over (risk-weighted) assets) falling below a certain barrier. This type of trigger is exclusively used in practice because of regulatory requirements, although it is widely criticized in the academic world (Flannery 2005, Haldane 2011, Pennacchi & Tschistyi 2015).[c] The reason for choosing the accounting ratio-based trigger nevertheless is that the European Union has implemented the BIS rule ruling out market-price-based triggers if the CoCo is to qualify as (AT1) capital (Capital Requirements Regulation 575/2013/EU 2013). As a consequence, no CoCos with market price-based triggers have been issued so far, at least not in the EU. A third type of trigger is the regulatory trigger, which allows the regulator to call for a conversion even when accounting ratios do not call yet for a conversion. All CoCos issued so far have a trigger mechanism which is a combination of an accounting trigger and a regulatory trigger, since that is required for the CoCo to count as regulatory capital in the European Union. The regulatory trigger has not been discussed in the asset pricing literature yet.

It is an open question whether regulators have better or earlier access to bank-specific information than markets have. Haldane (2011) in particular has argued for the opposite view. But whatever the merit of that point of view (regulatory inaction is not the same as regulators not being informed...), the situation is more than likely different now. Supervision has tightened considerably after the Lehman crisis, and bank management is personally liable these days when they are found not or not timely to have transmitted information pertinent to the solvency of their bank. And on-site supervision is much more easily resorted to than it used to. We therefore assume that the regulatory trigger is based on the actual capital ratio of the bank rather than on the noisy accounting image of it.

2.2. *The conversion mechanism*

The conversion mechanism specifies what happens at the moment of conversion: either a (partial) principal write-down or a conversion into shares. In case of a (partial) principal write-down (PWD) mechanism, the principal of the CoCo bond is (partially) written down at the moment of conversion, to strengthen the capital position of the issuing bank. In case of a conversion into shares, the principal of the CoCo bond is converted into a prespecified number of shares or at a prespecified price. The latter can either be determined at the time of issue or be linked to the market price prevailing at the time of conversion. Sundaresan & Wang (2015) warned that a market price-based conversion can also create incentives to short sell the stock to dilute existing shareholders. Presumably to avoid this, European law requires a floor under the conversion price if the CoCo is to count as capital.

2.3. *The maximum distributable amount (MDA) trigger*

To qualify as capital under Basel III regulations, Contingent Convertible bonds need to have a so called Maximum Distributable Amount (MDA) trigger, which

[c] Against this point of view, Sundaresan & Wang (2015), Glasserman & Nouri (2016) argued that if the terms of conversion are beneficial to CoCo holders, a market trigger could encourage CoCo holders to short-sell shares of the issuing bank to profit from a conversion, which could subsequently lead to a “death spiral”.

blocks earnings distributions (dividends or CoCo coupon payments) when the bank's capital would become too low after the earnings distribution, see Kiewiet *et al.* (2017) for a detailed discussion of the MDA trigger for coupons. The way this precedence is implemented is by requiring the so-called MDA capital buffer to be on top of other (T1 and P2)[d] buffers, so the MDA trigger will unavoidably be set off earlier than the conversion trigger. The MDA trigger has not been considered before in the asset pricing literature on CoCos, but will be introduced explicitly in this paper.

3. Relation to the Existing Literature

The existing pricing literature on CoCos can be grouped in three categories (Wilkens & Bethke 2014): structural models, equity derivative models and credit risk or reduced form models. In a structural model, one starts by describing the value of the assets of a firm by a stochastic process. Then the liabilities are introduced and equity is the difference between the assets and those liabilities. A credit derivative approach is a reduced-form approach where a conversion arrival intensity exists by assumption and is subsequently modeled as a function of latent state variables or predictors of future conversions. This approach is appealing for its tractability but is difficult to apply empirically for the simple reason that conversions have not yet occurred in practice, making the latent variable approach untestable in practice as of the date of writing this paper. A third approach, described in Wilkens & Bethke (2014), is the equity derivative approach where one tries to replicate the CoCo pay off by using equity derivatives directly. The CoCo is seen as a straight bond plus Knock-in Forwards minus Binary Down-in options. The long position in Knock-in Forwards corresponds to the possible purchase of shares at the stipulated conversion price in case the trigger event takes place (i.e. when the forwards knock in). The short position in Binary Down-in options reflects the loss of (parts of) the coupon payments once the trigger event occurs. The main problem with both the credit derivative and the equity derivative approach is that they unavoidably have to assume trigger events conditional on market price-based triggers, which is as we saw counterfactual. We therefore choose to use the structural approach and will review only the corresponding part of the literature.

In a structural setup, conversion of CoCos occurs when the market value of the firm's assets or the firm's capital ratio falls below a predetermined value (the conversion trigger). Liquidation of the firm can be incorporated in the model by assuming that the equity holders liquidate the firm when the value of assets falls below some optimal threshold, the insolvency trigger, chosen by the shareholders to maximize equity value, like in Leland (1994). Because CoCos count as capital even before they have converted, this insolvency trigger will obviously not depend

[d]T1: pure equity going concern buffer; P2 (Pillar 2) capital requirement covers risks which are not adequately covered by the minimum capital requirement (the P1 buffers).

on whether the CoCo has converted or not, the CoCo counts as capital either way. Furthermore, Leland (1994) argued that the insolvency trigger will always fall below the debt ratio, so it follows that default cannot occur *before* conversion. An early example of such a structural model is Albul *et al.* (2012), where the asset price is modeled as a Geometric Brownian motion (GBM). The CoCo converts into equity the first time the asset value A_t falls below some threshold α_c, so the conversion time is $\tau(\alpha_c) = \inf\{t \geq 0 : A_t \leq \alpha_c\}$. In their set up, the CoCo converts into equity valued at market prices at a specified conversion ratio λ, where $\lambda = 1$ means that the CoCo holder receives equity with a market value equal to the face value of the CoCo at issue. Pennacchi (2011) used a similar model but adds proportional jump processes by adding a compound Poisson process to the firm value dynamics. Chen *et al.* (2013) also used a GBM process with Poisson jumps added in, but adds a distinction between market-wide and firm-specific jumps. In contrast to the variable conversion share price featured in Albul *et al.* (2012), Pennacchi (2011), in Chen *et al.* (2013) the CoCo holders receive a fixed number of shares for every Euro of principal when the CoCo converts, which is the way most CoCos with a conversion into shares are set up in practice, see for example Avdjiev *et al.* (2020). Chen *et al.* (2013) also introduced finite maturity debt and the associated potential debt rollover problems. This feature has a significant effect on risk taking behavior before conversion. Pennacchi & Tschistyi (2015) reverted to a straight GBM process driving asset values, and focuses on existence and uniqueness of a price equilibrium when conversion involves a wealth transfer favoring either the CoCo holder (dilutive CoCos) or the old equity holder (nondilutive CoCos). All these models have in common that a conversion trigger based on market values is used.

Glasserman & Nouri (2012) do distinguish market- and accounting-based valuation: they assume that the ratio between the market value of equity and the accounting value of equity follows a GBM process. But a key assumption in Glasserman & Nouri (2012) is that markets and accountants always agree on whether the firm is solvent. Haldane (2011) strongly argued that this assumption is counterfactual. Another issue is that they assume that all processes can be observed continuously, while in practice regulatory capital ratios are calculated on a quarterly basis only.

We contribute to the literature by addressing both issues. We assume that firm values are driven by a GBM, but without jumps, for reasons explained in Sec. 4. We do not introduce a separate independent accounting process either, like Brigo *et al.* (2015) do; it surely is implausible that accounting reports are not at all informative about the market value of the firm. Instead we assume that accounting reports are noisy but informative observations of the underlying processes, like in Duffie & Lando (2001). We stipulate an underlying GBM process for the dynamic evolvement of the firm's asset valuation, but incorporate explicitly that that process is not directly observable: noisy information (an accounting report) is brought out at discrete-time instants. Moreover, we assume some persistence in the noise, by assuming that the noise term in the accounting report is serially correlated. Berg & Kaserer (2015) exactly adopted the Duffie–Lando model in the context of CoCos, with as a consequence that they can only analyze CoCos with just regulatory triggers. In contrast, we significantly extend the Duffie & Lando (2001) model in such a way that it allows us to analyze the richer array of triggers used in practice. In Sec. 4, we describe the model we use in detail and discuss how it differs from other approaches.

4. The Model

This section starts with the model description, in Sec. 4.1. We derive the density of asset values, conditional on accounting information (Sec. 4.2). We then use these results to derive the valuation of CoCos for the different trigger events (regulatory triggers and accounting triggers) and for the different conversion schemes (PWD CoCos and Equity Converters).

4.1. *Model description and the Firm's debt structure*

The value of assets of the firm, denoted by V_t, is modeled by a geometric Brownian motion, that is

$$\frac{\mathrm{d}V_t}{V_t} = \mu \mathrm{d}t + \sigma \mathrm{d}W_t, \tag{4.1}$$

for some $\mu \in \mathbb{R}$, $\sigma > 0$. We will not include jumps in the asset value process, because this does not make sense in the noisy accounting information framework, as it would not be possible to distinguish a big price movement caused by the dynamics of the asset process from a reaction to the accounting information. Define $Z_t = \log V_t$ and $m = \mu - \sigma^2/2$, then Z is a drifted Brownian motion with drift m and volatility σ, that is

$$Z_t = Z_0 + mt + \sigma W_t. \tag{4.2}$$

Investors do not observe the real asset value, instead they receive imperfect accounting information at known observation times $t_1 < t_2 < \cdots$ (typically every three months). At every observation date t_i there arrives an imperfect accounting report of the real asset value V_{t_i}, denoted by $\hat{V}_{t_i}$. We let

$$Y_{t_i} := \log \hat{V}_{t_i} = Z_{t_i} + U_{t_i}, \tag{4.3}$$

where U_{t_i} is normally distributed and independent of Z_{t_i}. In the following, we will use the notation $Y_i := Y_{t_i}$ and similar notations for Z and U. Following Duffie & Lando (2001), we assume there is some correlation between the accounting noise over time $U_1, U_2, \ldots$:

$$U_i = \kappa U_{i-1} + \epsilon_i, \tag{4.4}$$

for some fixed $\kappa \in \mathbb{R}$ and independent and identically distributed $\epsilon_1, \epsilon_2, \ldots,$ which have a normal distribution with mean $\mu_\epsilon \in \mathbb{R}$ and variance $\sigma_\epsilon^2 > 0$, and are independent of Z.

The firm issues straight debt and contingent convertible debt. The total par value of straight debt outstanding is denoted by P_1, over which coupons are paid continuously at rate c_1. Furthermore, the straight bonds have a perpetual maturity and it is assumed that default occurs the first time the log-value of assets falls below some threshold z_b (or equivalently, the first time asset value falls below $v_b := \exp(z_b)$), such that the default time is defined by

$$\tau_b = \inf\{t \geq 0 : Z_t \leq z_b\}. \tag{4.5}$$

At the moment of default a fraction $(1 - \alpha)$, for $\alpha \in (0, 1)$, of the firm's asset value is lost to bankruptcy costs, so a fraction α of the asset value is recovered and distributed among the senior debt holders. Following Leland (1994), the default trigger is set at the value where equity holders refuse to supply more capital when the asset value falls below the trigger, which in turn always will be lower than the ratio of debt to initial asset value. An important question is whether unconverted CoCos also count as debt when setting z_b (or equivalently v_b for the ratio, remember that $Z = \log V$). The answer is clear from the regulatory requirements for eligibility of the CoCo as AT1 capital (Capital Requirements Regulation 575/2013/EU 2013, ECB 2010). For eligibility, the CoCo needs to be a perpetual, with only the bank having the right to call the CoCo, not the CoCo holder; and the issuing bank also needs to have the discretion to suspend coupon payments without this constituting an event of default. This implies that $v_b < P_1/V_0$ both before and after conversion of any CoCos issued.

The total par value of CoCos outstanding is denoted by P_2, over which coupons are paid continuously at rate c_2. Furthermore, the maturity of the contingent convertible bonds is denoted by T. In our accounting report framework, we will consider two different types of conversion triggers. The first type of conversion trigger that will be looked into is the regulatory trigger, the trigger that governs when regulators will decide the bank has reached a so-called Point of Non-Viability and force conversion of the CoCo. Whether bank regulators have more direct access to information about the bank's asset value than market parties is open for dispute, see in particular Haldane (2011). However, since the Lehman crisis, banks in the Euro Area have the obligation to report immediately, i.e. without waiting for accounting dates, to their supervisor when they are approaching a trigger, and bank management is personally liable if they fail to do so. Since all CoCos ever issued are from after the tightening of supervision rules after the Lehman crisis and the vast majority from after the establishment of the Single Supervisory Mechanism (SSM) establishing the ECB as bank regulator in the Eurozone, we think it best reflects reality to assume that regulators have access to the underlying information on the bank assets. We therefore assume that the regulator will call for conversion when the true asset value falls below the bankruptcy trigger. Of course, this type of conversion can also happen in between accounting report dates. This type of conversion thus is triggered when the log-value of assets falls for the first time below a conversion threshold z_c (or equivalently, the first time the asset value falls below $v_c := \exp(z_c)$), i.e. the conversion time is given by

$$\tau_c = \inf\{t \geq 0 : Z_t \leq z_c\}. \tag{4.6}$$

The regulatory requirements for eligibility as AT1 capital also set a minimum trigger ratio for T1 capital of 5.125%. This means that the trigger will be set off when there is still some equity value left, asset values as a percentage of the original $t = 0$ asset value cannot yet have fallen below the original debt ratio. Given that we also have $v_b < P_1/V_0$, it follows that $z_b < z_c$, and that conversion will always happen before default, i.e. $\tau_c < \tau_b$.

There are also CoCos whose conversion trigger solely depends on accounting reports. An example is the CoCo issued by Barclays on 3 March 2017 (Barclays 2017). This means that conversion happens when the reported value of the capital ratio falls below some threshold and hence conversion can only happen at one of the accounting report dates $t_1, t_2, \ldots$. This corresponds to a setting in which the conversion time is defined as

$$\tau_c^A = \inf\{t_i \geq 0 : Y_{t_i} \leq y_c\}, \tag{4.7}$$

for some threshold $y_c \geq 0$.

In case we consider CoCos with regulatory triggers, the information available to investors at time t is described by the filtration $\mathcal{H}_t$, where

$$\mathcal{H}_t = \sigma(\{Y_{t_1}, \ldots, Y_{t_n}, \mathbf{1}_{\{\tau_c \leq s\}}, \mathbf{1}_{\{\tau_b \leq s\}} : s \leq t\}), \quad \text{for } t_n \leq t < t_{n+1}. \tag{4.8}$$

Here, the indicators are included to ensure that it is also observed in the market whether conversion has already occurred or the firm is liquidated before time t. In case we deal with CoCos with an accounting trigger, the market information is described by the filtration

$$\mathcal{H}_t^b = \sigma(Y_{t_1}, \ldots, Y_{t_n}, \mathbf{1}_{\{\tau_b \leq s\}} : s \leq t), \quad \text{for } t_n \leq t < t_{n+1}. \tag{4.9}$$

We note that Berg & Kaserer (2015) also use a setup derived from Duffie & Lando (2001), but they only consider one type of trigger (our regulatory trigger), which enables them to directly use results from that paper but precludes analysis of the accounting and MDA triggers. We additionally introduce the accounting trigger solely based on accounting reports, allowing us to distinguish between accounting triggers and regulatory triggers and include the MDA trigger. These multiple triggers have as a consequence that we have to deal with different observation filtrations and that the conditional density of the asset value as in Duffie & Lando (2001) cannot be directly applied. In the following sections, we provide further mathematical substantiation of our extended model and also provide the results for the case of more than one accounting report. Furthermore, all our new pricing formulas are of semi-analytical form, written as integrals weighted by conditional densities for which analytical formulas are provided that are new as well.

4.2. *The density of the asset value conditional on accounting information*

We consider two different types of conversion triggers. For the first one, the regulatory trigger, the conversion time is determined by the process Z falling below some threshold. In order to compute the market value of CoCos with such a trigger, we need to be able to compute the probability of conversion conditional on the market information $\mathcal{H}_t$. In order to do so, we will need the conditional density of Z, given the market information $\mathcal{H}_t$. In this section, we derive an expression for this conditional density, which is intensively used in the remainder of this paper.

Consider $t > 0$ such that $t_n \leq t < t_{n+1}$ and conversion did not happen until time t, that is $\tau_c > t$. The goal in this section is to find an expression for the conditional distribution of Z_t, given $\mathcal{H}_t$, which we will denote by $f(t, \cdot)$. Most of the results in this section can be found in the paper by Duffie & Lando (2001), but we will consider them shortly, to illustrate how the particular density is derived and we will provide some additional explicit formulas.

Consider the following notation for the relevant random vectors and their realisations:

$$\begin{aligned} &Z^{(n)} = (Z_1, Z_2, \ldots, Z_n) \quad \text{and its realisation } z^{(n)} = (z_1, z_2, \ldots, z_n), \\ &Y^{(n)} = (Y_1, Y_2, \ldots, Y_n) \quad \text{and its realisation } y^{(n)} = (y_1, y_2, \ldots, y_n), \\ &U^{(n)} = Y^{(n)} - Z^{(n)} \quad \text{and its realisation } u^{(n)} = y^{(n)} - z^{(n)}. \end{aligned} \tag{4.10}$$

We want to compute $f(t, \cdot)$, the conditional density of Z_t given $Y^{(n)}$ and $\tau_c > t$. In order to do so, we first compute the conditional density of Z_{t_n} at the report time t_n, which we will denote by $g_{t_n}(\cdot \mid Y^{(n)}, \tau_c > t_n)$. To this end, we introduce some functions. First, we need an expression for the probability $\psi(z_0, x, \sigma\sqrt{t})$ that $\min\{Z_s : s \leq t\} > 0$, conditional on $Z_0 = z_0 > 0$ and $Z_t = x > 0$. This expression is stated in the following lemma and can also be found in the paper by Duffie & Lando (2001).

Lemma 4.1. *The probability* $\psi(z_0, x, \sigma\sqrt{t})$ *that* $\min\{Z_s : s \leq t\} > 0$, *conditional on* $Z_0 = z_0 > 0$ *and* $Z_t = x > 0$, *is given by*

$$\psi(z_0, x, \sigma\sqrt{t}) = 1 - \exp\left(-\frac{2z_0 x}{\sigma^2 t}\right). \tag{4.11}$$

Consider the conditional probability of the intersection $\{Z^{(n)} \leq z^{(n)}\} \cap \{\tau_c > t_n\}$ given $Y^{(n)}$. We denote by $b_n(\cdot \mid Y^{(n)})$ its partial derivative with respect to $z^{(n)}$. Note that $(Z_n)_{n \in \mathbb{N}}$ and $(U_n)_{n \in \mathbb{N}}$ are Markov processes and denoted by $p_Z(z_n \mid z_{n-1})$ and $p_U(u_n \mid u_{n-1})$ their respective transition densities for realizations $z^{(n)}, u^{(n)}$. Furthermore, denoted by $p_Y(y_n \mid y^{(n-1)})$ the conditional density of Y_n given $Y^{(n-1)} = y^{(n-1)}$. It is then possible (Duffie & Lando 2001) to write $b_n(z^{(n)} \mid y^{(n)})$ in a recursive way

$$b_n(z^{(n)} \mid y^{(n)}) = \frac{\begin{array}{c}\psi(z_{n-1} - z_c, z_n - z_c, \sigma\sqrt{t_n - t_{n-1}}) p_Z(z_n \mid z_{n-1}) \\ \times\ p_U(y_n - z_n \mid y_{n-1} - z_{n-1}) b_{n-1}(z^{(n-1)} \mid y^{(n-1)})\end{array}}{p_Y(y_n \mid y^{(n-1)})}. \quad (4.12)$$

It now follows that the conditional density $g_{t_n}(\cdot \mid Y^{(n)}, \tau_c > t_n)$ of $Z^{(n)}$ is given by

$$g_{t_n}(z^{(n)} \mid y^{(n)}, \tau_c > t_n) = \frac{b_n(z^{(n)} \mid y^{(n)})}{\int_{(z_c,\infty)^n} b_n(z^{(n)} \mid y^{(n)}) \mathrm{d}z^{(n)}}. \quad (4.13)$$

It should be noted that there is no explicit expression for the integral in the denominator of Eq. (4.13), but note the important fact that we know the density up to a normalizing constant. Now the marginal conditional density of Z_n at time t_n is given by

$$g_{t_n}(z_n \mid y^{(n)}, \tau_c > t_n) = \int_{(z_c,\infty)^{n-1}} g_{t_n}(z^{(n)} \mid y^{(n)}, \tau_c > t_n) \mathrm{d}z^{(n-1)}. \quad (4.14)$$

Now that we found the conditional density for a report time t_n, we can use this to find the conditional density $f(t, \cdot)$ for a general time $t > 0$. For this we will need the $\mathcal{H}_t$-conditional density of Z_t, at a time before the first accounting report has arrived. Complementing Duffie & Lando (2001), we will now give an explicit expression for this density.

