COVARIANT EQUATIONS OF MOTION FOR A CHARGED PARTICLE WITH A MAGNETIC DIPOLE MOMENT

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By choosing a covariant position and spin operator equations of motion are derived for a Dirac particle in a non-uniform and time dependent field.

In *classical theory* one can derive [1] equations of motion and of inner angular momentum for a composite particle with charge e and magnetic dipole moment tensor $M^{\alpha\beta} = \kappa S^{\alpha\beta}$ ($S^{\alpha\beta}$ the inner angular momentum) in an external field $F^{\alpha\beta}$, using the condition $P^{\alpha} S_{\alpha\beta} = 0$ [2] (P^{α} the momentum); if only terms of zeroth and first order in the field are retained one obtains

$$\mathrm{d}P^{\alpha}/\mathrm{d}s = c^{-1} e F^{\alpha\beta} U_{\beta} + \tfrac{1}{2}(\partial^{\alpha} F^{\beta\gamma}) M_{\beta\gamma} , \tag{1}$$

$$\mathrm{d}S^{\alpha\beta}/\mathrm{d}s = P^{\alpha} U^{\beta} - P^{\beta} U^{\alpha} + F^{\alpha\gamma} M_{\gamma}^{\cdot\beta} - F^{\beta\gamma} M_{\gamma}^{\cdot\alpha} ; \tag{2}$$

$$P^{\alpha} \equiv M U^{\alpha} - c^{-2} M^{\alpha\beta} F_{\beta\gamma} U^{\gamma} + (e/M^* c^3) S^{\alpha\beta} F_{\beta\gamma} U^{\gamma} + (1/2M^* c^2) S^{\alpha\beta} (\partial_{\beta} F_{\gamma\epsilon}) M^{\gamma\epsilon} , \tag{3}$$

where $M^* \equiv M + \frac{1}{2} c^{-2} F_{\alpha\beta} M^{\alpha\beta}$ and $S^{\alpha\beta} S_{\alpha\beta}$ are conserved quantities. Eqs. (1)-(3) show similarity with eqs. (24)-(26) of ref. [3]; however, in eq. (26) the term $c^{-2} S^{\alpha\beta} \dot{U}_{\beta}$ occurs instead of the last two terms in eq. (3). The latter terms are small for atoms, since then the 'normal' magnetic moment $(e/M^* c) S^{\alpha\beta}$, associated with the inner angular momentum, is small compared with the magnetic moment $M^{\alpha\beta}$. Likewise one can show that for atoms the term $c^{-2} S^{\alpha\beta} \dot{U}_{\beta}$ may be neglected in (26). (The remaining terms in eq. (3) or (26) read in the rest frame P^0, $\boldsymbol{P} = Mc$, $c^{-1} \boldsymbol{M} \times \boldsymbol{E}$, cf [4].)

For a single particle the 'normal' magnetic moment is of the same order as the total magnetic moment. Thus one might surmise that for a Dirac particle a term $c^{-1} \boldsymbol{M}_{\mathrm{a}} \times \boldsymbol{E}$ will occur, where $\boldsymbol{M}_{\mathrm{a}}$ is the anomalous magnetic moment. The derivation of the equations of motion and of spin for a Dirac particle in a non-uniform and time dependent field * proceeds by choosing a representation in which the Hamiltonian commutes with the Dirac matrix β. In the Dirac picture one has $H = c\boldsymbol{\alpha} \cdot \boldsymbol{\pi} + \beta mc^2 + e\varphi + (g-2)\frac{1}{2}\mu_{\mathrm{B}} (\mathrm{i}\beta\boldsymbol{\alpha} \cdot \boldsymbol{E} - \beta\boldsymbol{\sigma} \cdot \boldsymbol{B})$ with $\boldsymbol{\pi} \equiv \boldsymbol{p} - e\boldsymbol{A}/c$, $\mu_{\mathrm{B}} \equiv e\hbar/2mc$ (the last term arises from the anomalous magnetic moment). It may be transformed into an expression that commutes with β, up to all orders in c^{-1}, if one is not interested in terms in e^2 and higher and in terms with derivatives of the fields ([6], cf [7]). In this picture the Weyl-transform of the Hamiltonian reads:

