From: STRONGLY COUPLED PLASMA PHYSICS
Edited by Forrest J. Rogers
and Hugh E. Dewitt
(Plenum Publishing Corporation, 1987)

COLLECTIVE MODES AND MODE COUPLING

FOR A DENSE PLASMA IN A MAGNETIC FIELD

L.G. Suttorp

Institute for Theoretical Physics
University of Amsterdam
Valckenierstraat 65
1018 XE Amsterdam, The Netherlands

1. INTRODUCTION

Collective modes play an important role in the dynamical response of macroscopic systems to external disturbances. In particular, the modes with small wavenumber determine the large-scale behaviour of the dynamical structure factor.

To derive collective modes various methods are available. From the point of view of kinetic theory the modes follow from the memory kernel of the formal kinetic equation for the one-particle time correlation function in momentum space. In a recent paper [Suttorp and Cohen, 1985] the complete set of modes for a dense plasma in a magnetic field has been derived along these lines. An alternative approach to the evaluation of the mode spectrum starts from the microscopic balance equations of particle number, momentum and energy. After establishing fluctuation formulae for the densities and the flows of these quantities the mode frequencies and the associated amplitudes may be derived by using projection operator techniques. This method has been employed before to determine the collective modes for a neutral fluid [Kadanoff and Swift, 1968; Résibois, 1972] and for an unmagnetized plasma [Marchetti and Kirkpatrick, 1985]. Recently, the collective modes of a magnetized plasma have been derived in this way [Suttorp and Schoolderman, 1986].

The collective modes of a plasma in a magnetic field depend on the angle between the wave vector and the field. Transverse and longitudinal modes can no longer be distinguished. As a consequence both the mode amplitudes and the mode frequencies are given by expressions that are rather more complicated than those for an unmagnetized plasma. In particular, the viscous modes and the plasmon modes of an unmagnetized plasma merge in a set of four mixed 'gyro-plasmon' modes, if a magnetic field is turned on.

The amplitudes and the frequencies of the collective modes are essential ingredients in the theory of mode-coupling, which may be used to analyse the long-time behaviour of time correlation functions like the velocity autocorrelation function of a tagged particle. For systems of neutral particles this method is well-established [Kawasaki, 1970; Ernst e.a., 1971,1976]. Furthermore it has been employed to determine the long-time tails of the

Green-Kubo integrands of an unmagnetized plasma [Marchetti and Kirkpatrick, 1985]. In a magnetized plasma the velocity autocorrelation function depends on the direction of the velocities owing to the anisotropy of the system; one should distinguish therefore a longitudinal and a transverse velocity autocorrelation function. Recently [Suttorp and Schoolderman, 1986] mode-coupling theory has been used to derive expressions for the long-time tails of both autocorrelation functions. It has been found that these tails are dominated by the coupling with the gyro-plasmon modes.

In the following a review will be given of the derivation of the collective modes and of the long-time behaviour of the velocity autocorrelation functions for a magnetized plasma. As a model we shall adopt the classical one-component plasma, consisting of charged particles which are immersed in a neutralizing inert background and which interact through a Coulomb potential. The external magnetic field is assumed to be static and uniform in space.

2. COLLECTIVE MODES

To obtain the modes for a one-component plasma in a magnetic field one may start from the microscopic balance equations of particle number, momentum and energy. These give the time derivative of the particle density $n(\vec{k})$, the momentum density $\vec{g}(\vec{k})$ and the energy density $\varepsilon(\vec{k})$ in Fourier space. These time derivatives are conveniently written in terms of the Liouville operator L in phase space, which determines for an arbitrary function F its time derivative as $\dot{F} = iLF$.

The microscopic momentum balance equation contains, apart from a pressure term, with a pressure tensor $\vec{\vec{\tau}}(\vec{k})$, and a consistent field term depending on the electric field generated by the charge density fluctuations, a Lorentz force term depending on the direction and the strength of the magnetic field, as given by the unit vector $\hat{B}$ and the Larmor frequency $\omega_B = eB/mc$, with e the charge and m the mass of the particles. The energy balance equation contains an energy flow $\vec{j}_\varepsilon(\vec{k})$, which is the sum of a kinetic and a potential contribution, as is the case for the pressure tensor.