Lemma 4.2. $\tilde{f}(t, \cdot, z_0)$, *the* $\mathcal{H}_t$*-conditional density of* Z_t*, at a time* $t < \tau_c$ *before the first accounting report has arrived, given that* Z *started in* z_0*, is given by*

$$\tilde{f}(t, x, z_0) = \frac{1}{\sigma\sqrt{t}} \times \frac{\exp\left(\frac{-m(z_0 - x)}{\sigma^2} - \frac{m^2 t}{2\sigma^2}\right)\left(\phi\left(\frac{z_0 - x}{\sigma\sqrt{t}}\right) - \phi\left(\frac{-z_0 - x + 2z_c}{\sigma\sqrt{t}}\right)\right)}{\Phi\left(\frac{z_0 - z_c + mt}{\sigma\sqrt{t}}\right) - e^{-2m(z_0 - z_c)/\sigma^2}\Phi\left(\frac{z_c - z_0 + mt}{\sigma\sqrt{t}}\right)}, \quad (4.15)$$

where ϕ *denotes the density of the standard normal distribution.*

Proof. The proof of this lemma can be found in Appendix B. □

Finally, we are now able to compute the conditional density $f(t,\cdot)$ for a general time $t > 0$, $t_n < t < t_{n+1}$ such that $\tau_c > t$. Using the stationarity of Z, the $\mathcal{H}_t$-conditional density of Z_t can be written as

$$f(t,x) = \int_{z_c}^{\infty} \tilde{f}(t - t_n, x, z_n) g_{t_n}(z_n \,|\, Y^{(n)}, \tau_c > t_n) \mathrm{d}z_n. \tag{4.16}$$

Equation (4.16) should be read as follows; until time t_n the process Z has stayed above z_c and ended in z_n, then on the time interval (t_n, t), in which no new accounting reports arrive, the process has to move from z_n to x and stay above z_c. Although we do not have an analytical expression for the density $f(t,\cdot)$, it is important to note at this point that $f(t,\cdot)$ is written as the integral of g_{t_n}, which is known up to normalizing constant, as can be seen from Eq. (4.13). This makes it possible to compute integrals with respect to $f(t,\cdot)$, using Monte Carlo Markov Chain simulations, which means that results that are stated as an integral weighted by the density $f(t,\cdot)$ can actually be computed. The necessary algorithms are described in Appendix A.

As a first use of the density $f(t,\cdot)$, we can for a time $s > t$, where $t < \tau_c$, define the $\mathcal{H}_t$-(CoCo) survival probability $p_c(t,s) = \mathbb{P}(\tau_c > s \,|\, \mathcal{H}_t)$. This probability is then given by

$$p_c(t,s) = \int_{z_c}^{\infty} (1 - \pi(s - t, x - z_c)) f(t,x) \mathrm{d}x, \tag{4.17}$$

whereas in Duffie & Lando (2001), $\pi(t,x)$ denotes the probability that Z hits 0 before time t, starting from $x > 0$. This probability is given by the following lemma, which follows from the well-known expression for the distribution of a Brownian motion's running minimum, see e.g. Harrison (1985), Sec. 1.8, Eq. (11).

Lemma 4.3. *The probability $\pi(t,x)$ that Z hits 0 before time t, starting from $x > 0$, is given by*

$$\pi(t,x) = 1 - \Phi\left(\frac{x + mt}{\sigma\sqrt{t}}\right) + e^{-2mx/\sigma^2} \Phi\left(\frac{-x + mt}{\sigma\sqrt{t}}\right), \tag{4.18}$$

where Φ denotes the distribution function of the standard normal distribution.

4.3. *Valuation of CoCos*

We can now derive the formulas for the market values of the different types of CoCos. First, in Sec. 4.3.1, CoCos whose principal is written down at conversion, are valued. We show how to incorporate the MDA-regulation potentially leading to early canceling of coupons in Sec. 4.3.2. In Sec. 4.3.3, we then extend the analysis to CoCos with a conversion into shares instead of a principal write down at conversion. In these first two cases, we assume a regulatory trigger and the results are all in the form of an integral weighted by the above-derived conditional density $f(t,\cdot)$. Then, in Sec. 4.3.4, PWD CoCos with only an accounting trigger are valued. We provide the algorithms necessary to compute all the integral expressions in Appendix A.

4.3.1. *Valuation of PWD CoCos with a regulatory trigger*

In this section, we will value CoCos with a regulatory trigger and a principal write down at conversion. At the end of the section we will also incorporate the MDA-trigger. Recall that in case of a regulatory trigger, the conversion date was defined as

$$\tau_c = \inf\{t \geq 0 : Z_t \leq z_c\}. \tag{4.19}$$

Also, recall that the firm pays coupons continuously at rate c_2 until either maturity or conversion. We consider a principal write down CoCo, which means a fraction $1-R$ of the principal value is written down at conversion, while a fraction R is recovered to the bond holder, with $R \in [0,1)$. In practice, we can always assume $R=0$, if it is not the CoCo can be split into straight debt and a CoCo that does have $R=0$. Furthermore it is assumed that the risk-free rate is constant, denoted by r.

Now the value at time $t < \tau_c$ of the CoCos, given the imperfect accounting information $\mathcal{H}_t$, is given by

$$\begin{aligned} C(t) &= \mathbb{E}(P_2 e^{-r(T-t)}\mathbf{1}_{\{\tau_c>T\}} \,|\, \mathcal{H}_t) + \mathbb{E}\left(\int_t^T c_2 P_2 e^{-r(u-t)}\mathbf{1}_{\{\tau_c>u\}}\mathrm{d}u \,|\, \mathcal{H}_t\right) \\ &\quad + \mathbb{E}(RP_2 e^{-r(\tau_c-t)}\mathbf{1}_{\{\tau_c\leq T\}} \,|\, \mathcal{H}_t) \\ &= P_2 e^{-r(T-t)} p_c(t,T) + c_2 P_2 \int_t^T e^{-r(u-t)} p_c(t,u)\mathrm{d}u \\ &\quad - RP_2 \int_t^T e^{-r(u-t)} p_c(t,\mathrm{d}u), \end{aligned} \tag{4.20}$$

where $p_c(t,\mathrm{d}u)$ indicates integration with respect to the survival probability $p_c(t,u)$.

In (4.20), the first term represents the payment of the principal, in case conversion does not happen before maturity, while the second term accounts for the payment of coupons until either conversion or maturity. The last term values the recovery of the principal at conversion. Note that every term is written in terms of the CoCo survival probability $p_c(t,s)$, which was given as an integral, weighted by the density $f(t,\cdot)$. Unsurprisingly, it turns out that these three terms together can be written as one integral weighted by the conditional density $f(t,\cdot)$, which was derived in the previous section. This leads to the main result of this section, which is proved in Appendix B.

Theorem 4.1 (Price of a PWD CoCo with a regulatory trigger). *The secondary market price of the CoCo at time $t < \tau_c$ is given by*

$$C(t) = \int_{z_c}^{\infty} h(x) f(t,x) \mathrm{d}x, \tag{4.21}$$

where $h(x)$ has the analytical expression given in Eq. (B.4).

4.3.2. *Including the MDA-trigger*

In the valuation of the firm's convertible debt in Eq. (4.20), it is assumed that coupons are paid until conversion. However, as pointed out before, CoCos are affected by the Maximum Distributable Amount (MDA), which requires regulators to stop earnings distributions when the firm's total capital falls below some trigger, higher than the conversion trigger. This we will incorporate in the model by introducing a trigger $z_{cc} > z_c$. If Z is below z_{cc} the firm will not pay coupons, while if Z is above z_{cc} the firm still pays coupons. To value the CoCo in this case, only the second term in Eq. (4.20) needs to be adjusted. In this case, coupons are only paid at time u if $Z_u > z_{cc}$, so the term

$$\mathbb{E}\left(\int_t^T c_2 P_2 e^{-r(u-t)} \mathbf{1}_{\{\tau_c > u\}} \mathrm{d}u \,|\, \mathcal{H}_t \right), \tag{4.22}$$

needs to be replaced with

$$\mathbb{E}\left(\int_t^T c_2 P_2 e^{-r(u-t)} \mathbf{1}_{\{\tau_c > u, Z_u > z_{cc}\}} \mathrm{d}u \,|\, \mathcal{H}_t \right). \tag{4.23}$$

For $\tau_c > t$ and $t_n \le t < t_{n+1}$, this term equals

$$c_2 P_2 \int_t^T e^{-r(u-t)} \mathbb{P}(\tau_c > u, Z_u > z_{cc} \,|\, Y^{(n)}, \tau_c > t) \mathrm{d}u. \tag{4.24}$$

Thus, to value the CoCos while including the effects of the MDA-trigger, the quantity we need to compute is $\mathbb{P}(\tau_c > u, Z_u > z_{cc} \,|\, Y^{(n)}, \tau_c > t)$, which can be written in a similar way as the CoCo survival probability $p_c(t,s)$. As the other two terms in Eq. (4.20) do not change, the following result is a relatively straightforward extension of Theorem 4.1 and is proved in Appendix B.

Theorem 4.2 (Price of a PWD CoCo with a regulatory trigger and MDA trigger). *The secondary market price of the CoCo at time $t < \tau_c$ is given by*

$$C(t) = \int_{z_c}^{\infty} (\tilde{h}(x) + I_{cc}(x)) f(t,x) \mathrm{d}x,$$

where $\tilde{h}$ has the analytical expression given in Eq. (B.10) *and I_{cc} is given by Eq.* (B.11).

4.3.3. *Valuation of CoCos with a conversion into shares and a regulatory trigger*

In this section, we consider the valuation of contingent convertible bonds which convert into equity at the conversion date. Recall that we assumed the firm issues two types of debts; straight debt and contingent convertible debt. The total par value of straight debt outstanding is denoted by P_1, over which coupons are paid continuously at rate c_1. Furthermore, the straight bonds have a perpetual maturity and default occurs at

$$\tau_b = \inf\{t \geq 0 : Z_t \leq z_b\}.$$

At the moment of default a fraction $(1-\alpha)$, for $\alpha \in (0,1)$, of the firm's asset value is lost to bankruptcy costs, so a fraction α of the asset value is recovered and distributed among the senior debt holders.

The total par value of CoCos outstanding is denoted by P_2, over which coupons are paid continuously at rate c_2. Furthermore, the maturity of the contingent convertible bonds is denoted by T. We consider a regulatory trigger, which means the conversion date is defined as

$$\tau_c = \inf\{t \geq 0 : Z_t \leq z_c\}, \tag{4.25}$$

where $z_c > z_b$, since we already saw that conversion if it happens will always take place before default. Following Chen *et al.* (2013), we will assume the CoCo holders receive Δ shares for every dollar of principal at the moment of conversion. This means that, if we normalize the number of shares before conversion to 1, the CoCo holders own a fraction $\rho = \frac{\Delta P_2}{\Delta P_2 + 1}$ of the firm's equity after conversion.

To recall, the information in the market at time t is described by the filtration

$$\mathcal{H}_t = \sigma(\{Y_{t_1}, \ldots, Y_{t_n}, \mathbf{1}_{\{\tau_c \leq s\}}, \mathbf{1}_{\{\tau_b \leq s\}} : s \leq t\}), \quad \text{for } t_n \leq t < t_{n+1}.$$

In analogy to Eq. (4.20), the market price of the CoCos is given by

$$\begin{aligned} C(t) = {} & \mathbb{E}(P_2 e^{-r(T-t)} \mathbf{1}_{\{\tau_c > T\}} \mid \mathcal{H}_t) + \mathbb{E}\left(\int_t^T c_2 P_2 e^{-r(u-t)} \mathbf{1}_{\{\tau_c > u\}} \mathrm{d}u \mid \mathcal{H}_t \right) \\ & + \mathbb{E}\left(\frac{\Delta P_2}{\Delta P_2 + 1} E^{PC}(\tau_c) e^{-r(\tau_c - t)} \mathbf{1}_{\{\tau_c \leq T\}} \mid \mathcal{H}_t \right). \end{aligned} \tag{4.26}$$

Only the third term has changed compared to Eq. (4.20), because this term describes what happens at the moment of conversion (if we want to include the possibility of early cancellation of coupons we need to replace the second term by the corresponding term in Eq. (4.23)). The third term now describes that the CoCo holders obtain a fraction $\frac{\Delta P_2}{\Delta P_2 + 1}$ of the firms post-conversion equity, denoted by $E^{PC}(\tau_c)$. This post conversion equity satisfies

$$E^{PC}(\tau_c) = V_{\tau_c} - D(\tau_c) - \mathbb{E}(e^{-r(\tau_b - \tau_c)}(1-\alpha) V_{\tau_b} \mid \mathcal{H}_{\tau_c}).$$

That is, the firm's value of assets minus the value of straight debt, denoted by $D(\tau_c)$, and bankruptcy costs, described by the last term. Note that the value of straight debt at conversion is given by

$$D(\tau_c) = \mathbb{E}\left(\int_{\tau_c}^{\infty} c_1 P_1 e^{-r(u-\tau_c)} \mathbf{1}_{\{\tau_b > u\}} \mathrm{d}u \,|\, \mathcal{H}_{\tau_c}\right) + \mathbb{E}(\alpha V_{\tau_b} e^{-r(\tau_b - \tau_c)} \,|\, \mathcal{H}_{\tau_c}),$$

where the first term accounts for the continuous payment of coupons and the second term describes the payment at default. After computing the value of the post-conversion equity, the CoCo price can also be computed. The result is summarized in the following theorem, of which the proof can be found in Appendix B.

Theorem 4.3 (Price with a regulatory trigger and a conversion into shares). *The secondary market price at time $t < \tau_c$ of the CoCo with a regulatory trigger and a conversion into shares is given by*

$$\begin{aligned} C(t) = &\int_{z_c}^{\infty} (h_0(x) + h_1(x)) f(t,x) \mathrm{d}x \\ &+ \int_{z_c}^{\infty} \int_{z_c}^{\infty} f(t,x) \hat{f}(x, z_c, \tilde{z}, T-t) h_2(\tilde{z}) \mathrm{d}\tilde{z} \mathrm{d}x, \end{aligned} \tag{4.27}$$

where h_0, h_1, h_2 and $\hat{f}$ are functions with analytical expressions that can be found in Eqs. (B.13), (B.23), (B.24) *and* (B.16).

4.3.4. *Valuation of PWD CoCos with an accounting trigger*

In this section, we will consider PWD CoCos which conversion trigger solely depends on accounting reports, such as for example the CoCos issued by Barclays. This means that conversion happens when the reported value of the capital ratio falls below some threshold and hence conversion can only happen at one of the accounting report dates $t_1, t_2, \ldots$. This corresponds to a setting in which the conversion time is defined as

$$\tau_c^A = \inf\{t_i \geq 0 : Y_{t_i} \leq y_c\}, \tag{4.28}$$

for some threshold $y_c \geq 0$. Of course, default can still happen in between accounting dates, corresponding to the default time

$$\tau_b = \inf\{t \geq 0 : Z_t \leq z_b\}. \tag{4.29}$$

In this case, the available information at time t reduces to

$$\mathcal{H}_t^b = \sigma(Y_{t_1}, \ldots, Y_{t_n}, \mathbf{1}_{\{\tau_b \leq s\}} : s \leq t), \quad t_n \leq t < t_{n+1}. \tag{4.30}$$

So where before the conditioning was on $Y^{(n)} = y^{(n)}, \tau_c > t$, this now changes into $Y^{(n)} = y^{(n)}, \tau_b > t$. This only changes the trigger z_c into z_b, so for example we still have the conditional density of Z_n, denoted by $g_{t_n}(z_n \,|\, y^{(n)}, \tau_b > t)$, which is given by Eq. (4.14), but now with z_b substituted for z_c. The same holds for $f(t,x)$, the conditional density of Z_t for a general time $t_n \leq t < t_{n+1}$, see Eq. (4.16). Similar to Eq. (4.20), at time $t < \min(\tau_b, \tau_c^A)$ the CoCo has secondary market price

$$C(t) = \mathbb{E}(P_2 e^{-r(T-t)} \mathbf{1}_{\{\tau_c^A > T, \tau_b > T\}} \,|\, \mathcal{H}_t^b) + \mathbb{E}\left(\int_t^T c_2 P_2 e^{-r(u-t)} \mathbf{1}_{\{\tau_c^A > u, \tau_b > u\}} \mathrm{d}u \,|\, \mathcal{H}_t^b\right)$$
$$+ \mathbb{E}(R P_2 e^{-r(\tau_c^A - t)} \mathbf{1}_{\{\tau_c^A \leq T, \tau_b > \tau_c^A\}} \,|\, \mathcal{H}_t^b), \tag{4.31}$$

of which the value is given by the following theorem, in terms of integrals that can be evaluated using the algorithms in Appendix A (in particular, the survival probability $p_b(t_n, t)$ is computed using Algorithm A.1, while the integrals with respect to $B_{n,i}$ are evaluated using Algorithm A.3).

Theorem 4.4 (Price of a PWD CoCo with a sole accounting trigger). *Let $t_n \leq t < t_{n+1}$, $T = t_{n+m}$ for some $m \in \mathbb{N}$ and $Y^{(n)} = y^{(n)}$, where $y_i > y_c, 1 \leq i \leq n$. Furthermore, denote $y^{(n+1,n+i)} = (y_{n+1}, \ldots, y_{n+i})$ and $z^{(n,n+i)} = (z_n, \ldots, z_{n+i})$. Then the market price $C(t)$ of the CoCo with an accounting trigger is given by*

$$\frac{1}{p_b(t_n, t)} \sum_{i=0}^{m} \int_{(y_c,\infty)^i} \int_{(z_b,\infty)^{i+1}} h_i(z_{n+i})$$
$$\times B_{n,i}(z^{(n,n+i)}, y^{(n+1,n+i)} | y^{(n)}, \tau_b > t_n) \mathrm{d}z^{(n,n+i)} \mathrm{d}y^{(n+1,n+i)}, \tag{4.32}$$

where $B_{n,i}$ is a density on $(z_b, \infty)^{i+1} \times \mathbb{R}^i$, as defined in Eq. (B.25) *and computed in Lemma* B.3; *h_i is a function with the analytical expression given by Eq.* (B.30) *and $p_b(t, s) = \mathbb{P}(\tau_b > s \,|\, \mathcal{H}_t)$ denotes the default survival probability, as given by Eq.* (4.17), *when substituting z_b for z_c.*

Remark 4.1. In expression (4.32), it is understood that the integral over $y^{(n+1,n+i)}$ disappears for $i = 0$ and that $B_{n,i}$ is a density in z_n only in that case, see also Lemma B.3.

