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Flux Quantization in Type II Superconductors

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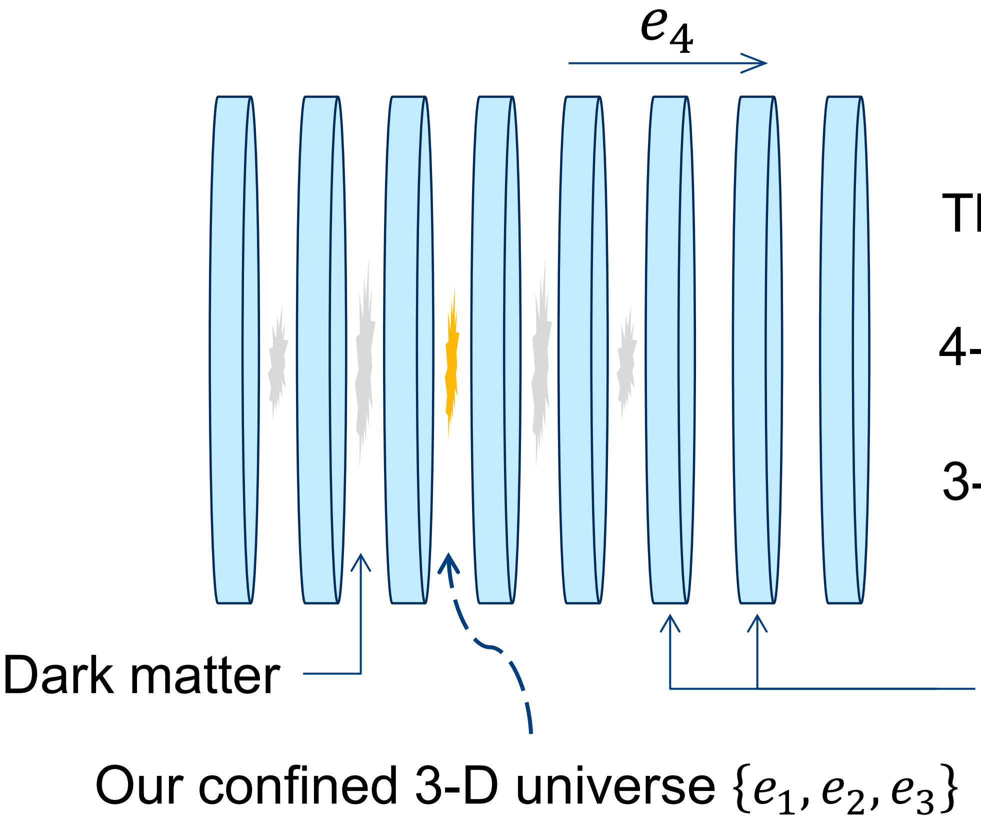
Motivation: An Approach to Large Extra Dimensions (*circa* 2000)



<u>Hypotheses</u>

There are four spatial dimensions.

4-D bulk is condensed matter.

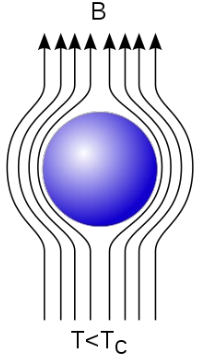
3-D hyperplanes are:

- magnetically superconducting
- transparent to gravity
- mostly confine EM fields

4-D Model Might Explain Quark Charges

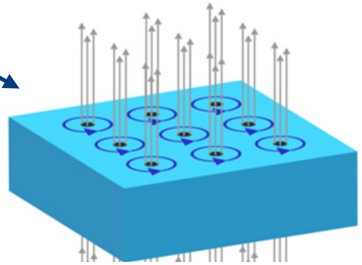
- Type I superconductors are effective EM shields.
- Type II superconductors also penetrated by flux quanta.
- Consider hypothetical, Type II, magnetic superconductors.
    - Can electric flux quanta be 1/3 or 2/3 of the electron flux?

Type I



Meissner

Type II



Abrikosov vortices

Can Type II Explain Fractional Charges of Quarks?