The collective modes are particular linear combinations of the particle density, the momentum density and the energy density. Let $a_i(\vec{k})$ denote a set of five independent linear combinations of these quantities, with adjoints $\bar{a}_i(\vec{k})$ such that

$$\frac{1}{V} < \bar{a}_i^*(\vec{k})\; a_j(\vec{k}) > = \delta_{ij} \quad . \tag{2.1}$$

Here the brackets denote a canonical ensemble average; V is the volume of the system. In the course of time $a_i(\vec{k})$ evolves into $a_i(\vec{k},t)$ of which the Laplace transform

$$a_i(\vec{k},z) = -\, i \int_0^\infty dt\; e^{izt}\; a_i(\vec{k},t) \tag{2.2}$$

satisfies the equation

$$(z+L)a_i(\vec{k},z) = a_i(\vec{k}) \quad . \tag{2.3}$$

Introducing a projection operator P by writing

$$Pf(\vec{k}) = \sum_i \frac{1}{V} < \bar{a}_i^*(\vec{k})f(\vec{k}) > a_i(\vec{k}) \tag{2.4}$$

for an arbitrary function $f(\vec{k})$ in phase space, one derives an equation for the hydrodynamic propagators:

$$G_{ij}(\vec{k},z) = \frac{1}{V} < \bar{a}_i^*(\vec{k}) \frac{1}{z+L} a_j(\vec{k}) > \tag{2.5}$$

in the form

$$\sum_\ell [z\delta_{i\ell} - \Omega_{i\ell}(\vec{k},z)]G_{\ell j}(\vec{k},z) = \delta_{ij} \quad . \tag{2.6}$$

The frequency matrix is given by

$$\Omega_{ij}(\vec{k},z) = \Omega_{ij}^{(1)}(\vec{k},z) + \Omega_{ij}^{(2)}(\vec{k},z) \quad , \tag{2.7}$$

where the direct and the indirect parts are

$$\Omega_{ij}^{(1)}(\vec{k},z) = -\frac{1}{V} < \bar{a}_i^*(\vec{k})L\, a_j(\vec{k}) > \quad , \tag{2.8}$$

$$\Omega_{ij}^{(2)}(\vec{k},z) = \frac{1}{V} < \bar{a}_i^*(\vec{k})LQ \frac{1}{z+QLQ} QL\, a_j(\vec{k}) > \quad , \tag{2.9}$$

with $Q = 1 - P$.

The collective mode frequencies follow as the eigenfrequencies of the frequency matrix for small values of the wave number $\vec{k}$. The modes themselves are the corresponding eigenvectors.

For vanishing wavenumber the five mode frequencies are:

$$z_T^{(0)} = 0 \quad , \tag{2.10}$$

$$z_{\lambda\rho}^{(0)} = \rho\, w_\lambda \quad , \tag{2.11}$$

with $\lambda = \pm 1$, $\rho = \pm 1$. Here w_λ is given by:

$$w_\lambda = \tfrac{1}{2}(\omega_p^{\,2} + \omega_B^{\,2} + 2\omega_p\omega_B \hat{k}_{/\!/})^{\frac{1}{2}} + \tfrac{1}{2}\lambda\,(\omega_p^{\,2} + \omega_B^{\,2} - 2\omega_p\omega_B \hat{k}_{/\!/})^{\frac{1}{2}} \quad , \tag{2.12}$$

with $\hat{k}_{/\!/} = \vec{k}\cdot\hat{\vec{B}}/k$ and ω_p the plasma frequency. Choosing as a basis set $k^{-1}n(\vec{k})$, $\vec{g}(\vec{k})$ and $\varepsilon(\vec{k})$, one finds for the modes up to first order in $\vec{k}$:

$$a_T(\vec{k}) = C_T(\hat{\vec{k}})\,[\varepsilon(\vec{k}) - hn(\vec{k})] \quad , \tag{2.13}$$

$$a_{\lambda\rho}(\vec{k}) = C_\lambda(\hat{\vec{k}})\Big[\frac{k_D}{k}\, n(\vec{k}) + \frac{1}{k_B T c_V}\left(\frac{1}{3}\, c_V + \tfrac{1}{2}k_B\right)\frac{k}{k_D}\,\varepsilon(\vec{k}) + \frac{1}{(mk_BT)^{\frac{1}{2}}}\, \vec{v}_{\lambda\rho}(\vec{k})\cdot\vec{g}(\vec{k})\Big] \quad . \tag{2.14}$$