5. Applying the Model

In this section, we use the model to shed light on a variety of questions related to the basic valuation model itself and its sensitivity to design and "environmental" variables such as volatility shocks. After setting up the parametrization of the base case in Sec. 5.1, we analyze in Sec. 5.2 a variety of accounting noise related items: accounting noise volatility and possible serial correlation, and time lapsed since the last accounting report. We then turn to the impact of various CoCo design parameters in Sec. 5.3. There we also analyze the impact of the MDA trigger and the coupon payment contingency on CoCo pricing and use our model to analyze the Deutsche Bank profit scare of February 2016 and its impact on CoCo prices. In Sec. 5.4, we study the impact of dilution and asset volatility on CoCo pricing.

Table 1. Base case parameters.

Parameter	Value
Initial asset value V_0	100
n, the number of accounting reports until time t	2
Conversion trigger v_c	80
Default trigger v_b	P_1-5
Recovery rate at default α	0.5
Total principal straight debt P_1	70
Coupon straight debt	0.04
Total principal CoCos P_2	5
Coupon CoCos c_2	0.07
Maturity CoCos T	t+5
Drift asset process m	0.01
Volatility asset process σ	0.1
Mean accounting noise μ_ϵ	0
Volatility accounting noise σ_ϵ	0.1
Risk-free rate	0.03

5.1. *Parametrization of the base case*

Table 1 lists the values of the base case parameters. For the choice of the base case parameters, some restraints should be taken into account. For example, the conversion trigger should be higher than the default trigger: we know from Leland (1994) that the default trigger should be below the debt ratio while regulation requires a conversion trigger at a positive T1/AT1 capital ratio of at least 5.125% and stipulates that CoCos themselves even when not yet converted can never trigger bankruptcy claims since coupons can contractually be suspended at the issuer's discretion. Also, a CoCo should pay a higher coupon than straight debt, to compensate for the higher risk. Furthermore, we have no empirical evidence for a reasonable level of accounting noise, so we set the volatility of accounting noise equal to the base case parameter chosen by Duffie & Lando (2001), where the accounting noise variance is chosen to match short run default probabilities implicit in short run CDS spreads.

In the base case, we will assume the CoCo has a regulatory trigger, i.e. the regulator has access to the true state of the bank and conversion can take place at any time, not just at accounting dates, but the market has to evaluate conversion probabilities given this trigger rule using accounting information only (see Sec. 4.3.3 for the mathematics of this trigger). We will also explore other trigger mechanisms.

Furthermore, we define the dilution ratio ρ (see Sec. 4.3.3) as the fraction shares owned by the CoCo holder post-conversion:

$$\rho = \frac{\Delta P_2}{\Delta P_2 + 1} \tag{5.1}$$

where P_2 is the face value of the CoCo before conversion, and Δ equals the number of shares the CoCo holder receives at conversion. The number of old shares is normalized to 1. A dilution ratio of $\rho = 0$ means that the CoCo suffers a principal write-down (PWD) at conversion, while $\rho = 1$ corresponds to the extreme case that the original shareholders are completely wiped out at conversion.

To compute prices for PWD CoCos, we make use of Theorem 4.1. The integral involved is approximated as in Eq. (A.2), for which the necessary sample is obtained by using Algorithm A.1. To compute prices for CoCos with a conversion into shares, we make use of Theorem 4.3, where the first term in the pricing formula follows again by using Algorithm A.1 and the second term is approximated as in Eq. (A.4), for which the necessary sample is obtained by execution of Algorithm A.2. Then the figures are produced by repeatedly following this procedure for different values of the parameters.

5.2. *Accounting noise and CoCo prices*

In this section, we study the impact of various aspects of accounting noise that are at the core of our approach to model the difference between market- and accounting values. In particular, we look at the impact of (changes in) accounting noise on CoCo prices as a function of different design parameters.

5.2.1. *Accounting noise shocks*

We consider the relationship between the accounting noise σ_ϵ and the price of a CoCo. In Fig. 2, CoCos with a regulatory trigger and CoCos with an accounting-based trigger are considered. We specifically look at a PWD CoCo ($\rho = 0$). The book value CoCo is priced by the formula given in Eq. (4.32). This value is computed using the approximation in Eq. (A.6), for which the necessary samples are obtained by using Algorithm A.3.

Figure 2 shows the importance of taking into account the trigger design for the pricing of the CoCo. The increase in accounting volatility has almost no impact on the value of the CoCo with a regulatory trigger (the solid line in Fig. 2); but the dashed line in the same figure shows that when the trigger depends on accounting reports the CoCo price is seriously (and obviously negatively) affected by accounting noise. This is in line with the results of Duffie & Lando (2001): they find that the default probability increases when the reports become more noisy. In our CoCo setting, this means that the probability of conversion increases when σ_ϵ increases, causing the CoCo price to go down.

5.2.2. *Accounting news and correlation in the accounting report error*

Consider next the impact of the correlation coefficient κ in the accounting noise error term. In Fig. 3, we show the price response of a PWD CoCo to a bad news accounting report. The set up is as follows. After the first report ($Y_1 = \log 100$), a second report is issued: $Y_2 = \log 85$. The conversion trigger is set at $\log 80$, with a PONV trigger type. The plots show a clear and immediate price response to the arrival of the bad news. Interestingly, a clear pattern emerges if the exercise

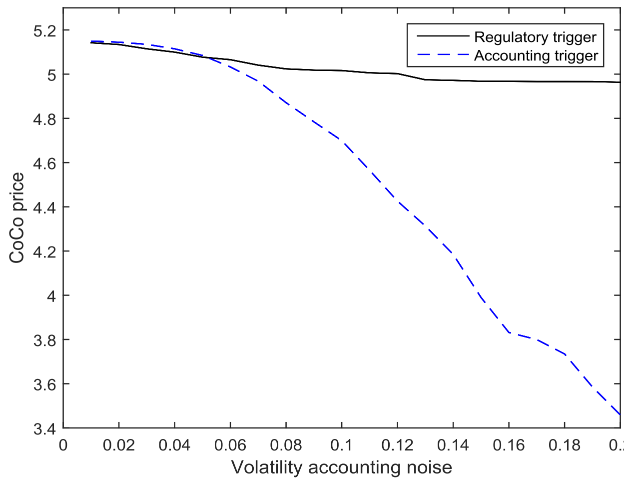


Fig. 2. How the CoCo price is affected by the volatility of accounting noise for two different trigger designs. Note that the accounting trigger CoCo is strongly affected by accounting noise, while the regulatory trigger is almost unaffected.

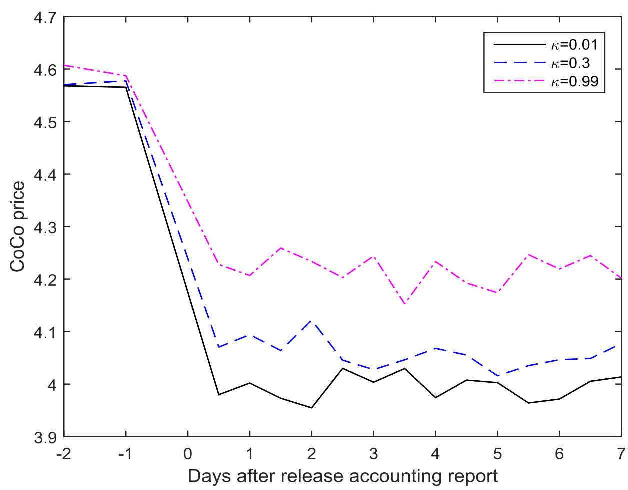


Fig. 3. Price response to "bad accounting news", for different values of the autocorrelation parameter κ. The figure shows that more autocorrelation in accounting noise mutes the price shock after a bad accounting report.

is repeated for different values of the autocorrelation parameter κ: although the pattern is similar over the entire range from almost no correlation in accounting noise ($\kappa = 0.01$) to almost complete persistence of accounting noise innovations ($\kappa = 0.99$), for higher values of the correlation parameter the price response is more muted.

Since the accounting report is known to be contaminated by accounting noise each time a new report is issued, a higher value of κ means that more of the past noise arrivals survive in the current one, while at the same time the variance of the accounting noise term U_i increases with κ, as it is, in a stationary regime, proportional to $1/(1-\kappa^2)$. This in turn lowers the information value of accounting news and explains why a bad (i.e. worse than the previous one) report leads to a smaller negative price response for higher κ : the signal is less informative so triggers a smaller price response.

5.2.3. *Time lapsed since last accounting report*

In Fig. 4, we report on a different experiment: we show how different CoCo designs are influenced by time lapsed since the last accounting report. The plot shows the value of three differently structured CoCos, each with a different degree of shareholder dilution after conversion as a function of time lapsed since the last accounting report. The black line represents a PWD CoCo where the CoCo is written off upon conversion and no subsequent dilution of the old shareholder takes place; the other two lines represent equity converters, one with partial dilution of the old shareholder ($\rho = 0.5$), the dashed-dotted line, and one where the old shareholder is completely wiped out after conversion ($\rho = 1$), the dashed line.

The plots show very little impact on the PWD CoCo while the two equity converters decline in value as the time since the last accounting report increases. A longer time lapse does not change the asset price dynamics but leads to a higher uncertainty as to where the asset value is at the time of valuation. This is similar to moving more weights in the tails. Since bankruptcy follows conversion, a higher

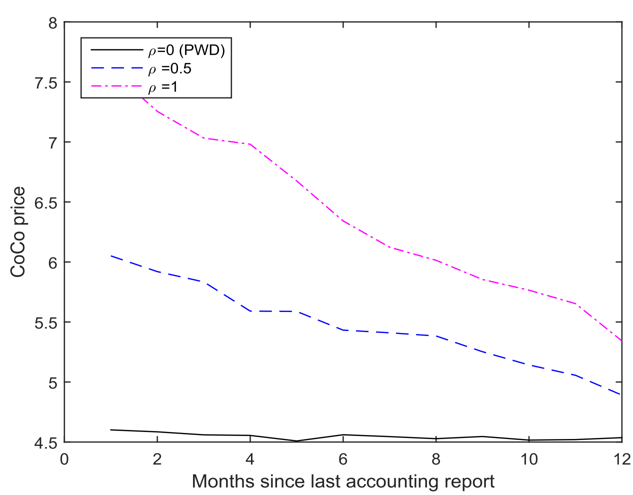


Fig. 4. How the CoCo price depends on the time since the last accounting report, for different values of the dilution ratio ρ as in Eq. (5.1).

probability of bankruptcy does not influence the PWD CoCo, their owners will then already have lost everything. But the more shares the CoCo holder receives upon conversion, the more she loses from a subsequent bankruptcy, so the price decline increases more for higher values of the dilution parameter ρ.

5.3. *Design parameters and CoCo valuation*

Consider next the impact on pricing of the main characteristics of the CoCo design: the trigger level and the number of shares received upon conversion.

5.3.1. *The conversion trigger*

In Fig. 5, the CoCo price is plotted against the conversion trigger for different degrees of dilution. The solid line corresponds to a PWD CoCo, the other lines to CoCos with varying degrees of dilution of the original shareholders upon conversion as specified in the legend.

As one would expect, the price of a PWD CoCo (the solid line) is lower for a higher conversion trigger[e]: a higher conversion trigger (i.e. a smaller required asset value decline) increases the probability of a principal write-down and its associated

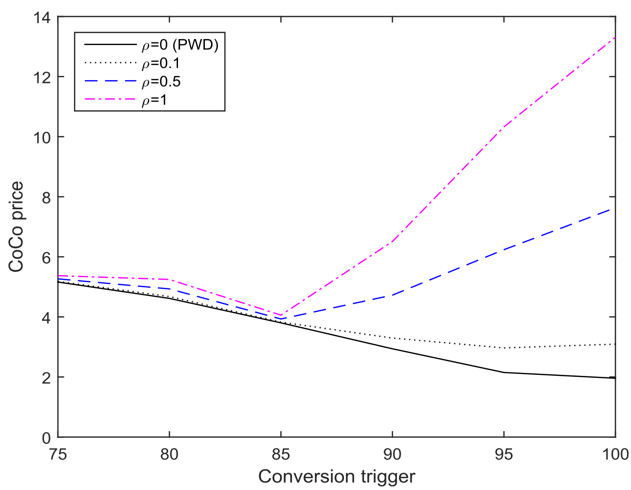


Fig. 5. How the CoCo price depends on the conversion trigger, for different values of the dilution ratio ρ as in Eq. (5.1).

[e]Note that the trigger is defined as a percentage of the asset value with the losses coming from the top (i.e. equity above debt on the liability side), so a higher trigger value means a higher probability of conversion, as is done in the rest of the academic literature. In the banking and supervision literature, it is more conventional to define the trigger value also as a percentage of (risk-weighted) assets, but with the losses coming from the bottom, with equity below debt; in that definition a higher trigger ratio leads to a lower probability of conversion.

loss of principal. However, the other lines show that if conversion terms are more favorable to the CoCo investor, the impact of the trigger level changes. Initially, getting shares after conversion barely outweighs the large loss at low trigger values, but eventually the price will increase with the conversion trigger if the dilution ratio favors the CoCo holder enough (i.e. ρ is close enough to one).

5.3.2. *The MDA trigger and the Deutsche Bank CoCo scare of February 2016*

In the literature, it is generally assumed that coupons are paid until conversion. However coupon payments are affected by the so-called Maximum Distributable Amount trigger, under which regulators stop the payment of coupons (and dividends) when the firm's capital value falls below some trigger that is higher than the conversion trigger. Coupon payments can start again when the capital value goes back up and exceeds the trigger value again. This means that in the valuation of a CoCo, we can apply Theorem 4.2 and Algorithm A.1.

To demonstrate the relevance of the inclusion of this trigger in the valuation of CoCos, consider the big price drop that the CoCos of Deutsche Bank suffered at the beginning of 2016. On January 28 Deutsche Bank reported a net loss of 2.1 Billion EUR over the last quarter of 2015. DB also reported a substantial decline in the value of its Risk-Weighted Assets, to 397 Billion EUR, down from 408 Billion EUR in the previous accounting report. As a consequence DB'sCET1 ratio fell from 11.5% to 11.1%, primarily reflecting the net loss over the quarter (Deutsche Bank 2016).

At this time, Deutsche Bank had four different CoCos outstanding:two in USD, one in EUR, one in GBP, all PWD CoCos. To avoid having to deal with an additional exchange rate risk factor, we will only consider the EUR CoCo. This CoCo's write-down is triggered when the CET1-ratio hits the level of 5.125% and it pays a coupon of 6%. As is clear from the above, the CET1-ratio did not even come close to the low trigger level. Still, the CoCo price tumbled 19.5% within the week after the announcement of the report. Market publications at the time widely argued that this happened out of fear for reaching the MDA trigger and the subsequent canceling of coupon payments.

The model developed in this paper is particularly relevant to analyze this case, as we can include the announcement of a bad accounting report in the valuation, as well as the possibility of early canceling of coupons when the MDA trigger is hit. The precise value of the MDA trigger is not publicly known, so it is not possible to use the actual value of the MDA trigger, but we can examine how much of a price drop the model can explain by taking the MDA trigger close to the values reported in the press (Kiewiet *et al.* 2017).

To estimate the parameters of DB's asset price process, we collect a year of daily stock price data (February 2015–February 2016), from which we obtain the relevant parameters of the stock price process. Then we use the Merton credit risk model to transform the drift and volatility of the equity prices into drift and volatility of the asset price process, which are our μ and σ. We take the overnight interbank rate (EONIA) at the time as risk-free rate r, see https://www.emmi-benchmarks.eu/euribor-eonia-org/eonia-rates.html. Before the bad accounting report arrives, we assume that the previous accounting report listed as asset value $Y_{t_1} = \text{EUR}\,408\,\text{bn}$. Then the new accounting report arrives, so we now have two accounting reports with values $Y_{t_1} = \text{EUR}\,408\,\text{bn}$ and $Y_{t_2} = \text{EUR}\,397\,\text{bn}$. The triggers are chosen such that they are consistent with the CET1 ratios at the moment of the accounting reports. That is, we choose v_c such that it corresponds to a CET1 ratio of 5.125%. We know the CET1 ratio is 11.1% where RWA is EUR 397 bn, so the total amount of debt (only CoCos and straight debt in the model) is EUR 397 bn $\times$ 0.889 = EUR 352.93 bn. So a CET1 ratio of 5.125% would then correspond to a RWA value of EUR 352.93/(1-0.05125) = EUR 372 bn, which is thus the value of the conversion trigger v_c. The value of the MDA trigger v_{cc} can be chosen in the same way, a MDA trigger at a CET1 ratio of 10% would correspond to a RWA value of EUR 352.93/(1-0.1) bn = EUR 392 bn. The coupon of the CoCo is $c_2 = 0.06$. As the relevant CoCo has a perpetual maturity, we take the maturity $T \to \infty$.

In Fig. 6, the price change after the announcement of a bad accounting report is illustrated for different choices of the MDA trigger. The solid line corresponds to the case where the MDA-trigger is not included in the model, in this case only a drop of 10.6% in the CoCo price occurs, when looking at the price just before the release of the accounting report and afterwards. However, if we add the MDA trigger to the model, a stronger negative price change follows. If we take the MDA trigger to be 10%, the price drops by 12.3%, which is illustrated by the dashed line. The dashed-dotted line corresponds to the case that we take the MDA trigger at 11%, i.e. just

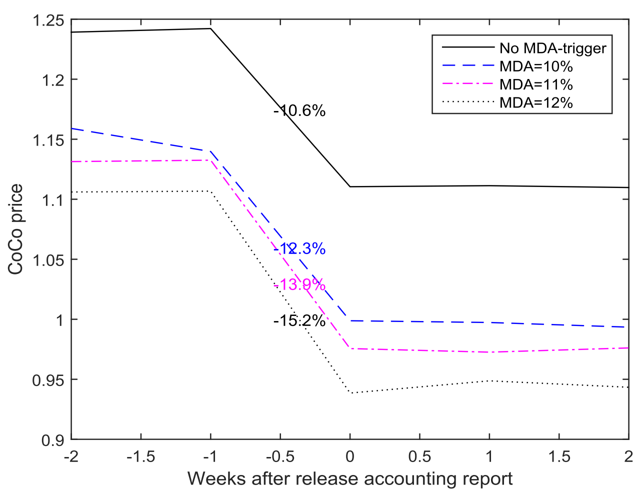


Fig. 6. CoCo price response after the release of the bad accounting report for different values of the MDA trigger.

beneath the reported CET1 value. This gives a price drop of 13.9%. If we set the MDA trigger at 12% (above the reported CET1 value and thus corresponding to a situation in which the MDA trigger was already breached) we see a price drop of 15.2% (see the dotted line). A price drop of 19.5% cannot be fully explained, but it is clear that a significant part of the price change is driven by the MDA trigger, not by the conversion trigger (a difference in price drop of 5 percent points comes out for reasonable levels of the MDA trigger). The above illustrates the added value of explicitly incorporating accounting reports into the analysis and also of taking the MDA trigger into account in the valuation of a CoCo, especially when the MDA trigger is coming close, but the conversion trigger is still far away.

5.4. *Dilution, leverage and asset volatility*

In Fig. 7, the price of a CoCo is plotted against ρ, the fraction of the total number of shares received at conversion per unit of principal, for different values of straight debt in the firm's capital structure. The case $\rho = 0$ corresponds to a principal write-down CoCo, while $\rho = 1$ corresponds to the case in which all of the original shareholders are wiped out at conversion and the CoCo investors are then the only shareholders left. Figure 7 clearly shows that the CoCo price increases with ρ. This is of course as expected, as a higher ρ means a higher payout at conversion. Furthermore, the figure shows that a CoCo with a conversion into shares has a higher price when there is a lower amount of straight debt issued. Hence, the CoCo is more valuable when the firm has a lower leverage. This can also easily be explained, as the CoCo investors receive a fraction of the firm's equity value at conversion and the equity value is higher in case there are less liabilities.