$$H \rightleftarrows \beta E_{\pi} + e\varphi - \beta\mu_{\mathrm{B}} \frac{mc^2}{E} \boldsymbol{\sigma} \cdot \boldsymbol{B} - \mu_{\mathrm{B}} \frac{mc^3}{E(E+mc^2)} (\boldsymbol{p} \times \boldsymbol{\sigma}) \cdot \boldsymbol{E} + - (g-2)\tfrac{1}{2}\mu_{\mathrm{B}} \left[\beta\boldsymbol{\sigma} \cdot \boldsymbol{B} - \frac{\beta c^2}{E(E+mc^2)} \boldsymbol{p} \cdot \boldsymbol{\sigma}\, \boldsymbol{p} \cdot \boldsymbol{B} + \frac{c}{E} (\boldsymbol{p} \times \boldsymbol{\sigma}) \cdot \boldsymbol{E} \right] , \tag{4}$$

where $E_{\pi} \equiv (\pi^2 c^2 + m^2 c^4)^{\frac{1}{2}}$. Furthermore we define now a position operator with covariant [8] properties that does not give rise to Zitterbewegung by writing its Weyl transform in the same picture as used for (4) as: $\boldsymbol{X} \rightleftarrows \boldsymbol{x} + \hbar\boldsymbol{\sigma} \times \boldsymbol{\pi}/2m(E_{\pi} + mc^2)$ and likewise a spin operator as $\boldsymbol{S} \leftrightarrows \frac{1}{2}\hbar\boldsymbol{\sigma} + \hbar\boldsymbol{\pi} \times (\boldsymbol{\sigma} \times \boldsymbol{\pi})/2m(E_{\pi} + mc^2)$. Then we obtain the equation of motion

* For this case no satisfactory derivation seems to have been obtained as yet. Plahte's [5] way of introducing proper time into Dirac theory does not solve the well-known difficulties of interpretation. Moreover, matrix operators do not possess a definite transformation character; expectation values only then if a local conservation law holds true (Klein's theorem).

$$m\, \mathrm{d}^2\boldsymbol{X}/\mathrm{d}t^2 \rightleftarrows \gamma^{-2}(\vec{\vec{\mathrm{U}}}-\boldsymbol{\beta}\boldsymbol{\beta})\cdot(\gamma e\boldsymbol{E}+\gamma e\boldsymbol{\beta}\times\boldsymbol{B}+\tfrac{1}{2}g\mu_{\mathrm{B}}\,[(\boldsymbol{\nabla}\boldsymbol{B})\cdot\boldsymbol{S}+(\boldsymbol{\nabla}\boldsymbol{E})\cdot(\boldsymbol{\beta}\times\boldsymbol{S})+ \gamma^2(\partial_0+\boldsymbol{\beta}\cdot\boldsymbol{\nabla})\{\boldsymbol{\beta}\boldsymbol{B}\cdot\boldsymbol{S}+\boldsymbol{\beta}\boldsymbol{E}\cdot(\boldsymbol{\beta}\times\boldsymbol{S})\}]+\tfrac{1}{2}(g-2)\mu_{\mathrm{B}}\gamma^2(\partial_0+\boldsymbol{\beta}\cdot\boldsymbol{\nabla})\{\boldsymbol{E}\times\boldsymbol{S}-(\boldsymbol{\beta}\times\boldsymbol{S})\boldsymbol{\beta}\cdot\boldsymbol{E}+(\boldsymbol{\beta}\times\boldsymbol{B})\times\boldsymbol{S}\}), \tag{5}$$