The heat mode (2.13) contains the enthalphy h per particle, which is related to the specific heat c_V and the isothermal compressibility κ_T of the plasma:

$$h = - k_B T + \frac{1}{3} c_V T + 3/(n\kappa_T) \quad , \tag{2.15}$$

with n the particle density and T the temperature. The vectors $\vec{v}_{\lambda\rho}$ occurring in the gyro-plasmon modes (2.14) are defined as:

$$\vec{v}_{\lambda\rho}(\vec{k}) = \frac{\rho\, w_\lambda\, \omega_p}{w_\lambda^2 - \omega_B^2} \hat{\vec{k}}_\perp + \frac{\rho\, \omega_p}{w_\lambda} \hat{\vec{k}}_{/\!/} - \frac{i\omega_p\, \omega_B}{w_\lambda^2 - \omega_B^2} \hat{\vec{k}} \wedge \hat{\vec{B}} \quad . \tag{2.16}$$

Here $\hat{\vec{k}}$ is a unit vector in the direction of the wave vector with components parallel and perpendicular to $\hat{\vec{B}}$ denoted by $\hat{\vec{k}}_{/\!/}$ and $\hat{\vec{k}}_\perp$, respectively. Furthermore k_D is the Debye wave vector, while $C_T(\vec{k})$ and $C_\lambda(\vec{k})$ are normalization constants that should be chosen such that (2.1) is satisfied.

The corrections to the mode frequencies that are of order k^2 follow by applying perturbation theory. Up to order k^2 one gets:

$$z_T = \frac{1}{V} < a_T^*(\vec{k}) LQ \frac{1}{z+QLQ} QLa_T(\vec{k}) > \quad , \tag{2.17}$$

$$z_{\lambda\rho} = \rho\, w_\lambda \left[1 + \tfrac{1}{2}k^2 c_s^2 \frac{w_\lambda^2 - \omega_B^2 \hat{k}_{/\!/}^2}{w_\lambda^2(\omega_p^2+\omega_B^2) - 2\omega_p^2 \omega_B^2 \hat{k}_{/\!/}^2}\right]$$

$$+ \frac{1}{V} < a_{\lambda\rho}^*(\vec{k}) LQ \frac{1}{z+QLQ} QLa_{\lambda\rho}(\vec{k}) > \quad . \tag{2.18}$$

with c_s the sound velocity. The Laplace variable z stands for the zeroth-order frequency $z^{(0)}$, as given by (2.10) and (2.11).

The dependence of the mode frequencies (2.17) and (2.18) on the wave vector $\vec{k}$ can be analyzed by using the balance equations and the symmetry properties of the system. The frequency of the thermal mode is found to contain the static thermal conductivities in the longitudinal and transverse directions:

$$z_T = \frac{-ik^2}{nc_V} (\hat{k}_\perp^2 \lambda_\perp + \hat{k}_{/\!/}^2 \lambda_{/\!/}) \quad . \tag{2.19}$$

These are defined by writing

$$\hat{k}_\perp^2 \lambda_\perp + \hat{k}_{/\!/}^2 \lambda_{/\!/} = \frac{i}{k_B T^2} \lim_{z \to i0} \lim_{k \to 0} \frac{1}{Vk^2}$$

$$\times < \vec{k}\cdot\vec{j}_\varepsilon^*(\vec{k})\, Q \frac{1}{z+QLQ} Q\, \vec{k}\cdot\vec{j}_\varepsilon(\vec{k}) > \quad . \tag{2.20}$$

The mode frequencies $z_{\lambda\rho}$ can be analyzed in a similar way. From (2.14) one obtains up to second order in $\vec{k}$:

$$\frac{1}{V} < a^*_{\lambda\rho}(\vec{k}) LQ \frac{1}{z+QLQ} QLa_{\lambda\rho}(\vec{k}) > =$$

$$= \frac{C_\lambda^2}{mk_BT} \frac{1}{V} < [\vec{k}\cdot\vec{\tau}(\vec{k})\cdot \vec{v}_{\lambda\rho}(\vec{k})]^* Q \frac{1}{z+QLQ} Q\vec{k}\cdot\vec{\tau}(\vec{k})\cdot \vec{v}_{\lambda\rho}(\vec{k}) > \quad . \qquad (2.21)$$