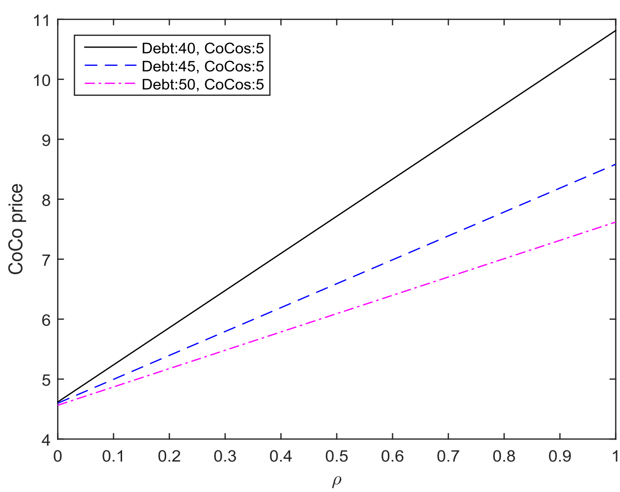


Fig. 7. How the CoCo price depends on the dilution ratio ρ as in Eq. (5.1), for different leverage ratios.

The lines for different leverages converge to the same point on the (left) vertical axis as $\rho \to 0$; for a PWD CoCo, leverage has no impact on the price since both the CoCo and equity are junior to debt. This result does depend on the assumption that the variance of the asset value process is exogenously chosen; if it would be endogenously chosen, higher leverage would lead to more risk taking and a higher variance, which would have an impact on the value of the CoCo even if it has a PWD structure (Chan & van Wijnbergen 2017).

This latter point becomes clear when we look at the price impact of changes in volatility of the underlying asset value process for different values of the dilution parameter ρ. In Fig. 8, several CoCo prices are plotted against the volatility of assets σ, see Eq. (4.1). The solid line corresponds to a PWD CoCo. Clearly, the price of a PWD CoCo decreases over the whole range considered when assets become more volatile. This is of course as one would expect, as a higher σ increases the probability of the principal write-down happening, causing the CoCo price to decrease. The dashed line, corresponding to $\rho = 0.5$, shows already that this negative effect from volatility on the CoCo price is weaker when terms of conversion are more favorable to the CoCo investor in that her loss is lower, at least some shares are received after conversion, although not yet enough to compensate for the loss of principal. In the extreme case that old shareholders are completely wiped out at conversion, corresponding to the dashed-dotted line, this negative effect is even partially reversed. In this case, the price first increases with volatility as the (now favorable) conversion becomes more likely. However, for higher volatility levels the increasing probability of default and associated costs of bankruptcy push the price down again.

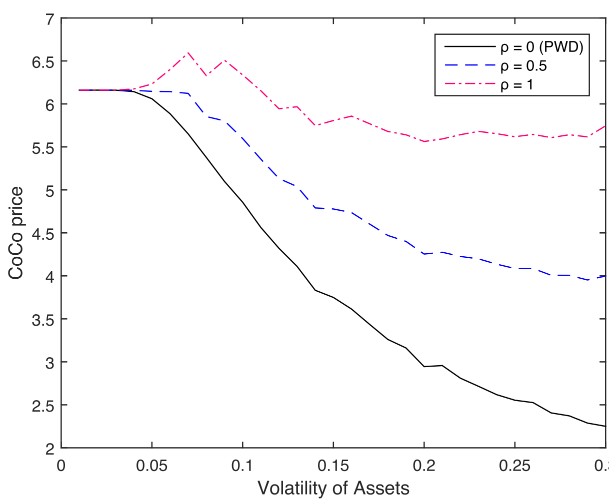


Fig. 8. How the CoCo price depends on asset volatility σ for different values of the dilution ratio ρ as in Eq. (5.1).

6. Capital Structure, Debt Overhang and Risk Taking Incentives

We have extensively discussed the impact of various features of CoCo design on CoCo pricing, and the interaction between CoCo structure and the issuing bank's risk-taking behavior. We now turn to the next question, why did banks issue CoCos to begin with? This question has not yet been addressed much, if at all, in the literature, nor why banks chose PWD CoCos to such a large extent. The practical experience with CoCo issuance over the past ten years or so does offer some clues as to the answer to both questions. CoCos were first proposed by Flannery (2005) as an ingenious way for banks to take on more leverage without increasing insolvency risk. Reverse Convertible Debentures, as CoCos were initially called, would allow market discipline without increasing the risk of incurring the social costs of default by automatically arranging for recapitalization in times of distress. Despite their apparent appeal, the issuance of CoCos did not really take off until the Lehman crisis triggered widespread debt overhang in commercial banks and banks were faced with substantially higher capital requirements in response.

Moreover, after the onset of the GFC in 2008/2009 the European banks were forced to not only increase their capital ratio, but their new regulator, the ECB, forced them to increase their capital ratio in a shorter time period than initially anticipated. The initial decision to require higher ratios for compliance with Basel III gave time until 2019, but in 2010 the ECB announced that banks would be excluded from the new ECB-based Single Supervisory Mechanism, the SSM, if they would not comply with the Basel III capital requirements at the starting date of the SSM, September 2014. Issuing of CoCos accelerated after that and have tapered off since banks reached those targets by the end of 2014, only to accelerate again when capital requirements were raised once more by the introduction of *gone concern*

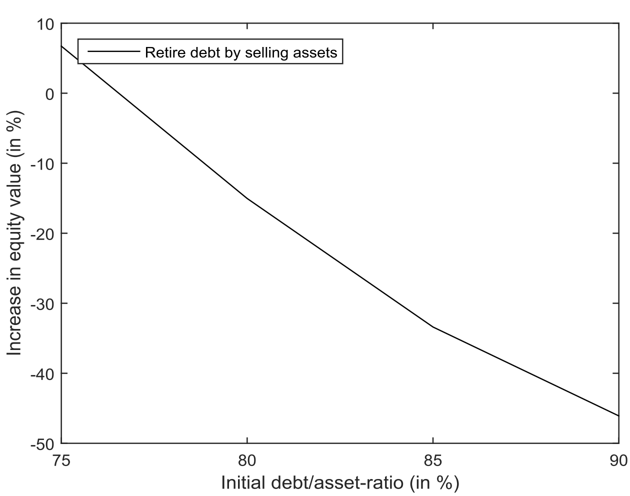


Fig. 9. Debt overhang and asset sales. The figure shows, as a function of initial leverage, the change in equity value when a bank sells assets and uses the proceeds to retire debt.

TLAC requirements. TLAC stands for Total Loss Absorption Capacity and refers to financial instruments that should be available during resolution to absorb losses and while the resolution process is ongoing.

This discussion leads to a possible answer to the capital structure question by placing it in the context of debt overhang, but that view immediately triggers two related follow-up questions. In the aftermath of the GFC regulators raised capital requirements while allowing CoCos to be used to meet those new targets; but did the regulators in this way achieve their higher level goal of lowering risk taking incentives for the banks they supervised? And are the negative investment incentives associated with debt overhang be likely to have disappeared once the higher capital requirements are implemented? Why banks issue CoCos is addressed in Sec. 6.1. In Sec. 6.2, we discuss whether replacing debt by CoCos reduce risk taking incentives and in Sec. 6.3 we discuss whether a CoCo-for-Debt swap solves the underinvestment problem caused by debt overhang.

6.1. *Debt overhang and capital structure decisions: Why do banks issue CoCos?*

Banks can raise their capital ratio by adjusting their asset portfolio or, if they choose not to do that, by operating on the liability side of their balance sheet. Consider the asset side option first: what are the consequences for equity holders when banks sell assets to reduce debt?[f] Figure 9 shows, as a function of initial leverage, the change in equity value when a bank sells assets and uses the proceeds to retire debt.

The diagram shows that the bank's equity *value* actually declines when the equity *ratio* is increased by selling assets and using the proceeds to retire debt. While this asset sale does raise the equity ratio, equity holders lose out and more so the higher the initial leverage. This is in line with Merton (1974)'s well-known credit risk approach: because of limited liability, higher leverage implies a larger put option written by (old) debt holders to the equity holders and thus higher losses when that put is diluted or reduced by what in effect comes down to a higher strike price (lower debt-to-assets ratio). There is also an offsetting effect, which we might label the Leland (1994) effect: for lower debt ratios we also get a lower insolvency ratio v_b. For very low levels of debt, the Leland effect dominates (see the upper left corner of the diagram), but as initial debt ratios increase, the Merton put effect takes over and asset sales to reduce debt lead to lower equity value.

So asset sales are an unattractive way to raise capital ratios.[g] Therefore, we next analyze the alternative to asset side restructuring: leaving assets untouched but issuing either a CoCo or new equity to retire debt and raising the capital ratio in that way. In Fig. 10, we once again show the resulting change in equity values after a debt swap and for the same range of initial leverage positions, but now in

[f] For our purposes, it does not matter whether the proceeds of asset sales are kept as liquid assets or used to reduce debt, what matters is net debt.
[g] Asset sales may be a less preferred option for other reasons too, like fears of setting off a fire sale.

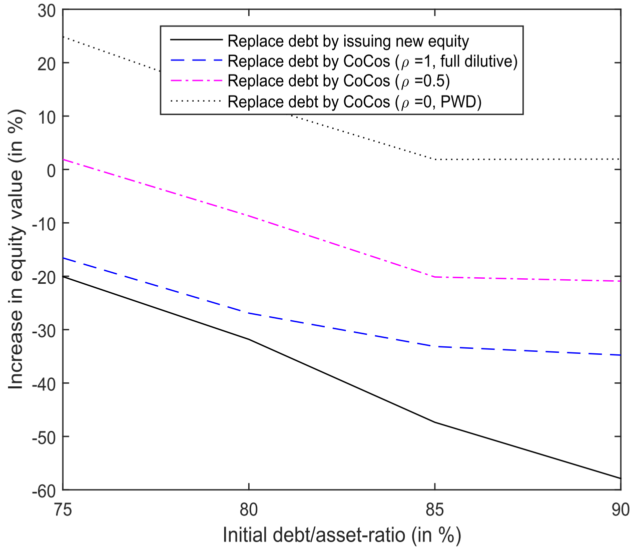


Fig. 10. Debt overhang and the choice between equity and CoCos. The figure shows the resulting change in equity value after a debt swap, as a function of initial leverage, in response to either issuing CoCos (of various degrees of dilution) or straight equity to increase capital ratios.

response to either issuing CoCos or straight equity to increase capital ratios. Each line represents a different means of doing so. Along the solid line debt is replaced by issuing new equity; along the other three lines, debt is being replaced by CoCos of different degrees of dilution, i.e. for full dilution ($\rho = 1$), partial dilution ($\rho = 0.5$) and no dilution at all ($\rho = 0$, i.e. a PWD CoCo is used).

Figure 10 shows three clear results. First and most importantly, raising the equity ratio by issuing new equity is much more damaging to an equity holder than doing so by issuing a new CoCo of any structure. This is also our first capital structure result: when faced with a call to raise equity ratios in whichever way they want, like European banks were after Lehman, banks will choose neither asset sales nor new equity to reduce debt, but will go for CoCos instead to the extent possible.[h] This explains the explosion of CoCos after the ECB required banks to accelerate their Basel-III implementation by giving them until the start of the SSM September 2014 instead of the original deadline of 2019 to comply with the new Basel-III standards.

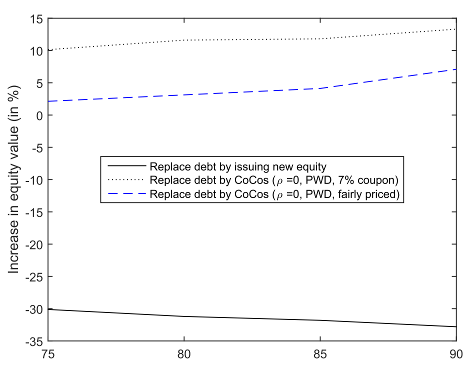
Second, the pattern of results displayed in Fig. 10 clearly supports our linking the explanation to dilution of the Merton credit risk put option: the more dilutive the CoCo the larger the loss/smaller the gain to the equity holder of meeting the leverage target by issuing CoCos. This in turn leads to a second capital structure result: our pricing model predicts that banks choosing the CoCo route will opt for

[h]There is a regulatory limit: CoCos cannot constitute more than a quarter of the total T1 capital.

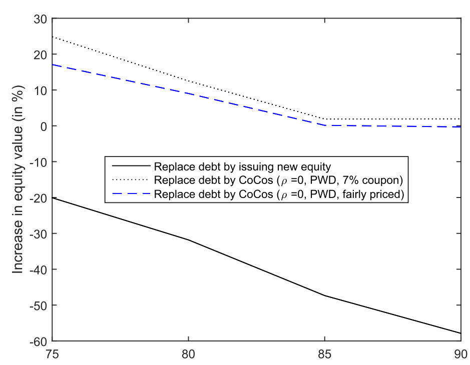
nondilutive CoCos, in extremis for PWD CoCos. The loss associated with raising the equity ratio and so reducing the Merton put received from the old debtholders transforms into a gain only if the CoCo dilution parameter is high enough. And, the third result speaking from Fig. 10, the dilution parameter needs to be higher for this sign change the higher the initial debt/asset ratio.

Consider next the issue of mispricing CoCos. To what extent are the capital structure results summarized in Figs. 9 and 10 due to the CoCo being mispriced? CoCos are complex and novel instruments, and mispricing may very well have occurred, certainly in the early days of the CoCo emission wave. However, none of the results of Sec. 5 depends on whether the CoCo was initially mispriced or not, since there we investigate the consequences of having issued CoCOs in the past. In the experiments of Sec. 5, the pricing of the CoCos is as it were a sunk cost. But in this section we explicitly compare the issue of new CoCos and equity, and there the pricing of course does matter. Is issuing a CoCo more attractive than issuing new equity simply because investors do not anticipate or misprice the risks embedded in this rather complicated debt-like instrument? To answer this question we rerun the same calculations but now making sure that the CoCo is fairly priced.

The results are presented in Fig. 11 and should be compared to those reported in Fig. 10. The outcome is clear: in Fig. 10, the size of the results was indeed exaggerated by mispricing the CoCo, but not the pattern. Issuing a PWD CoCo in a situation of debt overhang is still much preferred over issuing new equity, and the difference increases with the degree of indebtedness, suggesting once again that debt overhang and the associated put option conferred to equity holders by limited liability are the driving forces behind this result. The resulting difference is smaller



(a) z_b fixed on 65.



(b) z_b anchored to P_1 ($z_b = P_1 - 5$).

Fig. 11. Debt overhang and the choice between equity and fairly priced PWD CoCos. The figures show the resulting change in equity value after a debt swap, as a function of initial leverage, in response to either issuing PWD CoCos or straight equity to increase capital ratios. In the left panel, the default trigger z_b is fixed, while in the right panel it is adjusted to the new lower debt level.

than in the previous set of runs because we now eliminate any gain from mispricing the CoCo, but the sign and the dependency on the degree of initial indebtedness is not affected. Clearly our capital structure results do not depend on mispricing the CoCos being used.

Comparison of the two subfigures in Fig. 11 yields another interesting insight. In the left panel, the default trigger ratio z_b (or v_b) is not adjusted to the debt repurchase and the debt ratio that results from that repurchase. In the panel on the right, the bankruptcy trigger is reoptimized and adjusted to the new lower debt level. Anchoring the default value to the (reduced) debt levels makes the Merton put much smaller when debt is reduced; as a consequence the loss for the equity holder when debt is repurchased using new equity is much larger for higher initial debt-to-asset ratios.

So, to summarize our capital structure results: overleveraged banks when faced with a call to raise their equity ratios will (A) choose CoCos to retire debt rather than go for asset sales or new equity, and (B) they will go for nondilutive CoCos as much as possible. This is exactly what happened in practice (Fatouh*et al.* 2020). The reason is that issuing sufficiently dilutive new CoCos to retire debt, even when the CoCos are fairly priced, more than offsets the loss of equity value that results when the debt ratio is reduced and the value of the Merton Put option implied by limited liability goes down as a consequence. All this leads unavoidably to two follow-up questions. First, since regulators insisted on higher capital ratios to reduce risk taking, do risk taking incentives really decline when debt is swapped out for newly issued nondilutive CoCos? And two, since the key economic problem in a situation of debt overhang is reduced investment incentives, does a Debt-for-CoCo swap actually reduce debt overhang and the associated low investment incentives? We consider each question in turn in Secs. 6.2 and 6.3.

6.2. *Does replacing debt by CoCos reduce risk taking incentives?*

Capital requirements were raised to reduce risk taking incentives. So what happens to risk taking incentives when banks meet those higher requirements by issuing nondilutive CoCos, which is what our capital structure Sec. 6.1 suggests they will do? First consider the case in which straight debt is replaced with CoCos.[i] In Fig. 12, we show the increase in equity value (on the vertical axis; a negative number is a decline) as a consequence of replacing 5 units of straight debt with 5 units of CoCos, set off against different increasing levels of asset return volatility.

The different lines correspond to different degrees of dilution, with parameter ρ ranging from 0 to 1, from no dilution at all ($\rho = 0$) to infinite dilution ($\rho = 1$). Figure 12 shows results that should concern regulators. Issuing very dilutive CoCos ($\rho = 1$) to replace debt leads to a decline in equity value, as expected, and a loss

[i] The computation of the prices and the production of the figures is performed following the same procedures as in Sec. 5.3.

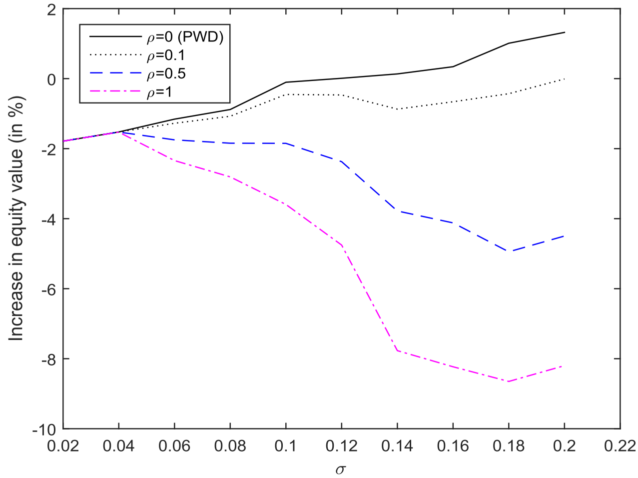


Fig. 12. Change in equity value as a function of asset return volatility when five units of debt are replaced with five units of CoCo (in market value terms), for different values of the dilution ratio ρ as in Eq. (5.1).

that increases with higher volatility. But these losses turn into gains in equity value, and gains that increase with higher asset return volatility, for a low enough dilution parameter ρ, see for example the upper two lines in Fig. 12 for, respectively, $\rho = 0.1$ and $\rho = 0$, i.e. PWD CoCos. So risk-taking incentives only decline when debt is swapped out for CoCos when they are sufficiently dilutive. It is worth mentioning here that the majority of CoCos issued so far are PWD CoCos, which are not dilutive at all.

These results are in line with the academic literature's advice to require dilutive CoCos (Calomiris & Herring 2013) when increasing capital ratios to reduce risk taking incentives; with a dilutive CoCo replacing debt banks clearly face reduced risk-taking incentives. However that is not what banks will choose when left to follow their own preferences, we just showed that banks when left to their own devices will choose nondilutive CoCos. Figure 12 shows that in that case risk-taking incentives actually worsen: the higher the asset volatility, the larger the equity gain when issuing nondilutive CoCos to meet the higher capital requirements, with as extreme the case of PWD CoCos (the solid black line in Fig. 12). When nondilutive CoCos are allowed for the swapping out of debt to increase capital ratios, equity holders gain but their risk taking incentives worsen, which presumably frustrates the purpose of the requirement to increase capital ratios to begin with.