where only terms up to the first order in e and containing not higher than first order derivatives of the fields have been included and where the fields are to be understood as functions of the position operator $\boldsymbol{X}$ and time t ($\boldsymbol{\beta} \equiv c\boldsymbol{p}/E$, $\gamma \equiv (1-\beta^2)^{-\frac{1}{2}}$, $\partial_0 \equiv c^{-1}\partial/\partial t$). For the time derivative of $\boldsymbol{S}$ we obtain

$$\mathrm{d}\boldsymbol{S}/\mathrm{d}t \rightleftarrows \gamma^{-1}[\tfrac{1}{2}g\mu_{\mathrm{B}}\{\boldsymbol{S}\times\boldsymbol{B}+(\boldsymbol{\beta}\times\boldsymbol{S})\times\boldsymbol{E}\}+\tfrac{1}{2}(g-2)\mu_{\mathrm{B}}(\gamma^2\boldsymbol{\beta}\cdot\boldsymbol{S}\boldsymbol{\beta}\times\boldsymbol{B}-\boldsymbol{S}\boldsymbol{\beta}\cdot\boldsymbol{E}+\gamma^2\boldsymbol{\beta}\cdot\boldsymbol{S}\boldsymbol{E}-\gamma^2\boldsymbol{\beta}\boldsymbol{\beta}\cdot\boldsymbol{S}\boldsymbol{\beta}\cdot\boldsymbol{E})], \tag{6}$$

where only terms up to first order in e and containing no derivatives of the fields have been included. Eqs. (5) and (6) are the quantummechanical counterparts of eqs. (1)-(3), although written in three-dimensional notation ‡.

‡ Blount [6], Shockley [9], and Van Vleck and Huang [10] use the even part of the Dirac position operator; in this way the Zitterbewegung is avoided but covariance is lost. Their resulting non-covariant equations contain $c^{-1}\boldsymbol{M}\times\boldsymbol{E}$ even for the normal magnetic moment.

References

1. W. G. Dixon, Nuovo Cim. 34 (1964) 317, 38 (1965) 1616; J. Math. Phys. 8 (1967) 1591.
2. W. Tulczyjew, Acta Phys. Polon. 18 (1959) 393.
3. S. R. De Groot and L. G. Suttorp, Physica 39 (1968) 84.
4. S. Coleman and J. H. Van Vleck, Phys. Rev. 171 (1968) 1370.
5. E. Plahte, Suppl. Nuovo Cim. 4 (1966) 246.
6. E. I. Blount, Phys. Rev. 128 (1962) 2454.
7. E. Eriksen and M. Kolsrud, Suppl. Nuovo Cim. 18 (1960) 1.
8. Cf. M. H. L. Pryce, Proc. Roy. Soc. A195 (1949) 62 for the field-free case.
9. W. Shockley, Phys. Letters 20 (1968) 343.
10. J. H. Van Vleck and N. L. Huang, Phys. Letters 28A (1969) 768.

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POLARISATION DYNAMIQUE DES PROTONS DANS LE GLYCOL ETHYLIQUE

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Proton polarizations of 50% have been obtained at 1.1°K and 25 kG in small samples of ethylene glycol containing a Cr^{V} complex.

Des cibles de protons polarisé sont utiles et parfois indispensables pour un grand nombre d'expériences en physique nucléaire et en physique des hautes énergies. En dépit d'efforts considérables, peu de progrès ont été réalisés en vue d'une cible plus riche en protons que le double nitrate de lanthane et de magnésium, bien connu sous le nom de LMN [1]. Le meilleur résultat dans cette direction a été obtenu avec le butanol [2].

Récemment, on a trouvé un complexe de Cr^{V} formé dans le glycol éthylique $(CH_2OH)_2$ par réduction du Cr^{VI} [3]. Ce complexe possède un spin effectif $S = \frac{1}{2}$ avec $g = 1.981$, mesuré dans le liquide à 300°K. Dans la solution gelée, la

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