Type II magnetic superconductor

Electric flux quantum $q = \frac{1}{3}e^{+}$

H atom: e^{-}, p^{+}

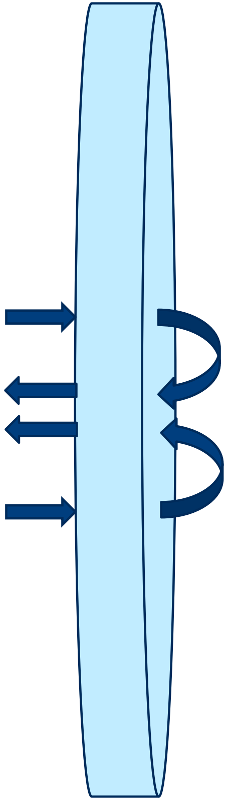
p^{+}:

$u = e^{+} - q$

$d = e^{-} + 2q$

$u = e^{+} - q$

Two of each!



Electric flux conserved

What Is Needed for Such Calculations?

- Type I superconductors, either electric or magnetic, 3-D or 4-D
  - Maxwell's equations
  - Klein-Gordon equation for charge carriers

  } Meissner effect w/London penetration depth } $\hat{B} \to 0$, $\bar{E} \to 0$

- Type II superconductors

  In 3-D

  - Current from K-G equation
  - Fundamental Theorem of Calculus (the Boundary Theorem)

  } Quantized flux tubes }

$$B: \quad \Phi_0 = 2\pi\hbar c/2e$$

$$E: \quad |\tilde{\Phi}_v| = \frac{s}{n_b n_d}\Phi_e$$

Notation

- A field $F = F(\bar{x}, t)$ in 3-D

$$F = \chi + \bar{E} + \widehat{B} + \widetilde{T}$$

- Denote grade by accents over the symbols (*ala* Terry Vold).

- Use trailing superscripts for the involutions of GA.
  - $F^{\wedge}$ for the main (or grade) involution:
  - $F^{\wedge} = \chi^{\wedge} + \bar{E}^{\wedge} + \widehat{B}^{\wedge} + \widetilde{T}^{\wedge} = \chi - \bar{E} + \widehat{B} - \widetilde{T}$

Geometric Calculus Extends Maxwell's Equations in 3-D

$$
\begin{aligned}
\partial_{\bar{x}} \cdot \bar{E} + c^{-1}\partial_t \chi &= 4\pi\rho_e, \\
\partial_{\bar{x}} \wedge \chi + \partial_{\bar{x}} \cdot \widehat{B} + c^{-1}\partial_t \bar{E} &= -\frac{4\pi}{c}\bar{J}_e \\
\partial_{\bar{x}} \wedge \bar{E} + \partial_{\bar{x}} \cdot \widetilde{T} + c^{-1}\partial_t \widehat{B} &= -\frac{4\pi}{c}\widehat{J}_m \\
\partial_{\bar{x}} \wedge \widehat{B} + c^{-1}\partial_t \widetilde{T} &= 4\pi\widetilde{\rho}_m
\end{aligned}
$$

$\bar{v}$ Vector electric currents

$\bar{v}$ Bivector magnetic currents

G.E. McClellan, "Geometric calculus-based postulates for the derivation and extension of the Maxwell equations", *Appl. Math. Inf.* 9, No. 1L, 1–10 (2015)

Easily Extended to 4-D Maxwell Equations

Fields: $F = \chi + \bar{E} + \widehat{B} + \widetilde{T} + \overset{+}{H}$

Sources: $D_{ex} = 4\pi c^{-1}(c\rho_e - \bar{J}_e - \widehat{J}_m + c\widetilde{\rho}_m - \overset{+}{J}_m)$

$$
\begin{aligned}
\partial_{\bar{x}} \cdot \bar{E} + c^{-1}\partial_t \chi &= 4\pi\rho_e, \\
\partial_{\bar{x}} \wedge \chi + \partial_{\bar{x}} \cdot \widehat{B} + c^{-1}\partial_t \bar{E} &= -\frac{4\pi}{c}\bar{J}_e \\
\partial_{\bar{x}} \wedge \bar{E} + \partial_{\bar{x}} \cdot \widetilde{T} + c^{-1}\partial_t \widehat{B} &= -\frac{4\pi}{c}\widehat{J}_m \\
\partial_{\bar{x}} \wedge \widehat{B} + \partial_{\bar{x}} \cdot \overset{+}{H} + c^{-1}\partial_t \widetilde{T} &= 4\pi\widetilde{\rho}_m \\
\partial_{\bar{x}} \wedge \widetilde{T} + c^{-1}\partial_t \overset{+}{H} &= -\frac{4\pi}{c}\overset{+}{J}_m
\end{aligned}
$$