6.3. *Does a CoCo-for-debt swap reduce debt overhang?*

If debt overhang creates incentives to replace debt by CoCos when faced with a call for higher capital ratios, it is natural to ask whether that swap actually reduces the debt overhang situation and leads to improved investment incentives. Debt overhang arises when the firm's loss absorption capacity has become too low to protect the debtholders from fluctuations in asset values (Merton 1974, Myers 1977), possibly to the point of arrears having emerged already. One consequence of debt overhang is that investment incentives are reduced for equity holders, since part of the benefits of a new project will in effect have to be shared with the creditors. Even if there are no actual arrears yet but debt is trading under par, part of the asset value increase will go into increased market value of the debt, at the (partial) expense of a higher market value for equity. In a structural model without CoCos, the shareholders then face a reduced incentive to invest. This underinvestment incentive is at its worst exactly when the firm most needs an increase in asset values, i.e. when the firm is near bankruptcy. Almost all the values of the investment will then be captured by the debt holders, as the value of debt increases when the probability of a bankruptcy is reduced. In which way, CoCos interact with debt overhang is an interesting question; CoCos introduce additional loss absorption capacity which is good for debt holders senior to the CoCo, but CoCos may also have their own impact on equity values and investment incentives: depending on the design of the CoCos, shareholders may have an increased incentive to make an investment to avoid conversion.

We can use our pricing model to look at debt overhang and the investment incentive problem by looking at what happens when assets are increased by one unit, financed through one unit of equity (issued at market value). If the total market value of equity goes up by more than one unit, the shareholders would make a profit when they invest, giving them an incentive to do so. However, when equity increases by less than one unit, the investment is apparently not sufficiently beneficial to shareholders to offset the expense, all or part of the benefits are apparently captured by debt holders. We therefore consider the case in which a new accounting report has just be released, with an asset value, see Eq. (4.3), of $Y_{t_n} = 100$; we can then examine what happens when this asset value increases by one unit. The profit of this investment of one unit is plotted against volatility in Fig. 13.

The solid black line is the benchmark case with only straight debt in addition to equity. The simulation shows the impact of debt overhang: without CoCos the shareholders do not make a profit when they invest, they actually suffer a small loss, for the entire range of volatilities on the horizontal axis. The four dashed lines represent the same experiment (one extra unit of investment financed by new equity), but part of the pre-existing equity has been replaced by CoCos of different degrees of dilution, as indicated in Fig. 13. The CoCos equal the equivalent of 5% of the asset value. These lines show that when the terms of conversion are favorable to shareholders (i.e. CoCo holders lose out upon conversion), the shareholders have even less of an incentive to engage in additional investment, actually worsening the debt overhang problem. This result confirms similar findings in Berg & Kaserer (2015, Sec. 4.2). The blue dashed line corresponds to the existence of a PWD CoCo in the capital structure of the firm and shows that the PWD CoCo indeed makes the investment incentive for shareholders more negative. So the strongest increase

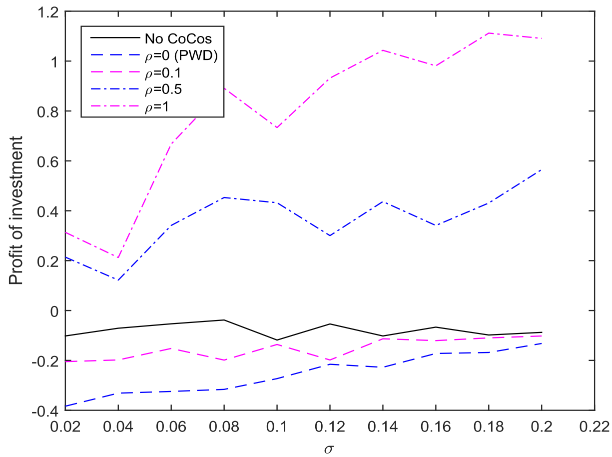


Fig. 13. Debt overhang, CoCos and investment incentives. The figure shows what happens when assets are increased by one unit, financed through one unit of equity (issued at market value). The profit of this investment is plotted against asset volatility σ, for different capital structure designs.

in debt overhang is with the CoCos that most favors shareholders, the CoCos with a principal write-down. The same happens to a somewhat lesser degree with CoCos at slightly less nondilutive terms but still favorable to shareholders. Thus, PWD or more generally insufficiently dilutive CoCos will not solve the problem of debt overhang.

However, highly dilutive CoCos do strengthen incentives for shareholders to invest because they want to avoid conversion. See in particular the dashed-dotted lines in Fig. 13, which correspond to highly dilutive CoCos; clearly such CoCos improve the investment incentives for shareholders because they wish to avoid conversion. To summarize, when terms of conversion are beneficial enough to CoCo investors instead of favoring the old shareholders, CoCos are capable of creating more of an investment incentive for the shareholders. However, PWD CoCos and in general less dilutive CoCos actually lead to lower investment incentives and worsen the debt overhang problem when compared to straight debt. From this perspective, our results from Sec. 6.1 are ominous, because apparently the very type of CoCos banks are most likely to choose do not reduce risk taking incentives and actually increase the negative impact of the initial debt overhang on investment incentives.

7. Conclusions

CoCos are debt instruments that are written down or converted into equity when the value of the issuing bank becomes too low. CoCos have taken European capital markets by storm since banks were required to increase capital ratios after the GFC. The asset pricing literature on CoCos that has rapidly developed since has almost exclusively focused on conversion triggers based on market prices. Yet, at least in the European Union, the UK and Switzerland market-based triggers disqualify the instrument as capital, so all CoCos issued so far base their conversion trigger on accounting ratios. In this paper, we have bridged the gap between the academic literature and actual practice by explicitly introducing different (although related) processes for accounting and market values and analyzing conversion triggers based on either one of the two. Also, we have extended the literature by introducing regulatory intervention and so-called MDA triggers, triggers that lead to suspension of coupon payments when breached. And finally, we have addressed arguably the most important question that has received insufficient attention in the literature, why do banks actually issue CoCos and why do they have such a revealed preference for Principal Write Down CoCos?

In order to do so we have modeled the basic stochastic process driving asset values as a standard geometric Brownian motion. Furthermore we have assumed that that process is not directly observable for market participants; information only reaches the market via noisy accounting reports which appear only at discrete moments in time. In this way, we can take into account differences between accounting and market values. The model does not lead to closed form solutions for CoCo prices, but Markov Chain Monte Carlo methods are used to compute prices. Finally regulators can inspect the true process and can force conversion if they think a Point of Non-Viability has been reached (the so-called PONV trigger).

Using the model as a tool of analysis yields a rich set of results on the relation between valuation, CoCo design, accounting noise characteristics or characteristics of the bank issuing the CoCo (asset volatility and initial leverage). The impact of higher accounting noise volatility is shown to depend on the trigger design: CoCos with only a PONV trigger are not significantly affected while a PWD CoCo with a bookvalue trigger drops precipitously after significant increases in accounting noise. The impact of serial correlation in accounting errors is interesting: bad news leads to bigger price drops for lower values of the serial correlation parameter. More serial correlation in effect lowers the information value of accounting news.

The impact of the conversion trigger is shown to depend on what happens once conversion takes place. Higher triggers (earlier conversion) lower the value of nondilutive CoCos (like in extremis Principal Write Down CoCos) but actually increase CoCo prices if dilution of old shareholders is high enough. Increases in asset volatility always decrease the value of PWD CoCos; CoCo holders are wiped out by a conversion, so shifting more weight in the tails lowers the CoCo price. For the opposite design (infinite dilution wiping out the old shareholders when converting) the response to small increases in volatility is positive as CoCo holders gain from conversion; but large increases in volatility bring a second, negative factor into play: an increase in the probability of bankruptcy. For larger increases in volatility, the second effect dominates and the price falls too, indicating the highly nonlinear structure of CoCo valuation.

We have next addressed a key question that is barely addressed in the literature at all, why do banks issue CoCos? We have taken our cue from the observation that CoCos were barely used before the GFC but took off once banks were forced to raise their capital ratios afterwards. We have done that by placing the capital structure decision in the context of a situation of debt overhang. We have shown that with debt overhang equity holders prefer CoCos over asset sales or issue of new equity when forced to raise capital ratios; and when left free to choose the type of CoCo, that they will opt for nondilutive ones, the most extreme variant of which is the Principal Write Down CoCo. This explains why most of the CoCos issued are in fact PWD CoCos.

But we have then shown also that, especially in situations of debt overhang, the insufficiently dilutive CoCos that banks will (and did) choose reduce investment incentives even more than regular debt would do and encourage more rather than less risk taking. This means that forcing banks to replace debt but allowing them to do that by issuing insufficiently dilutive CoCos actually worsens the negative incentive effects of debt overhang, encourages more rather than less risk taking and in this way would seem to defeat the purpose of the capital increase that triggered the increase in CoCo issues to begin with.

Appendix A. MCMC Algorithms for Simulating the Model

In this section, the algorithms that are necessary to compute all the derived CoCo values, are provided. The results in Sec. 4 contain three kinds of expressions, for which three different algorithms are proposed in this section.

A.1. *First type of expressions*

The first expressions we will consider are those of the form

$$\int_{z_c}^{\infty} h(x)f(t,x)\mathrm{d}x.$$

That is, integrals of a function h, weighted by the density $f(t,\cdot)$. This type of expression is needed in the valuation of a PWD CoCo with a regulatory trigger (see Theorem 4.1), when we include the MDA trigger (see Theorem 4.2), in the first part of the formula for the value of a CoCo with a conversion into shares (see Theorem 4.3) and to compute the survival probability $p_b(t_n,t)$ in Theorem 4.4.

First note that we can write

$$\begin{aligned}
&\int_{z_c}^{\infty} h(x)f(t,x)\mathrm{d}x \\
&\quad = \int_{z_c}^{\infty} h(x)\int_{z_c}^{\infty} \tilde{f}(t-t_n,x,z_n)g_{t_n}(z_n \mid Y^{(n)},\tau_c>t_n)\mathrm{d}z_n\mathrm{d}x \\
&\quad = \int_{z_c}^{\infty}\int_{(z_c,\infty)^n} h(x)\tilde{f}(t-t_n,x,z_n)g_{t_n}(z^{(n)} \mid Y^{(n)},\tau_c>t_n)\mathrm{d}z^{(n)}\mathrm{d}x \\
&\quad = \int_{(z_c,\infty)^{n+1}} h(z_{n+1})\tilde{f}(t-t_n,z_{n+1},z_n)g_{t_n}(z^{(n)} \mid Y^{(n)},\tau_c>t_n)\mathrm{d}z^{(n+1)}. \quad (\mathrm{A.1})
\end{aligned}$$

So we will need a sample $((z^{(n+1)})^1, \ldots, (z^{n+1})^G)$ from the $(n+1)$-dimensional distribution on $(z_c, \infty)^{n+1}$ with density $\tilde{f}(t - t_n, z_{n+1}, z_n) g_{t_n}(z^{(n)} \,|\, Y^{(n)}, \tau_c > t_n)$, in order to approximate this integral as

$$C(t) \approx \frac{1}{G} \sum_{g=1}^{G} h(z_{n+1}^g). \tag{A.2}$$

The algorithm used to obtain the sample, is the following MCMC-algorithm.

Algorithm A.1. (1) In each iteration g, $g = 1, \ldots, n_0 + G$, given the current value $(z^{(n+1)})^g$, the proposal $(z^{n+1})'$ is drawn according to

$$(z^{(n+1)})' = (z^{(n+1)})^g + X, \quad \text{for } X \sim \mathrm{N}_{n+1}(0, \Sigma),$$

where the $(n+1) \times (n+1)$-covariance matrix Σ is chosen to reach some desired acceptance rate.

(2) Set

$$(z^{(n+1)})^{g+1} = \begin{cases} (z^{(n+1)})' & \text{with prob. } \alpha((z^{(n+1)})^g, (z^{(n+1)})') \\ (z^{(n+1)})^g & \text{with prob. } 1 - \alpha((z^{(n+1)})^g, (z^{(n+1)})') \end{cases},$$

where the acceptance-probability $\alpha(z^{(n+1)}, (z^{(n+1)})')$ is given by

$$\begin{aligned} \alpha(z^{(n+1)}, (z^{(n+1)})') &= \min\left\{1, \frac{\tilde{f}(t - t_n, z'_{n+1}, z'_n) g_{t_n}((z^{(n)})' \,|\, y^{(n)}, \tau_c > t_n)}{\tilde{f}(t - t_n, z_{n+1}, z_n) g_{t_n}(z^{(n)} \,|\, y^{(n)}, \tau_c > t_n)}\right\} \\ &= \min\left\{1, \frac{\tilde{f}(t - t_n, z'_{n+1}, z'_n) b_n((z^{(n)})' \,|\, y^{(n)})}{\tilde{f}(t - t_n, z_{n+1}, z_n) b_n(z^{(n)} \,|\, y^{(n)})}\right\}. \end{aligned}$$

(3) Discard the draws from the first n_0 iterations and save the sample $(z^{(n+1)})^{n_0+1}, \ldots, (z^{(n+1)})^{n_0+G}$.

The acceptance probability involves the term $\frac{b_n((z^{(n)})' \,|\, y^{(n)})}{b_n(z^{(n)} \,|\, y^{(n)})}$. It follows from Eq. (4.12) that this fraction is explicitly given by

$$\frac{b_n((z^{(n)})' \,|\, y^{(n)})}{b_n(z^{(n)} \,|\, y^{(n)})} = \frac{\begin{array}{l} \prod_{i=1}^{n} \psi(z'_{i-1} - z_c, z'_i - z_c, \sigma\sqrt{t_i - t_{i-1}}) \\ \quad \times\, p_Z(z'_i \,|\, z'_{i-1}) p_U(y_i - z'_i \,|\, y_{i-1} - z'_{i-1}) \end{array}}{\begin{array}{l} \prod_{i=1}^{n} \psi(z_{i-1} - z_c, z_i - z_c, \sigma\sqrt{t_i - t_{i-1}}) \\ \quad \times\, p_Z(z_i \,|\, z_{i-1}) p_U(y_i - z_i \,|\, y_{i-1} - z_{i-1}) \end{array}},$$

under the convention that $t_0 = 0$ and $p_U(\cdot \,|\, u_0) = p_U(\cdot)$ is a Gaussian density with mean μ_ϵ and variance σ_ϵ^2. Note that $p_Z(z_i \,|\, z_{i-1})$ is a Gaussian density with mean $z_{i-1} + m(t_i - t_{i-1})$ and variance $\sigma^2(t_i - t_{i-1})$, that $p_U(u_i \,|\, u_{i-1})$ is a Gaussian density with mean $\kappa u_{i-1} + \mu_\epsilon$ and variance σ_ϵ^2 and that an expression for ψ is provided in Lemma 4.1. Algorithm A.1, in combination with Eqs. (A.2) and (A.1), allows us to compute all expressions which are of the form of an integral of a function, weighted by the density $f(t, \cdot)$.

A.2. *Second kind of expressions*

The second expression that occurs in the valuation of CoCos in Sec. 4, is the second part of the solution for the CoCo price with a regulatory trigger and a conversion into shares, see Theorem 4.3. In Eq. (4.27) we have to evaluate the double integral

$$\int_{z_c}^{\infty}\int_{z_c}^{\infty} f(t,x)\hat{f}(x,z_c,\tilde{z},T-t)h_2(\tilde{z})\mathrm{d}\tilde{z}\mathrm{d}x.$$

Note that this integral can be, similarly to the above, written as

$$\begin{aligned}
&\int_{z_c}^{\infty}\int_{(z_c,\infty)^{n+1}} h_2(\tilde{z})\hat{f}(z_{n+1},z_c,\tilde{z},T-t)\tilde{f}(t-t_n,z_{n+1},z_n)\\
&\qquad\times g_{t_n}(z^{(n)}\,|\,Y^{(n)},\tau_c>t_n)\mathrm{d}z^{(n+1)}\mathrm{d}\tilde{z}\\
&\quad=\int_{(z_c,\infty)^{n+2}} h_2(z_{n+2})\hat{f}(z_{n+1},z_c,z_{n+2},T-t)\tilde{f}(t-t_n,z_{n+1},z_n)\\
&\qquad\times g_{t_n}(z^{(n)}\,|\,Y^{(n)},\tau_c>t_n)\mathrm{d}z^{(n+2)}.
\end{aligned}\tag{A.3}$$

By definition of $\hat{f}$, it holds that

$$\begin{aligned}
&\int_{z_c}^{\infty}\hat{f}(z_{n+1},z_c,z_{n+2},T-t)\mathrm{d}z_{n+2}\\
&\quad=\int_{z_c}^{\infty}\mathbb{P}\left(\inf_{0\le s\le T-t} Z_s>z_c, Z_{T-t}\in\mathrm{d}z_{n+2}\,\middle|\, Z_0=z_{n+1}\right)\\
&\quad=\mathbb{P}\left(\inf_{0\le s\le T-t} Z_s>z_c\,\middle|\, Z_0=z_{n+1}\right)\\
&\quad=1-\pi(T-t,z_{n+1}-z_c).
\end{aligned}$$

Hence, $\hat{f}(z_{n+1},z_c,z_{n+2},T-t)\tilde{f}(t-t_n,z_{n+1},z_n)g_{t_n}(z^{(n)}\,|\,Y^{(n)},\tau_c>t_n)$ is not a density function on $(z_c,\infty)^{n+2}$, so it is not possible to proceed in the same way as in the previous case. However, by the above we know that

$$\frac{\hat{f}(z_{n+1},z_c,z_{n+2},T-t)\tilde{f}(t-t_n,z_{n+1},z_n)g_{t_n}(z^{(n)}\,|\,Y^{(n)},\tau_c>t_n)}{1-\pi(T-t,z_{n+1}-z_c)}$$

is a density function on $(z_c,\infty)^{n+2}$.

So if we have a sample $((z^{(n+2)})^1, \dots, (z^{n+2})^G)$ from the $(n+2)$-dimensional distribution with this density, we can approximate the integral in Eq. (A.3) by

$$\frac{1}{G}\sum_{g=1}^{G} h_2(z_{n+2}^g)(1-\pi(T-t, z_{n+1}^g - z_c)). \tag{A.4}$$

This sample is, in analogy to Algorithm A.1, obtained by the following MCMC-algorithm.

Algorithm A.2. (1) In each iteration g, $g = 1, \dots, n_0 + G$, given the current value $(z^{(n+2)})^g$, the proposal $(z^{n+2})'$ is drawn according to

$$(z^{(n+2)})' = (z^{(n+2)})^g + X, \quad \text{for } X \sim \mathrm{N}_{n+2}(0, \Sigma),$$

where the $(n+2)\times(n+2)$-covariance matrix Σ is chosen to reach some desired acceptance rate.

(2) Set

$$(z^{(n+2)})^{g+1} = \begin{cases} (z^{(n+2)})' & \text{with prob. } \alpha((z^{(n+2)})^g, (z^{(n+2)})') \\ (z^{(n+2)})^g & \text{with prob. } 1-\alpha((z^{(n+2)})^g, (z^{(n+2)})') \end{cases},$$

where the acceptance-probability $\alpha(z^{(n+2)}, (z^{(n+2)})')$ is given by

$$\min\left\{1, \frac{\begin{array}{c}\hat{f}(z'_{n+1}, z_c, z'_{n+2}, T-t)\tilde{f}(t-t_n, z'_{n+1}, z'_n) \\ \times\ b_n((z^{(n)})' \mid y^{(n)})(1-\pi(T-t, z_{n+1}-z_c))\end{array}}{\begin{array}{c}\hat{f}(z_{n+1}, z_c, z_{n+2}, T-t)\tilde{f}(t-t_n, z_{n+1}, z_n) \\ \times\ b_n(z^{(n)} \mid y^{(n)})(1-\pi(T-t, z'_{n+1}-z_c))\end{array}}\right\}.$$

(3) Discard the draws from the first n_0 iterations and save the sample $(z^{(n+2)})^{n_0+1}, \dots, (z^{(n+2)})^{n_0+G}$.