Employing again the symmetry properties one may write up to second order in $\vec{k}$:

$$\frac{1}{V} < \vec{k}\cdot\vec{\tau}^*(\vec{k}) Q \frac{1}{z+QLQ} Q\vec{k}\cdot\vec{\tau}(\vec{k}) > = - ik_BTk^2 \vec{T}(\hat{\vec{k}},z) \quad , \qquad (2.22)$$

with

$$T_{ij}(\hat{\vec{k}},z) = f_1(z)\delta_{ij} + f_2(z)\hat{k}_i\hat{k}_j$$
$$+ f_3(z)(\hat{k}_i\hat{B}_j + \hat{k}_j\hat{B}_i)\hat{\vec{k}}\cdot\hat{\vec{B}} + f_4(z)[\hat{B}_i\hat{B}_j + \delta_{ij}(\hat{\vec{k}}\cdot\hat{\vec{B}})^2]$$
$$+ f_5(z)\hat{B}_i\hat{B}_j(\hat{\vec{k}}\cdot\hat{\vec{B}})^2 + f_6(z)[\hat{k}_i(\hat{\vec{k}}\wedge\hat{\vec{B}})_j - (\hat{\vec{k}}\wedge\hat{\vec{B}})_i\hat{k}_j - \varepsilon_{ijm}\hat{B}_m]$$
$$+ f_7(z)[\hat{B}_i(\hat{\vec{k}}\wedge\hat{\vec{B}})_j - (\hat{\vec{k}}\wedge\hat{\vec{B}})_i\hat{B}_j - \varepsilon_{ijm}\hat{B}_m\hat{\vec{k}}\cdot\hat{\vec{B}}]\hat{\vec{k}}\cdot\hat{\vec{B}} \quad . \qquad (2.23)$$

The coefficients f_i depend on z and on the Larmor frequency ω_B . Instead of f_i one may introduce dynamical viscosity coefficients by writing

$$f_1 = -\eta_1 + 2\eta_2 \quad , \quad f_2 = \frac{1}{3}\eta_1 + \eta_V - 2\zeta \quad , \quad f_3 = -\eta_1 + \eta_3 + 3\zeta \quad ,$$
$$f_4 = \eta_1 - 2\eta_2 + \eta_3 \quad , \quad f_5 = 2\eta_1 + 2\eta_2 - 4\eta_3 \quad ,$$
$$f_6 = \tfrac{1}{2}\eta_4 \quad , \quad f_7 = -\tfrac{1}{2}\eta_4 - \eta_5 \quad . \qquad (2.24)$$

The coefficients $\eta_1,\dots,\eta_5$ are the shear viscosities, η_V is the volume viscosity, while ζ describes a cross effect between shear stresses and volume strains and vice versa [de Groot and Mazur, 1962].

Substituting (2.16) and (2.22), with (2.23) and (2.24), into (2.18) with (2.21) we obtain the mode frequencies:

$$z_{\lambda\rho} = \rho w_\lambda [1 + \tfrac{1}{2} k^2 c_s^2 \frac{w_\lambda^2 - \omega_B^2 \hat{k}_{//}^2}{w_\lambda^2(\omega_p^2+\omega_B^2) - 2\omega_p^2\omega_B^2\hat{k}_{//}^2}]$$
$$+ \frac{k^2}{2nm[w_\lambda^2(\omega_p^2+\omega_B^2) - 2\omega_p^2\omega_B^2\hat{k}_{//}^2]} \{ \rho w_\lambda^3\omega_B[2(\eta_4+\eta_5)\hat{k}_{//}^2 - 2\eta_4]$$
$$+ iw_\lambda^2\{\omega_p^2[-2(\eta_1+\eta_2-2\eta_3)\hat{k}_{//}^4 + 2(2\eta_2-2\eta_3-3\zeta)\hat{k}_{//}^2 + \frac{2}{3}\eta_1 - 2\eta_2 - \eta_V + 2\zeta]$$
$$+ \omega_B^2[(-\frac{5}{3}\eta_1 + 4\eta_2 - 2\eta_3 + \eta_V - 2\zeta)\hat{k}_{//}^2 + \frac{5}{3}\eta_1 - 4\eta_2 - \eta_V + 2\zeta]\}$$
$$+ \rho w_\lambda \omega_p^2\omega_B \hat{k}_{//}^2[-2(\eta_4+2\eta_5)\hat{k}_{//}^2 + 2(\eta_4+\eta_5)]$$
$$+ i\omega_p^2\omega_B^2\hat{k}_{//}^2[(3\eta_1 - 4\eta_2 + \eta_3 + 6\zeta)\hat{k}_{//}^2 - \frac{5}{3}\eta_1 + 4\eta_2 + \eta_3 + \eta_V - 2\zeta] \} \quad , \qquad (2.25)$$

where the viscosities η_i, η_V, ζ are to be evaluated at $z = \rho\, w_\lambda$.

The expressions (2.19) and (2.25) for the mode frequencies may be compared to those obtained by alternative methods. A macroscopic magnetohydrodynamical treatment leads to expressions for the mode frequencies of a similar form. However, they contain phenomenological transport coefficients that are static real quantities, defined at frequency zero. On the other hand, in (2.25) dynamical complex-valued viscosities at the finite frequency $\rho\, w_\lambda$ show up.