Can have either bivector or quadvector magnetic currents

Reduce to Familiar 3-D Set In 3-D Subspace of 4-D

$$F = \cancel{\chi} + \bar{E} + \widehat{B} + \cancel{\widetilde{T}} + \cancel{\overset{+}{H}}$$

$$D_{ex} = 4\pi c^{-1}(\cancel{c\rho_e} - \bar{J}_e - \widehat{J}_m + \cancel{c\widetilde{\rho}_m} - \overset{+}{J}_m)$$

$$\begin{aligned}
\partial_{\bar{x}} \cdot \bar{E} + \cancel{c^{-1}\partial_t \chi} &= \cancel{4\pi\rho_e,} \\
\cancel{\partial_{\bar{x}} \wedge \chi} + \partial_{\bar{x}} \cdot \widehat{B} + \cancel{c^{-1}\partial_t \bar{E}} &= -\frac{4\pi}{c}\bar{J}_e \\
\partial_{\bar{x}} \wedge \bar{E} + \cancel{\partial_{\bar{x}} \cdot \widetilde{T}} + \cancel{c^{-1}\partial_t \widehat{B}} &= -\frac{4\pi}{c}\widehat{J}_m \\
\partial_{\bar{x}} \wedge \widehat{B} + \cancel{\partial_{\bar{x}} \cdot \overset{+}{H}} + \cancel{c^{-1}\partial_t \widetilde{T}} &= \cancel{4\pi\widetilde{\rho}_m} \\
\cancel{\partial_{\bar{x}} \wedge \widetilde{T}} + \cancel{c^{-1}\partial_t \overset{+}{H}} &= \cancel{-\frac{4\pi}{c}\overset{+}{J}_m}
\end{aligned}$$

- Only $\bar{E}$ and $\widehat{B}$ fields
- No free charge densities
- Quasistatic
- Selected bivector magnetic current
  - charge carriers: $\widetilde{g}_m \sim e_r e_\phi e_4$
  - azimuthal velocity: $\bar{v} \sim e_\phi$
  - current: $\widehat{J}_m \sim e_r e_4$

Need the Potentials Generated by Currents

$$\partial_{\bar{x}} \cdot \bar{E} = 0$$

(Vector potential)

$$\partial_{\bar{x}} \cdot \widehat{B} = -\frac{4\pi}{c}\bar{J}_e \quad \rightarrow \quad \bar{A} \quad \rightarrow \quad \widehat{B} = \partial_{\bar{x}} \wedge \bar{A}$$

$$\partial_{\bar{x}} \wedge \bar{E} = -\frac{4\pi}{c}\widehat{J}_m \quad \rightarrow \quad \widehat{M} \quad \rightarrow \quad \bar{E} = \partial_{\bar{x}} \cdot \widehat{M}$$

$$\partial_{\bar{x}} \wedge \widehat{B} = 0$$

(Bivector potential)

Boson Charge Carriers Obey the Klein-Gordon Equation

- Relativistic free-field equation from the canonical substitution
  - $$(\partial_{\bar{x}}^2 - c^{-2}\partial_t^2)\psi = (\hbar/mc)\psi$$
- Electromagnetic potentials incorporated with the "minimal substitution" from gauge theory

$$\bar{p} \to \bar{p} - \frac{q}{c}\bar{A} \quad \text{or} \quad \bar{p} \to \bar{p} - \frac{\tilde{q}_m}{c}\widehat{M} \qquad \bar{p} \to -I\hbar\partial_{\bar{x}}$$

- With a vector potential, the conserved electric current is:

$$\bar{J}_e = \frac{q}{2m}[\psi^\wedge(-I\hbar\partial_{\bar{x}} - \frac{q}{c}\bar{A})\psi - \psi(-I\hbar\partial_{\bar{x}} + \frac{q}{c}\bar{A})\psi^\wedge]$$