A.3. *Third kind of expressions*

We move to the last type of expression that occurs in the valuation of CoCos in Sec. 4, is the integral weighted by the density $B_{n,i}$, as in Theorem 4.4, which is the following integral: Equation (A.5) has similar issues as (4.32). Better is Equation (A.5) has similar issues as (4.32). Better is

$$\begin{aligned}&\int_{(y_c,\infty)^i}\int_{(z_b,\infty)^{i+1}} h(z_{n+i}) \\ &\qquad \times B_{n,i}(z^{(n,n+i)}, y^{(n+1,n+i)}|y^{(n)}, \tau_b > t_n)\mathrm{d}z^{(n,n+i)}\mathrm{d}y^{(n+1,n+i)} \\ &\quad = \int_{(y_c,\infty)^i}\int_{(z_b,\infty)^{i+n}} h(z_{n+i}) \\ &\qquad \times B_{n,i}(z^{(n+i)}, y^{(n+1,n+i)}|y^{(n)}, \tau_b > t_n)\mathrm{d}z^{(n+i)}\mathrm{d}y^{(n+1,n+i)}\end{aligned} \tag{A.5}$$

for a function h, where in the second line, in analogy to Eq. (B.25) and Lemma B.3

$$\begin{aligned}
&B_{n,i}(z^{(n+i)}, y^{(n+1,n+i)} \mid y^{(n)}, \tau_b > t_n)\mathrm{d}z^{(n+i)}\mathrm{d}y^{(n+1,n+i)} \\
&\quad := \mathbb{P}(Z^{(n+i)} \in \mathrm{d}z^{(n+i)}, Y^{(n+1,n+i)} \in \mathrm{d}y^{(n+1,n+i)}, \tau_b > t_{n+i} \mid Y^{(n)} = y^{(n)}, \tau_b > t_n) \\
&\quad = \prod_{j=0}^{i} [\psi(z_{n+j} - z_b, z_{n+j-1} - z_b, \sigma\sqrt{\delta t}) p_U(y_{n+j} - z_{n+j} \mid y_{n+j-1} \\
&\qquad - z_{n+j-1}) p_Z(z_{n+j} \mid z_{n+j-1})] \times g_{t_n}(z^{(n)} \mid y^{(n)}, \tau_b > t_n),
\end{aligned}$$

Note that this $B_{n,i}(z^{(n+i)}, y^{(n+1,n+i)} \mid y^{(n)}, \tau_b > t_n)$ is a density on $(z_b, \infty)^{i+n} \times \mathbb{R}^i$. So if we have a sample $((z^{(n+i)}, y^{(n+1,n+i)})^1, \ldots, (z^{(n+i)}, y^{(n+1,n+i)})^G)$ from the $(2i+n)$-dimensional distribution with this density, we can approximate the integral in Eq. (A.5) by

$$\frac{1}{G} \sum_{g=1}^{G} h(z_{n+i}^g) \mathbf{1}_{\{y_{n+1}^g \geq y_c, \ldots, y_{n+i}^g \geq y_c\}}. \tag{A.6}$$

The necessary sample is again obtained using a MCMC-algorithm, as follows:

Algorithm A.3. (1) In each iteration g, $g = 1, \ldots, n_0 + G$, given the current value $(z^{(n+i)}, y^{(n+1,n+i)})^g$, the proposal $(z^{(n+i)}, y^{(n+1,n+i)})'$ is drawn according to

$$(z^{(n+i)}, y^{(n+1,n+i)})' = (z^{(n+i)}, y^{(n+1,n+i)})^g + X, \quad \text{for } X \sim \mathrm{N}_{2i+n}(0, \Sigma),$$

where the $(2i+n) \times (2i+n)$-covariance matrix Σ is chosen to reach some desired acceptance rate.

(2) Set

$$\begin{aligned}
&(z^{(n+i)}, y^{(n+1,n+i)})^{g+1} \\
&\quad = \begin{cases} (z^{(n+i)}, y^{(n+1,n+i)})' & \text{w/p. } \alpha((z^{(n+i)}, y^{(n+1,n+i)})^g, \\ & \qquad (z^{(n+i)}, y^{(n+1,n+i)})') \\ (z^{(n+i)}, y^{(n+1,n+i)})^g & \text{w/p. } 1 - \alpha((z^{(n+i)}, y^{(n+1,n+i)})^g, \\ & \qquad (z^{(n+i)}, y^{(n+1,n+i)})') \end{cases},
\end{aligned}$$

where the acceptance-probability $\alpha(z^{(n)}, (z^{(n)})')$ is given by

$$\begin{aligned}
&\alpha((z^{(n+i)}, y^{(n+1,n+i)}), (z^{(n+i)}, y^{(n+1,n+i)})') \\
&\quad = \min\left\{1, \frac{B_{n,i}((z^{(n+i)}, y^{(n+1,n+i)})' \mid y^{(n)}, \tau_b > t_n)}{B_{n,i}((z^{(n+i)}, y^{(n+1,n+i)}) \mid y^{(n)}, \tau_b > t_n)}\right\}
\end{aligned}$$

(3) Discard the draws from the first n_0 iterations and save the sample $(z^{(n)})^{n_0+1}, \ldots, (z^{(n)})^{n_0+G}$.

Appendix B. Formulas and Proofs

In this section, all the mathematical details and proofs that are left out in the main text, are provided.

B.1. *Proof of Lemma* 4.2

$\tilde{f}(t, \cdot, z_0)$ is defined by

$$\mathbb{P}(Z_t \in \mathrm{d}x \,|\, \tau_b > t) = \tilde{f}(t, x, z_0)\mathrm{d}x.$$

By Bayes' rule we can write

$$\mathbb{P}(Z_t \in \mathrm{d}x \,|\, \tau_b > t) = \frac{\mathbb{P}(Z_t \in \mathrm{d}x, \tau_b > t)}{\mathbb{P}(\tau_b > t)}.$$

The denominator of this expression is given by

$$\begin{aligned}\mathbb{P}(\tau_b > t) = 1 - \pi(t, z_0 - z_b) = \Phi\left(\frac{z_0 - z_b + mt}{\sigma\sqrt{t}}\right) \\ - e^{-2m(z_0 - z_b)/\sigma^2}\Phi\left(\frac{z_b - z_0 + mt}{\sigma\sqrt{t}}\right).\end{aligned}$$

In order to compute the numerator, we will rely on the following result by Harrison (1985), which can be found in Sec. 1.8, Proposition 1. Denote by X_t a Brownian motion with drift μ, variance σ^2 and $X_0 = 0$. Furthermore define $M_t := \max\{X_s : 0 \le s \le t\}$. Then the joint distribution of X_t and M_t satisfies

$$\begin{aligned}&\mathbb{P}(X_t \in \mathrm{d}x, M_t \le y) \\ &\quad = \frac{1}{\sigma\sqrt{t}} \exp\left(\frac{\mu x}{\sigma^2} - \frac{\mu^2 t}{2\sigma^2}\right)\left(\phi\left(\frac{x}{\sigma\sqrt{t}}\right) - \phi\left(\frac{x - 2y}{\sigma\sqrt{t}}\right)\right)\mathrm{d}x,\end{aligned} \tag{B.1}$$

where ϕ denotes the standard normal density function. Now, denote $X_t = -Z_t + z_0$, which is a Brownian motion with drift $-m$, variance σ^2 and $X_0 = 0$. Furthermore, denote $M_t = \max\{X_s : 0 \le s \le t\}$. Then Eq. (B.1) implies that

$$\begin{aligned}\mathbb{P}(Z_t \in \mathrm{d}x, \tau_b > t) &= \mathbb{P}\left(Z_t \in \mathrm{d}x, \inf_{0 \le s \le t} Z_s > z_b\right) \\ &= \mathbb{P}\left(X_t \in \mathrm{d}(z_0 - x), M_t \le z_0 - z_b\right) \\ &= \frac{1}{\sigma\sqrt{t}} \exp\left(\frac{-m(z_0 - x)}{\sigma^2} - \frac{m^2 t}{2\sigma^2}\right) \\ &\quad \times \left(\phi\left(\frac{z_0 - x}{\sigma\sqrt{t}}\right) - \phi\left(\frac{-z_0 - x + 2z_b}{\sigma\sqrt{t}}\right)\right)\mathrm{d}x.\end{aligned} \tag{B.2}$$

So we conclude that

$$\tilde{f}(t,x,z_0)=\frac{1}{\sigma\sqrt{t}}\frac{\exp\left(\dfrac{-m(z_0-x)}{\sigma^2}-\dfrac{m^2t}{2\sigma^2}\right)\left(\phi\left(\dfrac{z_0-x}{\sigma\sqrt{t}}\right)-\phi\left(\dfrac{-z_0-x+2z_b}{\sigma\sqrt{t}}\right)\right)}{\Phi\left(\dfrac{z_0-z_b+mt}{\sigma\sqrt{t}}\right)-e^{-2m(z_0-z_b)/\sigma^2}\Phi\left(\dfrac{z_b-z_0+mt}{\sigma\sqrt{t}}\right)}.$$

□

B.2. *Proof of Theorem* 4.1

Recall from Eq. (4.20) that the CoCo price was written as

$$\begin{aligned}C(t)=P_2e^{-r(T-t)}p_c(t,T)+c_2P_2\int_t^T e^{-r(u-t)}p_c(t,u)\mathrm{d}u\\ -RP_2\int_t^T e^{-r(u-t)}p_c(t,\mathrm{d}u).\end{aligned}\tag{B.3}$$

The integral in this last term can be written as

$$\begin{aligned}&\int_t^T e^{-r(u-t)}p_c(t,\mathrm{d}u)\\ &\quad=\int_t^T e^{-r(u-t)}\frac{\partial}{\partial u}p_c(t,u)\mathrm{d}u\\ &\quad=\int_t^T e^{-r(u-t)}\int_{z_c}^\infty\frac{\partial}{\partial u}(1-\pi(u-t,x-z_c))f(t,x,\omega)\mathrm{d}x\mathrm{d}u\\ &\quad=\int_{z_c}^\infty f(t,x,\omega)\int_t^T e^{-r(u-t)}\frac{\partial}{\partial u}(1-\pi(u-t,x-z_c))\mathrm{d}u\mathrm{d}x\\ &\quad=\int_{z_c}^\infty f(t,x,\omega)I(x)\mathrm{d}x,\end{aligned}$$

where

$$I(x)=\int_t^T e^{-r(u-t)}\frac{\partial}{\partial u}(1-\pi(u-t,x-z_c))\mathrm{d}u.$$

Furthermore, the integral in the second term of Eq. (B.3) can be written as

$$\begin{aligned}&\int_t^T e^{-r(u-t)}p_c(t,u)\mathrm{d}u\\ &\quad=\int_t^T e^{-r(u-t)}\int_{z_c}^\infty(1-\pi(u-t,x-z_c))f(t,x,\omega)\mathrm{d}x\mathrm{d}u\\ &\quad=\int_{z_c}^\infty f(t,x,\omega)\int_t^T e^{-r(u-t)}(1-\pi(u-t,x-z_c))\mathrm{d}u\mathrm{d}x\\ &\quad=\int_{z_c}^\infty f(t,x,\omega)\tilde{I}(x)\mathrm{d}x,\end{aligned}$$

where

$$
\begin{aligned}
\tilde{I}(x) &= \int_t^T e^{-r(u-t)}(1-\pi(u-t,x-z_c))\mathrm{d}u \\
&= \left[-\frac{1}{r}e^{-r(u-t)}(1-\pi(u-t,x-z_c))\right]_{u=t}^T + \frac{1}{r}I(x) \\
&= -\frac{1}{r}e^{-r(T-t)}(1-\pi(T-t,x-z_c)) + \frac{1}{r} + \frac{1}{r}I(x).
\end{aligned}
$$

Putting the above together allows us to write the CoCo price $C(t)$ as a single integral, weighted by the density $f(t,x)$, as follows:

$$
\begin{aligned}
C(t) &= \int_{z_c}^\infty (P_2 e^{-r(T-t)}(1-\pi(T-t,x-z_c)) + c_2P_2\tilde{I}(x) - RP_2I(x))f(t,xa)\mathrm{d}x \\
&= \int_{z_c}^\infty \left(\frac{r-c_2}{r}P_2e^{-r(T-t)}(1-\pi(T-t,x-z_c))\right. \\
&\quad \left. + \frac{c_2}{r}P_2 + \left(\frac{c_2P_2}{r} - RP_2\right)I(x)\right)f(t,x)\mathrm{d}x \\
&= \int_{z_c}^\infty h(x)f(t,x)\mathrm{d}x,
\end{aligned}
$$

where

$$
\begin{aligned}
h(x) := {}& \frac{r-c_2}{r}P_2e^{-r(T-t)}(1-\pi(T-t,x-z_c)) \\
& + \frac{c_2}{r}P_2 + \left(\frac{c_2P_2}{r} - RP_2\right)I(x). \qquad \text{(B.4)}
\end{aligned}
$$

It now remains to find an analytical expression for $I(x)$. First consider

$$
\begin{aligned}
&\frac{\partial}{\partial u}(1-\pi(u-t,x-z_c)) \\
&\quad = \frac{\partial}{\partial u}\left(\Phi\left(\frac{x-z_c+m(u-t)}{\sigma\sqrt{u-t}}\right) - e^{-2m(x-z_c)/\sigma^2}\Phi\left(\frac{-(x-z_c)+m(u-t)}{\sigma\sqrt{u-t}}\right)\right) \\
&\quad = \phi\left(\frac{x-z_c+m(u-t)}{\sigma\sqrt{u-t}}\right)\left(\frac{m}{2\sigma\sqrt{u-t}} - \frac{x-z_c}{2\sigma(u-t)^{3/2}}\right) \\
&\qquad - e^{-2m(x-z_c)/\sigma^2}\phi\left(\frac{-(x-z_c)+m(u-t)}{\sigma\sqrt{u-t}}\right)\left(\frac{m}{2\sigma\sqrt{u-t}} + \frac{x-z_c}{2\sigma(u-t)^{3/2}}\right) \\
&\quad = \frac{z_c-x}{\sigma(u-t)^{3/2}}\phi\left(\frac{x-z_c+m(u-t)}{\sigma\sqrt{u-t}}\right),
\end{aligned}
$$

which implies

$$
\begin{aligned}
I(x) &= \int_t^T e^{-r(u-t)} \frac{z_c - x}{\sigma(u-t)^{3/2}} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x - z_c + m(u-t))^2}{2\sigma^2(u-t)}\right) \mathrm{d}u \\
&= \frac{z_c - x}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{m(x-z_c)}{\sigma^2}\right) \int_0^{T-t} e^{-ru} \frac{1}{u^{3/2}} \exp\left(-\frac{(x-z_c)^2}{2\sigma^2 u} - \frac{m^2 u}{2\sigma^2}\right) \mathrm{d}u \\
&= \frac{z_c - x}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{m(x-z_c)}{\sigma^2}\right) \int_0^{T-t} \frac{1}{u^{3/2}} \\
&\quad \times \exp\left(-\frac{(x-z_c)^2}{2\sigma^2}\frac{1}{u} - \left(\frac{m^2}{2\sigma^2} + r\right) u\right) \mathrm{d}u \\
&= 2\frac{z_c - x}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{m(x-z_c)}{\sigma^2}\right) \int_{(T-t)^{-1/2}}^{\infty} \exp\left(-Av^2 - B\frac{1}{v^2}\right) \mathrm{d}v, \qquad \text{(B.5)}
\end{aligned}
$$

where the last line follows by substitution of $v = u^{-1/2}$ and by setting

$$
A = \frac{(x-z_c)^2}{2\sigma^2}, \quad B = \frac{m^2}{2\sigma^2} + r.
$$

Now, by noting that $(Av^2 + B/v^2) = (\sqrt{A}v - \sqrt{B}/v)^2 + 2\sqrt{AB}$, as well as $(Av^2 + B/v^2) = (\sqrt{A}v + \sqrt{B}/v)^2 - 2\sqrt{AB}$, the remaining integral can be evaluated, by doing the substitutions $u = \sqrt{A}v - \sqrt{B}/v$ and $u = \sqrt{A}v + \sqrt{B}/v$, as follows:

$$
\begin{aligned}
&\int_{(T-t)^{-1/2}}^{\infty} \exp\left(-Av^2 - B\frac{1}{v^2}\right) \mathrm{d}v \\
&\quad = \frac{1}{2\sqrt{A}} \int_{(T-t)^{-1/2}}^{\infty} \exp\left(-\left(\sqrt{A}v - \frac{\sqrt{B}}{v}\right)^2 - 2\sqrt{AB}\right) \left(\sqrt{A} + \sqrt{B}\frac{1}{v^2}\right) \mathrm{d}v \\
&\qquad + \frac{1}{2\sqrt{A}} \int_{(T-t)^{-1/2}}^{\infty} \exp\left(-\left(\sqrt{A}v + \frac{\sqrt{B}}{v}\right)^2 + 2\sqrt{AB}\right) \left(\sqrt{A} - \sqrt{B}\frac{1}{v^2}\right) \mathrm{d}v \\
&\quad = \frac{1}{2\sqrt{A}} e^{-2\sqrt{AB}} \int_{\sqrt{A/(T-t)} - \sqrt{B(T-t)}}^{\infty} e^{-u^2} \mathrm{d}u \\
&\qquad + \frac{1}{2\sqrt{A}} e^{2\sqrt{AB}} \int_{\sqrt{A/(T-t)} + \sqrt{B(T-t)}}^{\infty} e^{-u^2} \mathrm{d}u \\
&\quad = \frac{\sqrt{\pi}}{4\sqrt{A}} \left(e^{-2\sqrt{AB}} \operatorname{erfc}\left(\sqrt{\frac{A}{(T-t)}} - \sqrt{B(T-t)}\right)\right. \\
&\qquad \left. + e^{2\sqrt{AB}} \operatorname{erfc}\left(\sqrt{\frac{A}{(T-t)}} + \sqrt{B(T-t)}\right)\right), \qquad \text{(B.6)}
\end{aligned}
$$

where $\operatorname{erfc}(x)$ is the complementary error function, which is defined by

$$\operatorname{erfc}(x) := \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-u^2} \mathrm{d}u$$

and satisfies

$$\frac{1}{2} \operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right) = 1 - \Phi(x).$$

Combining Eqs. (B.5) and (B.6) and substituting back the expressions for A and B, finally leads to the expression for $I(x)$:

$$\begin{aligned} I(x) &= 2\frac{z_c - x}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{m(x-z_c)}{\sigma^2}\right) \int_{(T-t)^{-1/2}}^{\infty} \exp\left(-Av^2 - B\frac{1}{v^2}\right) \mathrm{d}v \\ &= \exp\left(-\frac{m(x-z_c)}{\sigma^2}\right)\left(-e^{-2\sqrt{AB}}\frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{A}{(T-t)}} - \sqrt{B(T-t)}\right)\right. \\ &\quad \left. - e^{2\sqrt{AB}}\frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{A}{(T-t)}} + \sqrt{B(T-t)}\right)\right) \\ &= \exp\left(-\frac{m(x-z_c) + (x-z_c)\sqrt{m^2+2r\sigma^2}}{\sigma^2}\right) \\ &\quad \times \left(\Phi\left(\frac{x - z_c - \sqrt{m^2+2r\sigma^2}(T-t)}{\sigma\sqrt{T-t}}\right) - 1\right) \\ &\quad + \exp\left(-\frac{m(x-z_c) - (x-z_c)\sqrt{m^2+2r\sigma^2}}{\sigma^2}\right) \\ &\quad \times \left(\Phi\left(\frac{x - z_c + \sqrt{m^2+2r\sigma^2}(T-t)}{\sigma\sqrt{T-t}}\right) - 1\right). \end{aligned} \tag{B.7}$$

□

B.3. *Proof of Theorem* 4.2

Recall that the MDA-trigger lead to the need to compute the integral

$$c_2 P_2 \int_t^T e^{-r(u-t)} \mathbb{P}(\tau_c > u, Z_u > z_{cc} \,|\, Y^{(n)}, \tau_c > t) \mathrm{d}u.$$

In order to compute $\mathbb{P}(\tau_c > u, Z_u > z_{cc} \,|\, Y^{(n)}, \tau_c > t)$ we need the following well-known result: the joint distribution of a drifted Brownian motion and its running minimum (Harrison 1985, Sec. 1.8, Corollary 7).