Another method to derive the mode spectrum is furnished by the kinetic theory for time correlation functions. The results obtained in this way contain frequency-dependent transport coefficients that are given in terms of matrix elements of a kinetic kernel. A detailed comparison [Suttorp and Schoolderman, 1986] shows that these frequency-dependent transport coefficients do not coincide with those introduced in the present treatment. The reason is that the projection operator used in kinetic theory has no simple relation to that defined in (2.4).

In a recent paper [Marchetti, Kirkpatrick and Dorfman, 1984] expressions for the frequencies of the oscillating modes of a strongly magnetized plasma have been presented. However, the terms of order k^2 are not given explicitly in terms of the seven anisotropic viscosity coefficients, so that a comparison is difficult. Expressions for the modes that might be compared to (2.13) and (2.14) are not given either.

3. MODE COUPLING AND LONG-TIME TAILS OF THE VELOCITY AUTOCORRELATION FUNCTION

The velocity autocorrelation function of a tagged particle is defined as

$$F(\vec{k},t) = \lim_{\vec{k} \to 0} \frac{1}{k^2} \left< \vec{k} \cdot \frac{\vec{g}_s^{\,*}(\vec{k})}{m}\, e^{iLt}\, \vec{k} \cdot \frac{\vec{g}_s(\vec{k})}{m} \right> \quad , \tag{3.1}$$

where the tagged-particle momentum density is given by

$$\vec{g}_s(\vec{k}) = \vec{p}_s\, e^{-i\vec{k}\cdot\vec{r}_s} \quad , \tag{3.2}$$

with $\vec{r}_s$ and $\vec{p}_s$ the position and momentum of the particle. The autocorrelation function (3.1) is anisotropic, as it depends on the angle between the wave vector $\vec{k}$ and the magnetic field $\vec{B}$. In fact, one may write:

$$F(\vec{k},t) = \hat{k}_{/\!/}^{\,2}\, F_{/\!/}(t) + \hat{k}_{\perp}^{\,2}\, F_{\perp}(t) \quad , \tag{3.3}$$

which defines the longitudinal and the transverse velocity autocorrelation functions $F_i(t)$, with $i = /\!/ , \perp$.

The long-time behaviour of the velocity autocorrelation functions $F_i(t)$ can be determined if one assumes it to be adequately described by mode-coupling theory. According to mode-coupling theory the long-time behaviour of the velocity autocorrelation function is dominated by contributions originating from the coupling of the tagged-particle momentum density to the product of a collective mode and of the tagged-particle density:

$$F(\vec{k},t) \simeq \lim_{\vec{k}\to 0} \frac{1}{k^2 V} \sum_i \sum_{\vec{q}} |A_i(\vec{k},\vec{q})|^2 \, e^{-i[z_i(\vec{q}) + z_s(\vec{k}-\vec{q})]t} \quad . \tag{3.4}$$

The sums are extended over the five collective modes (with label $i = T, \lambda\rho$) and over all values of the wave vector $\vec{q}$ of these modes. The amplitudes A_i are given as:

$$A_i(\vec{k},\vec{q}) = \langle a_i^*(\vec{q}) \; a_s^*(\vec{k}-\vec{q}) \; \vec{k} \cdot \frac{\vec{g}_s(\vec{k})}{m} \rangle \quad . \tag{3.5}$$

The collective modes $a_i(\vec{q})$ and the corresponding frequencies $z_i(\vec{q})$ have been discussed in the first part of this paper. The tagged-particle density mode:

$$a_s(\vec{q}) = e^{-i\vec{q}\cdot\vec{r}_s} \tag{3.6}$$

is a dissipative mode, with a frequency

$$z_s(\vec{q}) = - iq_{/\!/}^2 \, D_{/\!/} - iq_{\perp}^2 \, D_{\perp} \tag{3.7}$$

that is determined by the longitudinal and the transverse self-diffusion constants, $D_{/\!/}$ and $D_{\perp}$.

