and available experimental data for R and D [6]. Also, it contains the values of Δn_B calculated from available measurements [5]. For the sake of comparison, we give the variations Δn_V and Δn_T calculated previously [9] from eqs. (2) and (3).

From table 1, the Δn_B -values calculated from eq. (6) are in all cases seen to be (on the average about twice) larger than those from eq. (7), the latter being in better agreement with the experimental Δn_B -values resulting from present optical birefringence measurements [5] and earlier data from the Kerr effect [10]. As to the values of the variations Δn_V due to electrostriction, they exceed Δn_B in weakly birefringent liquids, but are smaller than Δn_B in strongly birefringent ones. The variations Δn_T due to the electrocaloric effect are negative and smaller in absolute value than Δn_V and Δn_B.

Thus, eqs. (6) and (7) lead smoothly and directly to Δn_B -values which are both reasonable and in accordance with nonlinear optical measurements [4,5]. In this way, and on the other hand recurring to formulae for B in molecular-statistical form [7,8], we can gain valuable information regarding both molecular optical anisotropy and molecular correlations in dense systems. Hence, work on the various nonlinear changes in refractive index should profitably proceed in conjunction with research on molecular light scattering in liquids [8]. The phenomena dicussed above, together with the experimental [11] and theoretical [12] investigation of multi-harmonic light scattering, provide the basis of nonlinear molecular optics.

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THE RELATIVISTIC ENERGY-MOMENTUM TENSOR IN POLARIZED MEDIA

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The relativistic energy-momentum tesnor of the electro-magnetic field in electric and magnetic dipole substances is derived and compared with the proposals of Lorentz, Einstein-Laub, Minkowski and Abraham.

Since 1903 the correct form of the electromagnetic energy-momentum tensor in polarized media has been a controversial issue. Forms, more or less based on electron theory, were obtained by Lorentz [1] (already before relativity theory existed) and by Einstein and Laub [2]; different expressions were put forward by Minkowski [3] and Abraham [4] and in a thermodynamic treatment [5].

A unique solution of the problem can be given if not only the field but also the material part of the energy-momentum tensor is considered. A derivation must start from the microscopic energy-momen-

tum laws of a system of point particles (electrons and nuclei) in an electromagnetic field. The point particles are supposed to be grouped into atoms k (or molecules, ions etc.), which carry both electric and magnetic dipoles, defined in momentary rest frames. One obtains thus energy-momentum laws on the "atomic level":

$$\partial_\beta \Big(\sum_k \rho_k u_{k\alpha} u_k^\beta + t^{\cdot\beta}_{\alpha(m)} + t^{\cdot\beta}_{\alpha(f)} \Big) = 0, \tag{1}$$

with

$$t^{\cdot\beta}_{\alpha(m)} = \tfrac{1}{2} \sum_k \Big\{ c^{-2} u_{k\alpha} \sigma_k^{\beta\lambda} Du_{k\lambda} + c^{-2} u_k^\beta \sigma^{\cdot\lambda}_{k\alpha} Du_{k\lambda} + \partial_\lambda \big(\sigma^{\cdot\lambda}_{k\alpha} u_k^\beta + \sigma_k^{\beta\lambda} u_{k\alpha} \big) - \Delta^\mu_{k\alpha} \Delta^\beta_{k\mu} \partial_\nu \big(\sigma^{\cdot\mu}_{k\lambda} u_k^\nu \big) \Big\} \tag{2}$$

and

$$t^{\cdot\beta}_{\alpha(f)} = \sum_{k \neq l} \Big\{ f_{k\alpha\lambda} h_l^{\beta\lambda} - \tfrac{1}{4} f_{k\lambda\mu} f_l^{\lambda\mu} \delta^\beta_\alpha + c^{-2} u_k^\beta \big(f_{k\alpha\lambda} h_l^{\lambda\mu} - h_{l\alpha\lambda} f_k^{\lambda\mu} \big) u_{k\mu} + \\ + c^{-4} u_{k\alpha} u_k^\beta u_k^\lambda f_{k\lambda\mu} \big(h_l^{\mu\nu} - f_l^{\mu\nu} \big) u_{k\nu} \Big\}, \tag{3}$$

where ρ_k is the mass density of atom k, $u_{k\alpha}$ its velocity four-vector, $D = u_k^\lambda \partial_\lambda$ the substantial derivative with respect to time, $\Delta_{k\alpha}{}^\beta = \delta_\alpha{}^\beta + c^{-2} u_{k\alpha} u_k{}^\beta$, $\sigma_{k\alpha\beta}$ the density of intrinsic angular momentum of atom k, $f_{k\alpha\beta}$ the "atomic" field tensor due to atom k, $m_{k\alpha\beta}$ its polarization tensor and $h_{k\alpha\beta} = f_{k\alpha\beta} + m_{k\alpha\beta}$ (we use the metric $g^{00} = -1$, $g^{ii} = 1$ ($i = 1, 2, 3$), $g^{\alpha\beta} = 0$ if $\alpha \neq \beta$).

(In a similar way one can derive the angular momentum law on the atomic level; in turns out to express the symmetry of the total energy-momentum tensor).