Lemma B.1. *The joint probability* $\tilde{\pi}(t,x,y)$ *that* Z*, starting from* $x>0$*, does not hit 0 before time* t *and that* $Z_t > y$ *is given by*

$$\begin{aligned}\tilde{\pi}(t,x,y) &:= \mathbb{P}(\inf_{0\le s\le t} Z_s > 0, Z_t > y)\\ &= \Phi\left(\frac{x-y+mt}{\sigma\sqrt{t}}\right) - e^{-2mx/\sigma^2}\Phi\left(\frac{-x-y+mt}{\sigma\sqrt{t}}\right). \end{aligned} \tag{B.8}$$

Now, similarly to Eq. (4.17), we can write

$$\begin{aligned}&\mathbb{P}(\tau_c > u, Z_u > z_{cc}\,|\,Y^{(n)}, \tau_c > t)\\ &\quad = \int_{z_c}^{\infty} \tilde{\pi}(u-t, x-z_c, z_{cc}-z_c) f(t,x)\mathrm{d}x, \end{aligned} \tag{B.9}$$

The other two terms in Eq. (4.20) do not change, so the CoCo price at time $t<\tau_c$ is given by the sum of the new term in (4.23) and the unchanged part

$$Pe^{-r(T-t)}p_c(t,T) - RP\int_t^T e^{-r(u-t)}p_c(t,\mathrm{d}u).$$

By an adaption of Eq. (4.21) it is seen that this unchanged part can be written as

$$\int_{z_c}^{\infty} \tilde{h}(x) f(t,x)\mathrm{d}x,$$

where

$$\tilde{h}(x) = Pe^{-r(T-t)}(1-\pi(T-t,x-z_c)) - RPI(x), \tag{B.10}$$

in which $I(x)$ is given by Eq. (B.7). The result now follows by defining

$$I_{cc}(x) = \int_t^T e^{-r(u-t)} c_2 P_2 \tilde{\pi}(u-t, x-z_c, z_{cc}-z_c)\mathrm{d}u. \tag{B.11}$$

□

B.4. *Proof of Theorem* 4.3

Recall from Eq. (4.26) that the CoCo price was given by

$$\begin{aligned} C(t) &= \mathbb{E}(P_2 e^{-r(T-t)}\mathbf{1}_{\{\tau_c > T\}}\,|\,\mathcal{H}_t) + \mathbb{E}\left(\int_t^T c_2 P_2 e^{-r(u-t)}\mathbf{1}_{\{\tau_c>u\}}\mathrm{d}u\,|\,\mathcal{H}_t\right)\\ &\quad + \mathbb{E}\left(\frac{\Delta P_2}{\Delta P_2 + 1}E^{PC}(\tau_c)e^{-r(\tau_c - t)}\mathbf{1}_{\{\tau_c\le T\}}\,|\,\mathcal{H}_t\right). \end{aligned} \tag{B.12}$$

The first two terms together, are captured in the integral

$$\int_{z_c}^{\infty} h_0(x) f(t,x)\mathrm{d}x,$$

for

$$h_0(x) = \frac{r-c_2}{r}P_2 e^{-r(T-t)}(1-\pi(T-t,x-z_c)) + \frac{c_2P_2}{r} + \frac{c_2P_2}{r}I(x), \tag{B.13}$$

as is clear from taking $R=0$ in the PWD case, see Theorem 4.1.

To compute the last term, we note that the post-conversion equity value is given by

$$
\begin{aligned}
E^{PC}(\tau_c) &= V_{\tau_c} - \mathbb{E}\left(\int_{\tau_c}^{\infty} c_1 P_1 e^{-r(u-\tau_c)}\mathbf{1}_{\{\tau_b>u\}}\mathrm{d}u \,|\, \mathcal{H}_{\tau_c}\right) - \mathbb{E}(e^{-r(\tau_b-\tau_c)}V_{\tau_b} \,|\, \mathcal{H}_{\tau_c}) \\
&= e^{z_c} - \mathbb{E}\left(\int_{t}^{\infty} c_1 P_1 e^{-r(u-\tau_c)}\mathbf{1}_{\{\tau_c\le u,\tau_b>u\}}\mathrm{d}u \,|\, \mathcal{H}_{\tau_c}\right) \\
&\quad - e^{z_b}\mathbb{E}(e^{-r(\tau_b-\tau_c)} \,|\, \mathcal{H}_{\tau_c}).
\end{aligned}
$$

So for $\tau_c > t$, the third term in Eq. (4.26) can be written as

$$
\begin{aligned}
&\mathbb{E}\left(\frac{\Delta P_2}{\Delta P_2+1}E^{PC}(\tau_c)e^{-r(\tau_c-t)}\mathbf{1}_{\{\tau_c\le T\}} \,|\, \mathcal{H}_t\right) \\
&\quad = \frac{\Delta P_2}{\Delta P_2+1}e^{z_c}\int_t^T e^{-r(u-t)}\mathbb{P}(\tau_c\in\mathrm{d}u \,|\, \tau_c>t, Y^{(n)}) \\
&\qquad - \frac{\Delta P_2 c_1 P_1}{\Delta P_2+1}\int_t^{\infty} e^{-r(u-t)}\mathbb{P}(\tau_c\le T\wedge u, \tau_b>u \,|\, \tau_c>t, Y^{(n)})\mathrm{d}u \\
&\qquad - \frac{\Delta P_2}{\Delta P_2+1}e^{z_b}\int_t^{\infty} e^{-r(u-t)}\mathbb{P}(\tau_c\le T, \tau_b\in\mathrm{d}u \,|\, \tau_c>t, Y^{(n)}).
\end{aligned}
\tag{B.14}
$$

So in this case, the key to valuation is finding an expression for the joint conditional distribution of τ_c and τ_b, as needed in the above integrals. Note that the first integral in this equation is already computed in the proof of Theorem 4.1 and given by

$$
e^{z_c}\int_t^T e^{-r(u-t)}\mathbb{P}(\tau_c\in\mathrm{d}u \,|\, \tau_c>t, Y^{(n)}) = -e^{z_c}\int_{z_c}^{\infty} f(t,x)I(x)\mathrm{d}x, \tag{B.15}
$$

where $I(x)$ is given by Eq. (B.7).

To compute the other integrals in Eq. (B.14), it is sufficient to find expressions for

$$
\mathbb{P}(\tau_c\le T, \tau_b>u \,|\, \tau_c>t, Y^{(n)}=y^{(n)}) \quad \text{and} \quad \mathbb{P}(\tau_c\le u, \tau_b>u \,|\, \tau_c>t, Y^{(n)}=y^{(n)}).
$$

In order to find expressions for these joint probabilities, we first need the following lemma.

Lemma B.2. *The joint probability $\gamma(x,y,z,t_1,t_2)$ that Z, starting from x, does not hit z before time t_1 but does hit y before time t_2, is for $x>y>z$ given by*

$$
\begin{aligned}
&\gamma(x,y,z,t_1,t_2) \\
&\quad = \mathbb{P}\left(\inf_{0\le s\le t_1} Z_s > z, \inf_{0\le s\le t_2} Z_s \le y\right) \\
&\quad = \begin{cases} \pi(t_2, x-y) - \pi(t_1, x-z) & \textit{for } t_1\le t_2, \\ 1-\pi(t_1,x-z) - \displaystyle\int_y^{\infty}(1-\pi(t_1-t_2,\tilde{z}-z))\hat{f}(x,y,\tilde{z},t_2)\mathrm{d}\tilde{z} & \textit{for } t_1>t_2, \end{cases}
\end{aligned}
$$

where

$$\hat{f}(x,y,\tilde{z},t_2) = \frac{1}{\sigma\sqrt{t_2}}\exp\left(\frac{-m(x-\tilde{z})}{\sigma^2} - \frac{m^2 t_2}{2\sigma^2}\right) \times \left(\phi\left(\frac{x-\tilde{z}}{\sigma\sqrt{t_2}}\right) - \phi\left(\frac{-x-\tilde{z}+2y}{\sigma\sqrt{t_2}}\right)\right) \tag{B.16}$$

Proof.

- For $t_1 \leq t_2$, we can write

$$\begin{aligned}
&\mathbb{P}\left(\inf_{0\leq s\leq t_1} Z_s > z, \inf_{0\leq s\leq t_2} Z_s \leq y\right)\\
&\quad = \mathbb{P}\left(\inf_{0\leq s\leq t_2} Z_s \leq y\right) - \mathbb{P}\left(\inf_{0\leq s\leq t_1} Z_s \leq z, \inf_{0\leq s\leq t_2} Z_s \leq y\right)\\
&\quad = \mathbb{P}\left(\inf_{0\leq s\leq t_2} Z_s \leq y\right) - \mathbb{P}\left(\inf_{0\leq s\leq t_1} Z_s \leq z\right)\\
&\quad = \pi(t_2, x-y) - \pi(t_1, x-z).
\end{aligned}$$

- For $t_1 > t_2$, note that

$$\begin{aligned}
&\mathbb{P}\left(\inf_{0\leq s\leq t_1} Z_s > z, \inf_{0\leq s\leq t_2} Z_s \leq y\right)\\
&\quad = \mathbb{P}\left(\inf_{0\leq s\leq t_1} Z_s > z\right) - \mathbb{P}\left(\inf_{0\leq s\leq t_1} Z_s > z, \inf_{0\leq s\leq t_2} Z_s > y\right)\\
&\quad = 1 - \pi(t_1, x-z) - \mathbb{P}\left(\inf_{0\leq s\leq t_1} Z_s > z, \inf_{0\leq s\leq t_2} Z_s > y\right),
\end{aligned}$$

  where

$$\begin{aligned}
&\mathbb{P}\left(\inf_{0\leq s\leq t_1} Z_s > z, \inf_{0\leq s\leq t_2} Z_s > y\right)\\
&\quad = \int_y^\infty \mathbb{P}\left(\inf_{t_2\leq s\leq t_1} Z_s > z, \inf_{0\leq s\leq t_2} Z_s > y \,|\, Z_{t_2} = \tilde{z}\right)\mathbb{P}(Z_{t_2} \in \mathrm{d}\tilde{z})\\
&\quad = \int_y^\infty \mathbb{P}\left(\inf_{t_2\leq s\leq t_1} Z_s - Z_{t_2} > z-\tilde{z}\right)\mathbb{P}\left(\inf_{0\leq s\leq t_2} Z_s > y, Z_{t_2} \in \mathrm{d}\tilde{z}\right)\\
&\quad = \int_y^\infty \mathbb{P}\left(\inf_{0\leq s\leq t_1-t_2} Z_s > z-\tilde{z}+x\right)\mathbb{P}\left(\inf_{0\leq s\leq t_2} Z_s > y, Z_{t_2} \in \mathrm{d}\tilde{z}\right)\\
&\quad = \int_y^\infty (1-\pi(t_1-t_2, \tilde{z}-z))\mathbb{P}\left(\inf_{0\leq s\leq t_2} Z_s > y, Z_{t_2} \in \mathrm{d}\tilde{z}\right),
\end{aligned}$$

  where is used that Z has independent and stationary increments.

Now the result follows by noting that by a modification of Eq. (B.2) to the current setting, it holds that

$$\mathbb{P}\left(\inf_{0\leq s\leq t_2} Z_s > y, Z_{t_2} \in \mathrm{d}\tilde{z}\right) = \hat{f}(x, y, \tilde{z}, t_2)\mathrm{d}\tilde{z}. \qquad \square$$

Now the desired probabilities are, in analogy to Eq. (4.17), given by

$$\mathbb{P}(\tau_c \leq T, \tau_b > u \,|\, \tau_c > t, Y^{(n)} = y^{(n)}) = \int_{z_c}^{\infty} \gamma(x, z_c, z_b, u-t, T-t) f(t,x)\mathrm{d}x$$

and

$$\mathbb{P}(\tau_c \leq u, \tau_b > u \,|\, \tau_c > t, Y^{(n)} = y^{(n)}) = \int_{z_c}^{\infty} \gamma(x, z_c, z_b, u-t, u-t) f(t,x)\mathrm{d}x.$$

Recall that the objective was to compute the last two integrals in Eq. (B.14). Let us first consider the second one, that is

$$\begin{aligned}
&-\int_t^{\infty} e^{-r(u-t)}\mathbb{P}(\tau_c \leq T, \tau_b \in \mathrm{d}u \,|\, \tau_c > t, Y^{(n)}) \\
&\quad = \int_t^{\infty} e^{-r(u-t)} \frac{\partial}{\partial u}\mathbb{P}(\tau_c \leq T, \tau_b > u \,|\, \tau_c > t, Y^{(n)})\mathrm{d}u \\
&\quad = (\mathrm{I}) + (\mathrm{II}),
\end{aligned}$$

where

$$\begin{aligned}
(\mathrm{I}) &= \int_t^{T} e^{-r(u-t)} \frac{\partial}{\partial u}\mathbb{P}(\tau_c \leq T, \tau_b > u \,|\, \tau_c > t, Y^{(n)})\mathrm{d}u \\
&= \int_{z_c}^{\infty} f(t,x) \int_t^{T} e^{-r(u-t)} \frac{\partial}{\partial u}\gamma(x, z_c, z_b, u-t, T-t)\mathrm{d}u\mathrm{d}x \\
&= \int_{z_c}^{\infty} f(t,x) \int_t^{T} e^{-r(u-t)} \frac{\partial}{\partial u}(-\pi(u-t, x-z_b))\mathrm{d}u\mathrm{d}x \\
&= \int_{z_c}^{\infty} f(t,x) I_b(x)\mathrm{d}x,
\end{aligned}$$

in which

$$\begin{aligned}
I_b(x) &= \int_t^{T} e^{-r(u-t)} \frac{\partial}{\partial u}(-\pi(u-t, x-z_b))\mathrm{d}u \\
&= \exp\left(-\frac{m(x-z_b) + (x-z_b)\sqrt{m^2 + 2r\sigma^2}}{\sigma^2}\right) \\
&\quad \times \left(\Phi\left(\frac{x - z_b - \sqrt{m^2+2r\sigma^2}(T-t)}{\sigma\sqrt{T-t}}\right) - 1\right)
\end{aligned}$$

$$+\exp\left(-\frac{m(x-z_b)-(x-z_b)\sqrt{m^2+2r\sigma^2}}{\sigma^2}\right)$$
$$\times\left(\Phi\left(\frac{x-z_b+\sqrt{m^2+2r\sigma^2}(T-t)}{\sigma\sqrt{T-t}}\right)-1\right), \tag{B.17}$$

which follows from Eq. (B.7), by replacing z_c by z_b. Furthermore, we have

$$\begin{aligned}
(\mathrm{II}) &= \int_T^\infty e^{-r(u-t)}\frac{\partial}{\partial u}\mathbb{P}(\tau_c \le T, \tau_b > u \,|\, \tau c > t, Y^{(n)})\mathrm{d}u \\
&= \int_{z_c}^\infty f(t,x)\int_T^\infty e^{-r(u-t)}\frac{\partial}{\partial u}\gamma(x, z_c, z_b, u-t, T-t)\mathrm{d}u\mathrm{d}x \\
&= \int_{z_c}^\infty f(t,x)\int_T^\infty e^{-r(u-t)}\frac{\partial}{\partial u}(-\pi(u-t, x-z_b))\mathrm{d}u\mathrm{d}x \\
&\quad -\int_{z_c}^\infty\int_{z_c}^\infty f(t,x)\hat{f}(x, z_c, \tilde{z}, T-t) \\
&\quad \times\int_T^\infty e^{-r(u-t)}\frac{\partial}{\partial u}(1-\pi(u-T, \tilde{z}-z_b))\mathrm{d}u\mathrm{d}\tilde{z}\mathrm{d}x \\
&= \int_{z_c}^\infty f(t,x)(J_b(x)-I_b(x))\mathrm{d}x \\
&\quad -\int_{z_c}^\infty\int_{z_c}^\infty f(t,x)\hat{f}(x, z_c, \tilde{z}, T-t)e^{-r(T-t)}J_b(\tilde{z}, T)\mathrm{d}\tilde{z}\mathrm{d}x,
\end{aligned}$$

where

$$\begin{aligned}
J_b(x) &= \int_t^\infty e^{-r(u-t)}\frac{\partial}{\partial u}(1-\pi(u-t, x-z_b))\mathrm{d}u \\
&= -\exp\left(-\frac{m(x-z_b)+(x-z_b)\sqrt{m^2+2r\sigma^2}}{\sigma^2}\right),
\end{aligned} \tag{B.18}$$

where the last line follows by taking $T \to \infty$ in Eq. (B.17). This leaves us with an expression for the last integral in Eq. (B.14).

Similarly, the other integral satisfies

$$\int_t^\infty e^{-r(u-t)}\mathbb{P}(\tau_c \le T \wedge u, \tau_b > u \,|\, \tau_c > t, Y^{(n)})\mathrm{d}u = (\mathrm{III}) + (\mathrm{IV}),$$

where

$$\begin{aligned}
(\mathrm{III}) &= \int_t^T e^{-r(u-t)}\mathbb{P}(\tau_c \le u, \tau_b > u \,|\, \tau_c > t, Y^{(n)})\mathrm{d}u \\
&= \int_{z_c}^\infty f(t,x)\int_t^T e^{-r(u-t)}\gamma(x, z_c, z_b, u-t, u-t)\mathrm{d}u\mathrm{d}x
\end{aligned}$$

$$
\begin{aligned}
&= \int_{z_c}^{\infty} f(t,x) \int_t^T e^{-r(u-t)}(\pi(u-t,x-z_c) - \pi(u-t,x-z_b))\mathrm{d}u\mathrm{d}x \\
&= \int_{z_c}^{\infty} f(t,x)(\tilde{I}_b(x) - \tilde{I}(x))\mathrm{d}x
\end{aligned}
$$

in which $\tilde{I}(x)$ is defined in the proof Theorem 4.1 and $\tilde{I}_b(x)$ is equivalently defined as

$$
\begin{aligned}
\tilde{I}_b(x) &= \int_t^T e^{-r(u-t)}(1 - \pi(u-t,x-z_b))\mathrm{d}u \\
&= \left[-\frac{1}{r}e^{-r(u-t)}(1-\pi(u-t,x-z_b))\right]_{u=t}^{T} + \frac{1}{r}I_b(x) \\
&= -\frac{1}{r}e^{-r(T-t)}(1-\pi(T-t,x-z_b)) + \frac{1}{r} + \frac{1}{r}I_b(x).
\end{aligned} \tag{B.19}
$$

Furthermore, we have

$$
\begin{aligned}
(\mathrm{IV}) &= \int_T^{\infty} e^{-r(u-t)}\mathbb{P}(\tau_c \le T, \tau_b > u \,|\, \tau_c > t, Y^{(n)})\mathrm{d}u \\
&= \int_{z_c}^{\infty}\int_T^{\infty} e^{-r(u-t)}\gamma(x,z_c,z_b,u-t,T-t)\mathrm{d}u\mathrm{d}x \\
&= \int_{z_c}^{\infty} f(t,x)\int_T^{\infty} e^{-r(u-t)}(1-\pi(u-t,x-z_b))\mathrm{d}u\mathrm{d}x \\
&\quad - \int_{z_c}^{\infty}\int_{z_c}^{\infty} f(t,x)\hat{f}(x,z_c,\tilde{z},T-t) \\
&\quad \times \int_T^{\infty} e^{-r(u-t)}(1-\pi(u-T,\tilde{z}-z_b))\mathrm{d}u\mathrm{d}\tilde{z}\mathrm{d}x \\
&= \int_{z_c}^{\infty} f(t,x)(\tilde{J}_b(x) - \tilde{I}_b(x))\mathrm{d}x \\
&\quad - \int_{z_c}^{\infty}\int_{z_c}^{\infty} f(t,x)\hat{f}(x,z_c,\tilde{z},T-t)e^{-r(T-t)}\tilde{J}_b(\tilde{z})\mathrm{d}\tilde{z}\mathrm{d}x,
\end{aligned}
$$

in which

$$
\begin{aligned}
\tilde{J}_b(x) &= \int_t^{\infty} e^{-r(u-t)}(1-\pi(u-t,x-z_b))\mathrm{d}u \\
&= \left[-\frac{1}{r}(1-\pi(u-t,x-z_b)\right]_{u=t}^{\infty} + \frac{1}{r}J_b(x) \\
&= \frac{1}{r} + \frac{1}{r}J_b(x).
\end{aligned} \tag{B.20}
$$