As a consequence of the symmetry of the momentum integration, which is implied in the average (3.5), only the gyro-plasmon modes, with $i = \lambda\rho$, contribute to the mode-coupling expression (3.4). The corresponding amplitudes are easily evaluated with the help of (2.14):

$$A_{\lambda\rho}(\vec{k},\vec{q}) = C_\lambda(\hat{q})(k_B T/m)^{\frac{1}{2}} \; \vec{v}_{\lambda\rho}^{\,*}(\hat{q}) \cdot \vec{k} \quad . \tag{3.8}$$

The mode frequencies $z_{\lambda\rho}(\vec{q})$ have the form:

$$z_{\lambda,1}(\vec{q}) = w_\lambda(\hat{q}) - iq^2 D_\lambda(\hat{q}) \quad , \quad z_{\lambda,-1}(\vec{q}) = - \left[z_{\lambda,1}(\vec{q})\right]^* \quad , \tag{3.9}$$

with (complex) damping coefficients D_λ . Inserting (3.8) and (3.9) in (3.4), taking the limit $\vec{k} \to 0$, averaging over the azimuthal angle of $\vec{q}$ (in a spherical coordinate system with a polar axis in the direction of the magnetic field) and integrating over $|\vec{q}|$ we obtain an expression for $F_i(\vec{k},t)$, or, with the help of (3.3), for $F_i(t)$:

$$F_i(t) \simeq \frac{k_B T}{m(4\pi t)^{3/2}} \sum_{\lambda = \pm 1} \mathrm{Re} \int_{-1}^{1} d\hat{q}_{/\!/} \, C_\lambda^2(\hat{q}) \Phi_i(\hat{q})$$
$$\times \frac{e^{-iw_\lambda(\hat{q})t}}{[\hat{q}_{/\!/}^2 \, D_{/\!/} + \hat{q}_{\perp}^2 \, D_{\perp} + D_\lambda(\hat{q})]^{3/2}} \quad . \tag{3.10}$$

Here we introduced the abbreviations:

$$\Phi_{/\!/}(\hat{q}) = \frac{\omega_p^2}{w^2} \, \hat{q}_{/\!/}^2 \quad , \tag{3.11}$$

$$\Phi_{\perp}(\vec{q}) = \tfrac{1}{2}\,\frac{(w^2+\omega_B{}^2)\omega_p{}^2}{(w^2-\omega_B{}^2)^2}\;\hat{q}_{\perp}{}^2 \quad , \tag{3.12}$$

with $w = w_{\lambda}(\hat{\vec{q}})$.

Let us consider first the contribution $F_i^{(+)}$ of the modes with $\lambda = +1$. Choosing the new integration variable $w = w_{\lambda}(\vec{q})$ we obtain:

$$F_i^{(+)}(t) \simeq \frac{k_B T}{nm(4\pi t)^{3/2}}\,\mathrm{Re}\int_{w_1}^{w_2} dw\,\frac{w^2-\omega_B{}^2}{\omega_p\omega_B(\omega_p{}^2+\omega_B{}^2-w^2)^{\frac{1}{2}}}\,\Phi_i\,\frac{e^{-iwt}}{(\hat{q}_{/\!/}{}^2 D_{/\!/}+\hat{q}_{\perp}{}^2 D_{\perp}+D_1)^{3/2}} , \tag{3.13}$$

with $w_1 = \omega_M \equiv \mathrm{Max}(\omega_p,\omega_B)$ and $w_2 = \omega_0 \equiv (\omega_p{}^2+\omega_B{}^2)^{\frac{1}{2}}$. In the integrand one should substitute:

$$\hat{q}_{/\!/} = \frac{w}{\omega_p\,\omega_B}\,(\omega_p{}^2+\omega_B{}^2-w^2)^{\frac{1}{2}} \tag{3.14}$$

and $\hat{q}_{\perp} = (1-\hat{q}_{/\!/}{}^2)^{\frac{1}{2}}$.

The integrand in (3.13) is a regular nonvanishing function for all w in the open interval (w_1,w_2). At the upper boundary of the integration domain the integrand is proportional to $(w_2-w)^{\frac{1}{2}}$ for $i = /\!/$ and to $(w_2-w)^{-\frac{1}{2}}$ for $i = \perp$. At the lower boundary it is proportional to $(w-\omega_B)$ for $i = /\!/$ and to $(w-\omega_p)$ for $i = \perp$.

For large values of t the contribution of the interior of the integration domain in (3.13) may be disregarded. In fact, as a consequence of the phase factor exp(-iwt) destructive interference damps all contributions from the interior region. The main contributions to the asymptotic expression for the integral originate from the boundaries of the integration domain, since there the interference is not completely destructive. One may derive the following asymptotic expression for $F_i^{(+)}(t)$:

$$F_i^{(+)}(t) \simeq A_{2,i}\,t^{-\nu_{2,i}}\cos(\omega_0 t+\theta_{2,i}) - A_{1,i}\,t^{-\nu_{1,i}}\cos(\omega_M t+\theta_{1,i}) , \tag{3.15}$$

valid for large t. The indices 1,2 indicate contributions from the boundaries at w_1,w_2, respectively. The exponent $\nu_{2,i}$ equals 3 for $i = /\!/$ and 2 for $i = \perp$. The other exponent $\nu_{1,i}$ depends on the relative magnitude of ω_p and ω_B ; for $\omega_p > \omega_B$ one has $\nu_{1,/\!/} = \frac{5}{2}$, $\nu_{1,\perp} = \frac{7}{2}$, while for $\omega_p < \omega_B$ these values are interchanged.