By means of a covariant statistical averaging procedure one obtains the macroscopic conservation laws of energy and momentum

$$\partial_\beta \Big(\rho U_\alpha U^\beta + T^{\cdot\beta}_{\alpha(m)} + T^{\cdot\beta}_{\alpha(f)} \Big) = 0. \tag{4}$$

Here ρc^2 is the rest mass energy and internal energy density in the rest frame and U_α the bulk four-velocity of matter. The first two terms between the brackets form the material energy-momentum tensor: $\rho U_\alpha U^\beta$ being due to bulk motion, $T^{\cdot\beta}_{\alpha(m)}$ containing correlations and velocity fluctuations. The latter is not simply the average of $t^{\cdot\beta}_{\alpha(m)}$, but includes also the correlation and velocity fluctuation parts, which arises from the other terms in (1). The field energy-momentum tensor is then found to be

$$T^{\cdot\beta}_{\alpha(f)} = F_{\alpha\lambda} H^{\beta\lambda} - \tfrac{1}{4} F_{\lambda\mu} F^{\lambda\mu} \delta^\beta_\alpha - c^{-2} U^\beta (F_{\alpha\lambda} H^{\lambda\mu} - H_{\alpha\lambda} F^{\lambda\mu}) U_\mu + \\ + c^{-4} U_\alpha U^\beta U^\lambda F_{\lambda\mu} (H^{\mu\nu} - F^{\mu\nu}) U_\nu, \tag{5}$$

where the field tensors $F^{\alpha\beta}$ and $H^{\alpha\beta}$ are the averages of $\sum_k f_k^{\alpha\beta}$ and $\sum_k h_k^{\alpha\beta}$ [6]. Let us write the energy-momentum tensor also in the rest frame, using three-dimensional notation (with $i, j = 1, 2, 3$):

$$T^{\alpha\beta}_{(f)} = \begin{pmatrix} \tfrac{1}{2}\boldsymbol{E}^2 + \tfrac{1}{2}\boldsymbol{B}^2 & (\boldsymbol{E}\times\boldsymbol{H})^i \\ (\boldsymbol{E}\times\boldsymbol{H})^i & -\boldsymbol{E}^i \boldsymbol{D}^j - \boldsymbol{H}^i \boldsymbol{B}^j + (\tfrac{1}{2}\boldsymbol{E}^2 + \tfrac{1}{2}\boldsymbol{B}^2 - \boldsymbol{B}\cdot\boldsymbol{M}) g^{ij} \end{pmatrix}, \tag{6}$$

where $T^{00}_{(f)}$ is the energy density, $c\, T^{0i}_{(f)}$ the energy flow (the Poynting vector), $c^{-1} T^{i0}_{(f)}$ the momentum density and $T_{(f)}{}^{ij}$ the momentum flow (the Maxwell pressure tensor).

This result differs from the expressions of Lorentz, Einstein-Laub and the thermodynamical theory only by magnetization terms. Minkowski's and Abraham's proposals differ in a fundamental way from the statistical result (6).

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SHELL MODEL FIT OF THE DIELECTRIC PROPERTIES OF SOLID KRYPTON

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The refractive index of solid krypton has been measured between 65° and 116°K and the results fitted to a theory of the shift of the ultraviolet excitation frequency based on the Dick-Overhauser shell model.

It has recently been suggested [1] that the density and frequency dependence of the dielectric constant ϵ of the solidified inert gases might be well represented by the Dick-Overhauser shell model [2] which has been successfully applied to the alkali halides. We have measured the refractive index n of solid krypton ($\epsilon = n^2$) at temperatures between 65° and 116°K by a method previously described by Smith [3]; and have fitted the data to a theory proposed by Doniach and Huggins [1], based on the shift of the ultraviolet excitation frequency with density. The results are shown in fig. 1.

According to Doniach the dielectric constant ϵ measured at frequency ω and density ρ may be represented by

$$\frac{3(\epsilon - 1)}{\epsilon + 2}\frac{1}{\rho} = \frac{\omega_\rho^2}{\omega_o^2 + V_o^3 - \omega^2} + \chi_1$$

where ω_ρ^2 represents the dipole-dipole interactions of the lowest atomic states, ω_o is the frequency of the non-interacting atomic oscillators and V_o^3 corresponds to the shift in ω_o^2 caused by shell-core forces. The term χ_1 takes into account the core-polarizability and contributions to the exchange integrals from higher excited states.

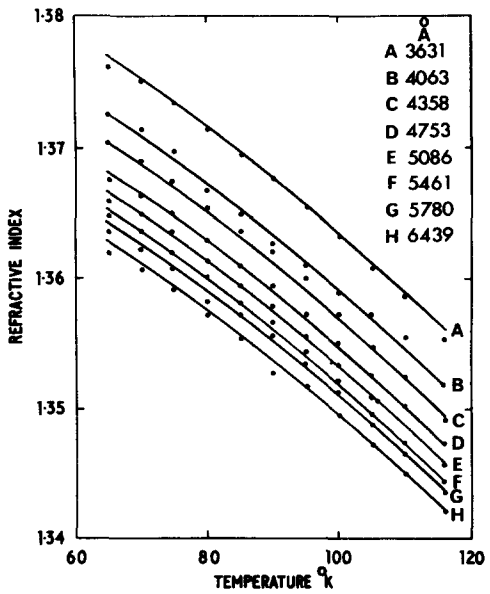

Fig. 1. Refractive index of solid krypton.

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