- With a bivector potential:

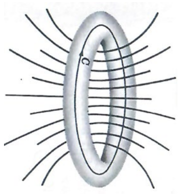
$$\widehat{J}_m = -\frac{\tilde{q}_m}{2m}[\psi^\wedge(-I\hbar\partial_{\bar{x}} - \frac{\tilde{q}_m}{c}\widehat{M})\psi - \psi(-I\hbar\partial_{\bar{x}} + \frac{\tilde{q}_m}{c}\widehat{M}))\psi^\wedge]$$

Split the Supercurrent into Canonical and "Electrokinetic" Components

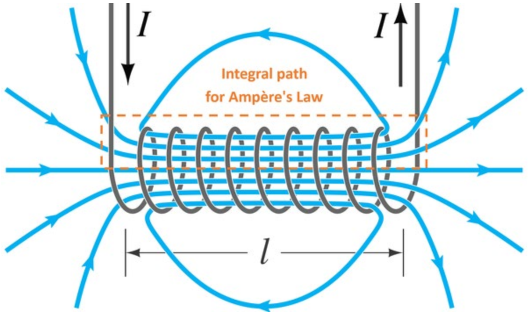
- Electric superconductor
  - Canonical current: $\bar{J}_C = \frac{q}{2m}[\psi^\wedge(-I\hbar\partial_{\bar{x}})\psi - \psi(-I\hbar\partial_{\bar{x}})\psi^\wedge]$
  - Electrokinetic current: $\bar{J}_A = \frac{q}{2m}[\psi^\wedge(-\frac{q}{c}\bar{A})\psi - \psi(\frac{q}{c}\bar{A})\psi^\wedge]$
  - Total electric supercurrent: $\bar{J}_e = \bar{J}_C + \bar{J}_A$
- Maxwell $\partial_{\bar{x}} \cdot \widehat{B} = -\frac{4\pi}{c}\bar{J}_e$, given $\widehat{B} \to 0$, requires $\bar{J}_e = 0$.
- Must balance canonical and electrokinetic currents.

Apply to a Tube in 3-D and a Hypertube in 4-D

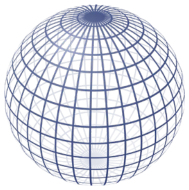
3-D:



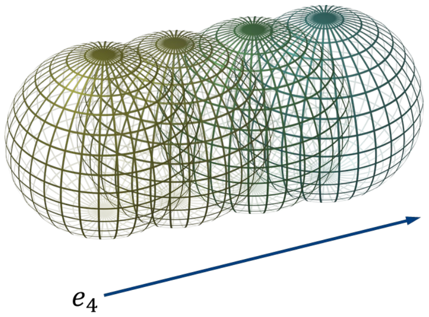
- Tube: Replicate current loop into a 3rd dimension.



4-D:

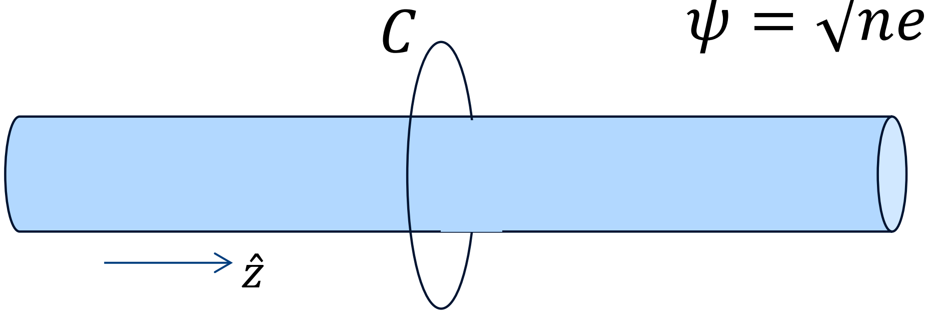


- Hypertube: Replicate sphere with surface current into a 4th dimension.