Likewise one may derive the asymptotic expression for the contribution $F_i^{(-)}(t)$ that results by taking $\lambda = -1$ in (3.10). As before the dominant contribution to the integral for large t stems from the boundaries of the integration domain. From the behaviour of the integrand at these boundaries one finds:

$$F_i^{(-)}(t) \propto t^{-\nu_i}\cos(\omega_m + \theta_i) \quad , \tag{3.16}$$

with a frequency $\omega_m = \mathrm{Min}(\omega_p,\omega_B)$ and exponents ν_i that are related to $\nu_{1,i}$ in (3.15) as $\nu_{/\!/} = \nu_{1,\perp}$ and $\nu_{\perp} = \nu_{1,/\!/}$.

Comparing the exponents we conclude that the dominant terms in $F_i(t)$ have the form:

$$F_{//}(t) \propto t^{-5/2} \cos(\omega_p t + \theta_{//}) \quad , \tag{3.17}$$

$$F_{\perp}(t) \propto t^{-2} \cos(\omega_0 t + \theta_{\perp}) \quad . \tag{3.18}$$

A detailed calculation leads to explicit expressions for the proportionality factors and for the phase angles. In fact, the result for the longitudinal velocity autocorrelation function is:

$$F_{//}(t) \simeq \frac{k_B T(\omega_B^2 - \omega_p^2)}{8\pi^{3/2} nm\omega_p \omega_B^2 t^{5/2}} \operatorname{Re}\left\{\frac{i\, e^{-i\omega_p t}}{\left[D_{//} + \frac{i}{2}\frac{c_s^2}{\omega_p} + \frac{1}{2nm}\left(\frac{4}{3}\eta_1 + \eta_V + 4\zeta\right)\right]^{3/2}}\right\} , \tag{3.19}$$

with the (complex) dynamical viscosities η_i , ζ at the frequency $z = \omega_p$ and the (real) static self-diffusion coefficient $D_{//}$. For the transverse velocity autocorrelation function one obtains:

$$F_{\perp}(t) \simeq \frac{k_B T(\omega_p^2 + 2\omega_B^2)}{16\sqrt{2}\ \pi\ nm\omega_p \omega_B \omega_0^{\frac{1}{2}}\ t^2}$$

$$\times \operatorname{Re}\left[\frac{\exp(i\frac{\pi}{4} - i\omega_0 t)}{\left\{D_{\perp} + \frac{i}{2}\frac{c_s^2}{\omega_0} - \frac{i\eta_4\omega_B}{nm\omega_0} - \frac{1}{2nm}\left[\frac{2}{3}\eta_1 - 2\eta_2 - \eta_V + 2\zeta + \frac{\omega_B^2}{\omega_0^2}(\eta_1 - 2\eta_2)\right]\right\}^{3/2}}\right] \tag{3.20}$$

with the dynamical viscosities η_i, ζ at the frequency $z = \omega_0$ and the static self-diffusion coefficient $D_{\perp}$.

In the case of resonance, with $\omega_p = \omega_B$, the asymptotic expression for $F_{//}(t)$ is no longer given by (3.17) or (3.19). Instead, the dominant term stems then from the upper boundary of the integral for $F_{//}^{(+)}(t)$. The asymptotic expression for $F_{//}(t)$ reads in this case:

$$F_{//}(t) \simeq \frac{k_B T}{8\pi\ 2^{\frac{1}{4}}\ nm\omega_p^{3/2}\ t^3}$$

$$\times \operatorname{Re}\left[\frac{i\ \exp(i\frac{\pi}{4} - i\sqrt{2}\ \omega_p t)}{\left\{D_{\perp} + \frac{i}{2}\frac{c_s^2}{\omega_0} - \frac{i\eta_4}{\sqrt{2}\ nm} - \frac{1}{2nm}\left(\frac{7}{6}\eta_1 - 3\eta_2 - \eta_V + 2\zeta\right)\right\}^{3/2}}\right] \quad . \tag{3.21}$$