Putting all the above together leads to an expression for the last two integrals in Eq. (B.14), given by

$$\begin{aligned}
-c_1 P_1 \int_t^{\infty} & e^{-r(u-t)} \mathbb{P}(\tau_c \leq T \wedge u, \tau_b > u \mid \tau_c > t, Y^{(n)}) \mathrm{d}u \\
&- e^{z_b} \int_t^{\infty} e^{-r(u-t)} \mathbb{P}(\tau_c \leq T, \tau_b \in \mathrm{d}u \mid \tau_c > t, Y^{(n)}) \\
&= e^{z_b}((I) + (II)) - c_1 P_1((III) + (IV)) \\
&= \int_{z_c}^{\infty} f(t,x) \left(e^{z_b} J_b(x) + c_1 P_1 \tilde{I}(x) - c_1 P_1 \tilde{J}_b(x) \right) \mathrm{d}x \\
&\quad + \int_{z_c}^{\infty} \int_{z_c}^{\infty} f(t,x) \hat{f}(x, z_c, \tilde{z}, T-t) e^{-r(T-t)} (c_1 P_1 \tilde{J}_b(\tilde{z}) - e^{z_b} J_b(\tilde{z})) \mathrm{d}\tilde{z} \mathrm{d}x.
\end{aligned} \tag{B.21}$$

Finally, by combining Eqs. (B.14), (B.15) and (B.21), it follows that the third term in Eq. (4.26), i.e.

$$\mathbb{E}\left(\frac{\Delta P_2}{\Delta P_2 + 1} E^{PC}(\tau_c) e^{-r(\tau_c - t)} \mathbf{1}_{\{\tau_c \leq T\}} \mid \mathcal{H}_t \right),$$

is given by

$$\int_{z_c}^{\infty} f(t,x) h_1(x) \mathrm{d}x + \int_{z_c}^{\infty} \int_{z_c}^{\infty} f(t,x) \hat{f}(x, z_c, \tilde{z}, T-t) h_2(\tilde{z}) \mathrm{d}\tilde{z} \mathrm{d}x, \tag{B.22}$$

where

$$h_1(x) = \frac{\Delta P_2}{\Delta P_2 + 1} \left(e^{z_b} J_b(x) + c_1 P_1 \tilde{I}(x) - c_1 P_1 \tilde{J}_b(x) - e^{z_c} I(x) \right), \tag{B.23}$$

$$h_2(\tilde{z}) = \frac{\Delta P_2}{\Delta P_2 + 1} e^{-r(T-t)} (c_1 P_1 \tilde{J}_b(\tilde{z}) - e^{z_b} J_b(\tilde{z})), \tag{B.24}$$

in which $I(x)$ is given by Eq. (B.7), $\tilde{I}(x)$ equals

$$\tilde{I}(x) = -\frac{1}{r} e^{-r(T-t)} (1 - \pi(T-t, x - z_c)) + \frac{1}{r} + \frac{1}{r} I(x),$$

$J_b(x)$ is given by

$$J_b(x) = -\exp\left(-\frac{m(x - z_b) + (x - z_b)\sqrt{m^2 + 2r\sigma^2}}{\sigma^2} \right)$$

and $\tilde{J}_b(x) = \frac{1}{r} + \frac{1}{r} J_b(x)$. □

B.5. *Proof of Theorem* 4.4

Here we outline the rather lengthy proof of Theorem 4.4 in a terse way. Recall that the CoCo's market price was given by Eq. (4.31), from which it is seen that we in fact need an expression for the following joint probability. For $t_n \le t < t_{n+1} < t_{n+i} \le u < t_{n+i+1}$, define the *i-step joint default/conversion survival probability* by

$$p_{bc}(t, t_{n+i}, u) = \mathbb{P}(\tau_c^A > t_{n+i}, \tau_b > u \,|\, Y^{(n)} = y^{(n)}, \tau_b > t), \quad i \ge 1$$

$$p_{bc}(t, t_{n+i}, u) = \mathbb{P}(\tau_b > u \,|\, Y^{(n)} = y^{(n)}, \tau_b > t), x \quad i = 0.$$

Recall from (4.10) the notations $y^{(n)} = (y_1, \ldots, y_n)$ (and similar for $z^{(n)}$) and $Z_n = Z_{t_n}$. Additionally, we introduce the notation $y^{(n,n+i)} = (y_n, \ldots, y_{n+i})$ (and similar for $z^{(n,n+i)}$) and write $t_{i+1} - t_i =: \delta t$. In order to find an expression for $p_{bc}(t, t_{n+i}, u)$, we first need an expression for the density $B_{n,i}(z^{(n,n+i)}, y^{(n+1,n+i)} \,|\, y^{(n)}, \tau_b > t_n)$, as defined by the relationship

$$\begin{aligned} &B_{n,i}(z^{(n,n+i)}, y^{(n+1,n+i)} \,|\, y^{(n)}, \tau_b > t_n) \mathrm{d}z^{(n,n+i)} \mathrm{d}y^{(n+1,n+i)} \\ &\quad = \mathbb{P}(Z^{(n,n+i)} \in \mathrm{d}z^{(n,n+i)}, Y^{(n+1,n+i)} \in \mathrm{d}y^{(n+1,n+i)}, \\ &\qquad\qquad \tau_b > t_{n+i} \,|\, Y^{(n)} = y^{(n)}, \tau_b > t_n). \end{aligned} \tag{B.25}$$

Lemma B.3. $B_{n,i}(z^{(n,n+i)}, y^{(n+1,n+i)} \,|\, y^{(n)}, \tau_b > t_n)$ *is given by*

$$\begin{aligned} &B_{n,i}(z^{(n,n+i)}, y^{(n+1,n+i)} \,|\, y^{(n)}, \tau_b > t_n) \\ &\quad = \prod_{j=0}^{i} [\psi(z_{n+j} - z_b, z_{n+j-1} - z_b, \sigma\sqrt{\delta t}) p_U(y_{n+j} - z_{n+j} \,|\, y_{n+j-1} \\ &\qquad - z_{n+j-1}) p_Z(z_{n+j} \,|\, z_{n+j-1})] \times g_{t_n}(z_n \,|\, y^{(n)}, \tau_b > t_n), \end{aligned}$$

where ψ is defined in Lemma 4.1, g_{t_n} is given by Eq. (4.14) (when making the substitution z_b for z_c) and p_U, p_Z are the transition densities corresponding to the Markov processes $(U_n)_{n\in\mathbb{N}}$ and $(Z_n)_{n\in\mathbb{N}}$.

Proof. In analogy to the computation of b_n in Eq. (4.12), we can write Three times incorrect splitting in (B.26). Better would be

$$\begin{aligned} &B_{n,i}(z^{(n,n+i)}, y^{(n+1,n+i)} | y^{(n)}, \tau_b > t_n) \mathrm{d}z^{(n,n+i)} \mathrm{d}y^{(n+1,n+i)} \\ &= \mathbb{P}(Z^{(n,n+i)} \in \mathrm{d}z^{(n,n+i)}, Y^{(n+1,n+i)} \in \mathrm{d}y^{(n+1,n+i)}, \tau_b > t_{n+i} | \\ &\qquad Y^{(n)} = y^{(n)}, \tau_b > t_n) \\ &= \mathbb{P}(Y_{n+i} \in \mathrm{d}y_{n+i}, Z_{n+i} \in \mathrm{d}z_{n+i}, \tau_b > t_{n+i} | \\ &\qquad Y^{(n+i-1)} = y^{(n+i-1)}, Z^{(n,n+i-1)} = z^{(n,n+i-1)}, \tau_b > t_{n+i-1}) \\ &\qquad \times B_{n,i-1}(z^{(n,n+i-1)}, y^{(n,n+i-1)} | y^{(n)}, \tau_b > t_n) \mathrm{d}z^{(n,n+i-1)} \mathrm{d}y^{(n+1,n+i-1)}. \end{aligned} \tag{B.26}$$

Now the result follows by noting that

$$\begin{aligned}&\mathbb{P}(Y_{n+i} \in \mathrm{d}y_{n+i}, Z_{n+i} \in \mathrm{d}z_{n+i}, \tau_b > t_{n+i} | \\ &\quad Y^{(n+i-1)} = y^{(n+i-1)}, Z^{(n,n+i-1)} = z^{(n,n+i-1)}, \tau_b > t_{n+i-1}) \\ &= \psi(z_{n+i} - z_b, z_{n+i-1} - z_b, \sigma\sqrt{\delta t}) p_U(y_{n+i} - z_{n+i} | y_{n+i-1} - z_{n+i-1}) p_Z(z_{n+i} | z_{n+i-1}) \mathrm{d}y_{n+i} \mathrm{d}z_{n+i}\end{aligned}$$

and

$$B_{n,0}(z_n \,|\, y^{(n)}, \tau_b > t_n) = g_{t_n}(z_n \,|\, y^{(n)}, \tau_b > t_n). \qquad \square$$

By definition of $B_{n,i}$, we see that the i-step joint default/conversion survival probability for $t = t_n$ and $u = t_{n+i}$ is given by

$$\begin{aligned}p_{bc}(t_n, t_{n+i}, t_{n+i}) = \int_{(y_c,\infty)^i} \int_{(z_b,\infty)^{i+1}} & B_{n,i}(z^{(n,n+i)}, \\ & y^{(n+1,n+i)} \,|\, y^{(n)}, \tau_b > t_n) \mathrm{d}z^{(n,n+i)} \mathrm{d}y^{(n+1,n+i)}.\end{aligned}$$

By stationarity of Z, the i-step joint default/conversion survival probability for $t = t_n$ and general $u \geq t_{n+i}$ then equals, see Lemma 4.3 for an explicit expression for $\pi(u - t_{n+i}, z_{n+i} - z_b)$,

$$\begin{aligned}p_{bc}(t_n, t_{n+i}, u) = \int_{(y_c,\infty)^i} \int_{(z_b,\infty)^{i+1}} & (1 - \pi(u - t_{n+i}, z_{n+i} - z_b)) B_{n,i}(z^{(n,n+i)}, \\ & \times y^{(n+1,n+i)} \,|\, y^{(n)}, \tau_b > t_n) \mathrm{d}z^{(n,n+i)} \mathrm{d}y^{(n+1,n+i)}. \qquad \text{(B.27)}\end{aligned}$$

Finally, we can now also compute $p(t, t_{n+i}, u)$ for general $t_n \leq t < t_{n+1}$, since by Bayes' rule

$$p_{bc}(t, t_{n+i}, u) = \frac{p_{bc}(t_n, t_{n+i}, u)}{p_b(t_n, t)}, \qquad \text{(B.28)}$$

where $p_b(t, s) = \mathbb{P}(\tau_b > s \,|\, Y^{(n)} = y^{(n)}, \tau_b > t)$ is the default survival probability, as given by Eq. (4.17) (making the substitution z_b for z_c).

For $t_n \leq t < t_{n+1}$, $T = t_{n+m}$ for some $m \in \mathbb{N}$ and $Y^{(n)} = y^{(n)}$, where $y_i > y_c, 1 \leq i \leq n$, the CoCo price can now be written as

$$\begin{aligned}C(t) &= P_2 e^{-r(T-t)} p_{bc}(t, t_{n+m}, t_{n+m}) \\ &\quad + \int_t^T c_2 P_2 e^{-r(u-t)} \mathbb{P}(\tau_c^A > u, \tau_b > u \,|\, Y^{(n)} = y^{(n)}, \tau_b > t) \mathrm{d}u \\ &\quad + R P_2 \sum_{i=1}^m e^{-r(t_{n+i}-t)} \mathbb{P}(\tau_c^A = t_{n+i}, \tau_b > t_{n+i} \,|\, Y^{(n)} = y^{(n)}, \tau_b > t) \\ &= P_2 e^{-r(T-t)} p_{bc}(t, t_{n+m}, t_{n+m})\end{aligned}$$

$$
\begin{aligned}
&+ c_2 P_2 \sum_{i=0}^{m-1} \int_{\min(t,t_{n+i})}^{t_{n+i+1}} e^{-r(u-t)} p_{bc}(t, t_{n+i}, u) \mathrm{d}u \\
&+ RP_2 \sum_{i=1}^{m} e^{-r(t_{n+i}-t)} (p_{bc}(t, t_{n+i-1}, t_{n+i}) - p_{bc}(t, t_{n+i}, t_{n+i})) \\
= {}& (1-R) P_2 e^{-r(T-t)} p_{bc}(t, t_{n+m}, t_{n+m}) \\
&+ c_2 P_2 \sum_{i=0}^{m-1} \int_{\min(t,t_{n+i})}^{t_{n+i+1}} e^{-r(u-t)} p_{bc}(t, t_{n+i}, u) \mathrm{d}u \\
&+ RP_2 \sum_{i=0}^{m-1} e^{-r(t_{n+i+1}-t)} p_{bc}(t, t_{n+i}, t_{n+i+1}) \\
&- RP_2 \sum_{i=1}^{m-1} e^{-r(t_{n+i}-t)} p_{bc}(t, t_{n+i}, t_{n+i}).
\end{aligned}
$$

So the only quantities left to compute are the integrals over time in the coupon payment term. To this end, we first consider the case for $i \geq 1$. Use (B.28) to get

$$
\int_{t_{n+i}}^{t_{n+i+1}} e^{-r(u-t)} p_{bc}(t, t_{n+i}, u) \mathrm{d}u = \frac{1}{p_b(t_n, t)} \int_{t_{n+i}}^{t_{n+i+1}} e^{-r(u-t)} p_{bc}(t_n, t_{n+i}, u) \mathrm{d}u
$$

and $p_{bc}(t_n, t_{n+i}, u)$ depends only on u through the function $(1 - \pi(u - t_{n+i}, z_{n+i} - z_b))$ (see Eq. (B.27)), hence

$$
\begin{aligned}
&\int_{t_{n+i}}^{t_{n+i+1}} e^{-r(u-t)} p_{bc}(t_n, t_{n+i}, u) \mathrm{d}u \\
&= \int\limits_{(y_c,\infty)^i} \int\limits_{(z_b,\infty)^{i+1}} \tilde{I}_b(t_{n+i}, t_{n+i+1}, z_{n+i}) \\
&\qquad \times B_{n,i}(z^{(n,n+i)}, y^{(n+1,n+i)} | y^{(n)}, \tau_b > t_n) \mathrm{d}z^{(n,n+i)} \mathrm{d}y^{(n+1,n+i)},
\end{aligned}
$$

where, in analogy to Eq. (B.19)

$$
\begin{aligned}
&\tilde{I}_b(t_{n+i}, t_{n+i+1}, z_{n+i}) \\
&\quad = e^{-r(t_{n+i}-t)} \int_{t_{n+i}}^{t_{n+i+1}} e^{-r(u-t_{n+i})} (1 - \pi(u - t_{n+i}, z_{n+i} - z_b)) \mathrm{d}u \\
&\quad = e^{-r(t_{n+i}-t)} \left(-\frac{1}{r} e^{-r\delta t} (1 - \pi(\delta t, z_{n+i} - z_b)) + \frac{1}{r} + \frac{1}{r} I_b(t_{n+i}, t_{n+i+1}, z_{n+i}) \right),
\end{aligned}
$$

in which (compare to Eq. (B.17))

$$
\begin{aligned}
&I_b(t_{n+i}, t_{n+i+1}, z_{n+i}) \\
&\qquad = \int_{t_{n+i}}^{t_{n+i+1}} e^{-r(u-t_{n+i})} \frac{\partial}{\partial u} (-\pi(u - t_{n+i}, z_{n+i} - z_b)) \mathrm{d}u
\end{aligned}
$$

$$
\begin{aligned}
&= \exp\left(-\frac{m(z_{n+i}-z_b)+(z_{n+i}-z_b)\sqrt{m^2+2r\sigma^2}}{\sigma^2}\right)\\
&\quad \times \left(\Phi\left(\frac{z_{n+i}-z_b-\delta t\sqrt{m^2+2r\sigma^2}}{\sigma\sqrt{\delta t}}\right)-1\right)\\
&\quad + \exp\left(-\frac{m(z_{n+i}-z_b)-(z_{n+i}-z_b)\sqrt{m^2+2r\sigma^2}}{\sigma^2}\right)\\
&\quad \times \left(\Phi\left(\frac{z_{n+i}-z_b+\delta t\sqrt{m^2+2r\sigma^2}}{\sigma\sqrt{\delta t}}\right)-1\right). \qquad \text{(B.29)}
\end{aligned}
$$

For the case $i = 0$, we see that

$$
\begin{aligned}
&\int_t^{t_{n+1}} e^{-r(u-t)} p_{bc}(t, t_n, u)\mathrm{d}u\\
&\quad = \frac{1}{p_b(t_n, t)} \int_t^{t_{n+1}} e^{-r(u-t)} p_b(t_n, u)\mathrm{d}u\\
&\quad = \frac{1}{p_b(t_n, t)} \int_{z_b}^{\infty} \tilde{I}_b(t_n, t_{n+1}, z_n) g_{t_n}(z_n \,|\, y^{(n)}, \tau_b > t_n)\mathrm{d}z_n.
\end{aligned}
$$

Finally, putting everything together now gives the CoCo market price

$$
\begin{aligned}
C(t) &= \frac{P_2}{p_b(t_n,t)} \int_{z_b}^{\infty} \left(Re^{-r(t_{n+1}-t)}(1-\pi(\delta t, z_n - z_b)) + c_2 \tilde{I}_b(t_n, t_{n+1}, z_n)\right) g_{t_n}(z_n | y^{(n)}, \tau_b > t_n)\mathrm{d}z_n\\
&\quad + \frac{P_2}{p_b(t_n,t)} \sum_{i=1}^{m-1} \int_{(y_c,\infty)^i} \int_{(z_b,\infty)^{i+1}} \xi(z_{n+i})\\
&\quad \times B_{n,i}(z^{(n,n+i)}, y^{(n+1,n+i)} | y^{(n)}, \tau_b > t_n)\mathrm{d}z^{(n,n+i)}\mathrm{d}y^{(n+1,n+i)}\\
&\quad + \frac{(1-R)P_2 e^{-r(T-t)}}{p_b(t_n,t)} \int_{(y_c,\infty)^m} \int_{(z_b,\infty)^{m+1}}\\
&\quad B_{n,m}(z^{(n,n+m)}, y^{(n+1,n+m)} | y^{(n)}, \tau_b > t_n)\mathrm{d}z^{(n,n+m)}\mathrm{d}y^{(n+1,n+m)},
\end{aligned}
$$

where

$$
\begin{aligned}
\xi(z_{n+i}) &= Re^{-r(t_{n+i+1}-t)}(1-\pi(\delta t, z_{n+i}-z_b)) - Re^{-r(t_{n+i}-t)}\\
&\quad + c_2 \tilde{I}_b(t_{n+i}, t_{n+i+1}, z_{n+i}),
\end{aligned}
$$

which completes the proof by setting

$$
h_i(z_{n+i}) = \begin{cases} P_2(Re^{-r(t_{n+1}-t)}(1-\pi(\delta t, z_n - z_b)) & \\ \quad + c_2 \tilde{I}_b(t_n, t_{n+1}, z_n)) & \text{for } i = 0,\\ P_2 \xi(z_{n+i}) & \text{for } 1 \le i \le m-1,\\ P_2(1-R)e^{-r(T-t)} & \text{for } i = m. \end{cases} \qquad \text{(B.30)}
$$

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