Tube in 3-D: Simplest ψ is Exponential "Ground State" Around the Tube

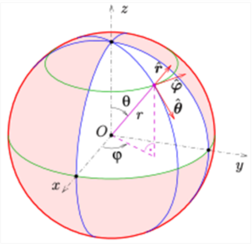
$$\psi = \sqrt{n}e^{I\phi}$$

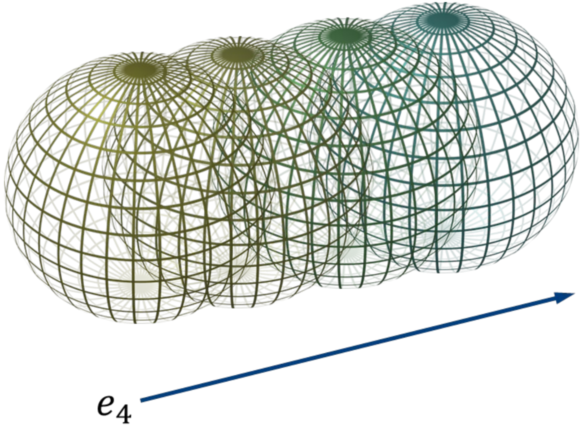


Hypertube in 4-D: Assume Spherical Harmonic "Ground State" ψ

$$\psi = \sqrt{n}Y_1^1(\theta,\phi) = -\sqrt{n}\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i_3\phi}$$

Cross section uses spherical coordinates (r,θ,ϕ) :



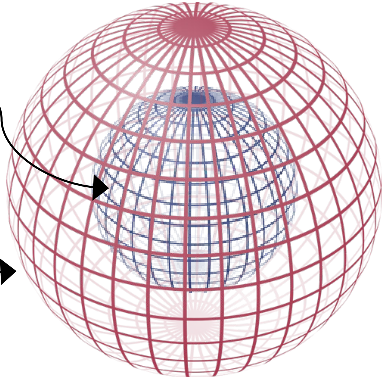


Calculate Magnetic Currents With the Spherical Harmonic ψ

Supercurrent sheet

- thin (London penetration depth)
- normal conductor inside
- superconductor outside

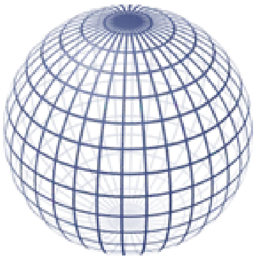
Integration surface S in the superconductor away from supercurrent sheet



Canonical: $\hat{J}_{C'} = e_r e_4 \frac{3g\hbar n}{8\pi m} \frac{\sin\theta}{r}$

Electrokinetic: $\hat{J}_M = -\frac{3g^2 n}{8\pi mc} \sin^2\theta \widehat{M}$

For Magnetic Superconductor, Need Quadvector Component of the Boundary Theorem



- Theorem: $$\iiint_V d\tilde{V}\,\partial_{\bar{x}} F = \oiint_S d\hat{\sigma} F$$

- Components

vector: $$\iiint_V d\tilde{V} \cdot (\partial_{\bar{x}} \wedge \bar{A}) = \oiint_S d\hat{\sigma} \cdot \bar{A}$$

quadvector: $$\iiint_V d\tilde{V} \wedge (\partial_{\bar{x}} \cdot \widehat{M}) = \oiint_S d\hat{\sigma} \wedge \widehat{M}$$ $\Leftarrow$

Integrate Currents Over the Boundary; Apply Boundary Theorem to Electrokinetic Result

- Canonical current

$$\oiint_S d\widehat{\sigma} \wedge \widehat{J}_{C'} = \frac{3g\hbar n}{8\pi m} \oiint_S d\widehat{\sigma} \wedge (e_r e_4 \frac{\sin\theta}{r}) = I_4 \frac{3ghn}{16m} r$$

- Electrokinetic current

$$-\oiint_S d\widehat{\sigma} \wedge \widehat{J}_M = \frac{3g^2 n}{8\pi mc} \oiint_S d\widehat{\sigma} \wedge (\sin^2\theta \widehat{M}) = \frac{3g^2 n}{8\pi mc} \iiint_V d\widetilde{V} \wedge [\partial_{\bar{x}} \cdot (\sin^2\theta \widehat{M})]$$

$$= \frac{3g^2 n}{8\pi mc} (\frac{3}{4}) \iiint_V d\widetilde{V} \wedge [\partial_{\bar{x}} \cdot \widehat{M}] = \underbrace{\frac{9g^2 n}{32\pi mc} \iiint_V d\widetilde{V} \wedge \bar{E}}_{\text{Flux quantum } \overset{+}{\Phi}_E}$$