As is well-known [Alder and Wainwright, 1970; Kawasaki, 1970; Ernst e.a., 1971, 1976] the velocity autocorrelation function for a fluid of neutral particles has a tail proportional to $t^{-d/2}$ with $d = 3$. For an unmagnetized one-component plasma the tail of the velocity autocorrelation function is the sum of a term proportional to $t^{-3/2}$ and a term proportional to $t^{-3/2}\cos(\omega_p t + \theta)$ [Gould and Mazenko, 1975; Giaquinta e.a., 1976; Varley, 1977; Gaskell, 1982; Marchetti and Kirkpatrick, 1985]. For a magnetized one-component plasma we have found that the tails behave qualitatively differently. The anisotropy of the mode spectrum leads to interference effects in the coupling of the modes. As a consequence the tails drop off more rapidly than those of the correlation functions for an unmagnetized plasma. Moreover, a second frequency, viz. $\omega_0 = (\omega_p^2+\omega_B^2)^{\frac{1}{2}}$ shows up on a par with the plasma frequency. In the general off-resonant case this frequency determines the oscillations of the tail of the transverse velocity autocorrelation function, whereas the tail of the longitudinal function still oscillates at the plasma frequency, as in the unmagnetized case. As a consequence one expects a peak at ω_0 in the power spectrum of the transverse autocorrelation function and similarly a peak at ω_p for the longitudinal function. The latter will be less pronounced, however, since the tail of the longitudinal function drops off more rapidly, so that its contribution to the power spectrum is less important.

In the particular case of resonance the frequency $\omega_0 = \sqrt{2}\,\omega_p$ determines the oscillations of the tails of both the transverse and the longitudinal autocorrelation functions. As before it is expected that in the power spectrum the peak at ω_0 is more pronounced for the transverse case; in the longitudinal case the tail is damped by an extra factor t^{-1} so that its influence is less important.

In a paper that appeared several years ago [Bernu, 1981] molecular dynamics computations for the velocity autocorrelation functions of a one-component plasma in a magnetic field have been reported on. It was found that the power spectrum of the transverse velocity autocorrelation function for strongly coupled plasmas ($\Gamma = 10$ or 100) in a magnetic field with a resonant Larmor frequency ($\omega_B = \omega_p$) indeed shows a peak structure at a frequency $\omega \simeq 1.3\,\omega_p$, which is quite near to $\sqrt{2}\,\omega_p$. The plasmon peak, which is present for vanishing magnetic fields, turned out to be suppressed completely in the resonant case. This result is corroborated by the mode-coupling calculation of the tails, as presented here. As to the power spectrum of the longitudinal velocity autocorrelation function, it turned out to be rather flat. Apparently, the influence of the tail, which would have led to a peak at $\sqrt{2}\,\omega_p$ as well, is rather weak, as was anticipated above, in view of the strong damping ($\propto t^{-3}$) of the tail in this case.

REFERENCES

Alder, B.J. and T.E. Wainwright, 1970, Phys.Rev., A1:18.

Bernu, B., 1981, J. de Phys.Lett., 42:L253.

Ernst, M.H., E.H. Hauge and J.M.J. van Leeuwen, 1971, Phys.Lett., 34A:419.

Ernst, M.H., E.H. Hauge and J.M.J. van Leeuwen, 1976, J.Stat.Phys., 15:7.

Gaskell, T., 1982, J.Phys., C15:1601.

Giaquinta, P.V., M. Parrinello, M.P. Tosi and N.H. March, 1976, Phys.Chem. Liquids, 5:197.

Gould, H. and G.F. Mazenko, 1975, Phys.Rev.Lett., 35:1455.

de Groot, S.R. and P. Mazur, 1962, "Non-equilibrium Thermodynamics", North-Holland, Amsterdam.

Kadanoff, L.P. and J. Swift, 1968, Phys.Rev., 166:89.

Kawasaki, K., 1970, Phys.Lett., 32A:379.

Marchetti, M.C., T.R. Kirkpatrick and J.R. Dorfman, 1984, Phys.Rev., A29:2960.

Marchetti, M.C. and T.R. Kirkpatrick, 1985, Phys.Rev., A32:2981.

Résibois, P., 1972, in: "Irreversibility in the Many-Body Problem", J. Biel and J. Rae, eds., Plenum, New York, 273.

Suttorp, L.G. and J.S. Cohen, 1985, Physica, 133A:357,370.

Suttorp, L.G. and A.J. Schoolderman, 1986, preprint.

Varley, R.L., 1977, Phys.Lett., 62A:340.