- Equate these current integrals to satisfy: $\widehat{J}_m = \widehat{J}_{C'} + \widehat{J}_M = 0$

Solve for Electric Flux Quantum

$$\overset{+}{\Phi}_E = \iiint_V d\widetilde{V} \wedge \bar{E} = \frac{32\pi mc}{9g^2 n} I_4 \frac{3ghn}{16m} r = I_4 \frac{2\pi hc}{3g} r$$

From abstract: $g \longrightarrow q_m = n_b n_d \frac{\hbar c}{2e} = n_b n_d \frac{hc}{(4\pi e)}$

n_d - Dirac's integer

n_b - # of Dirac charges in a boson

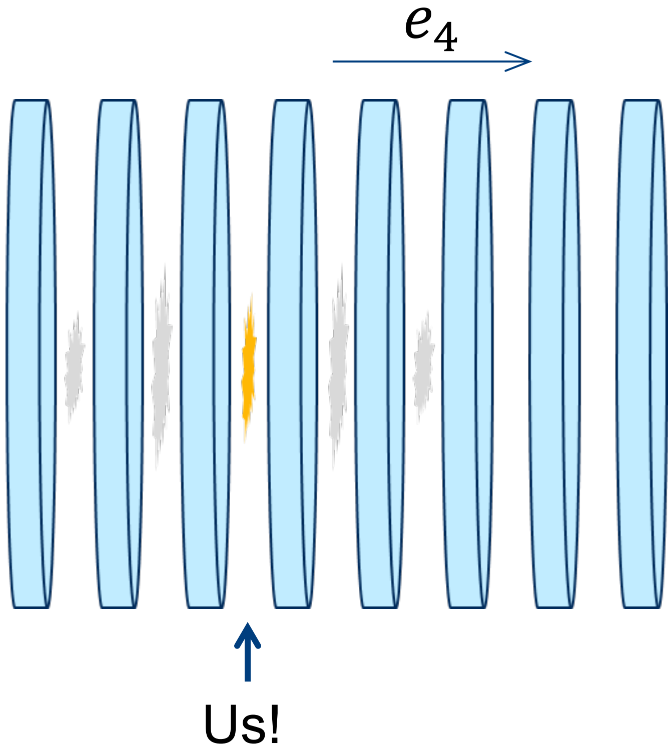
so: $\overset{+}{\Phi}_E = I_4 \frac{(2\pi)^2 2e}{3n_d n_b} r = I_4 \frac{(4\pi e)}{3n_d n_b} \underbrace{(2\pi r)}_{?}$

Results

- See abstract for 3-D superconductors:
  - Obtain correct, experimentally observed, magnetic flux quantum
  - Obtain analogous electric flux quantum result that allows 1/3 e+
- Analyzing 4-D superconductors
  - Same techniques of geometric calculus can be applied
  - Tantalizing but issues remain
  - No conclusion yet

Remaining Issues

- What is the relationship between
  - trivector flux in 3-D
  - quadvector flux in 4-D
- Why does the boundary integral in 4-D yield a factor of r?

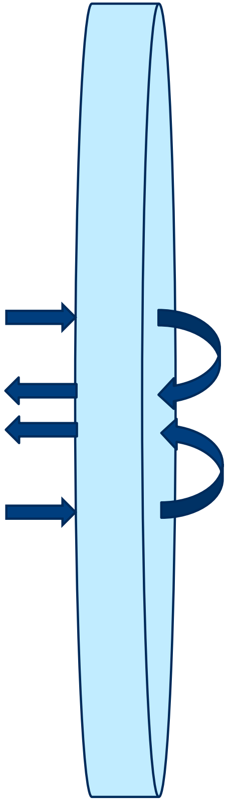


Thank you!

In any case, the value of geometric calculus is apparent!

$$e^-$$

$$p^+ \begin{cases} u = e^+ - q \\ d = e^- + 2q \\ u = e^+ - q \end{cases}$$



Questions?