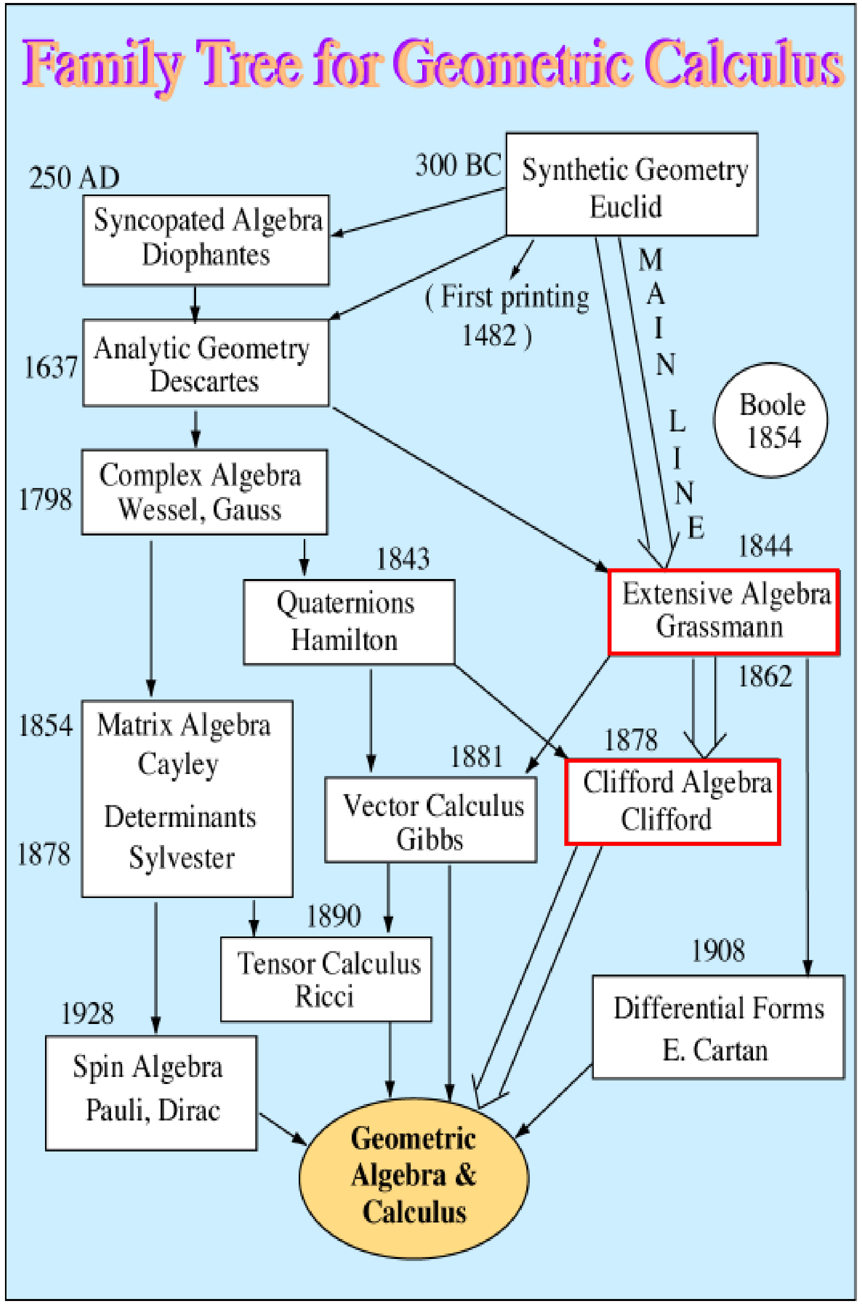
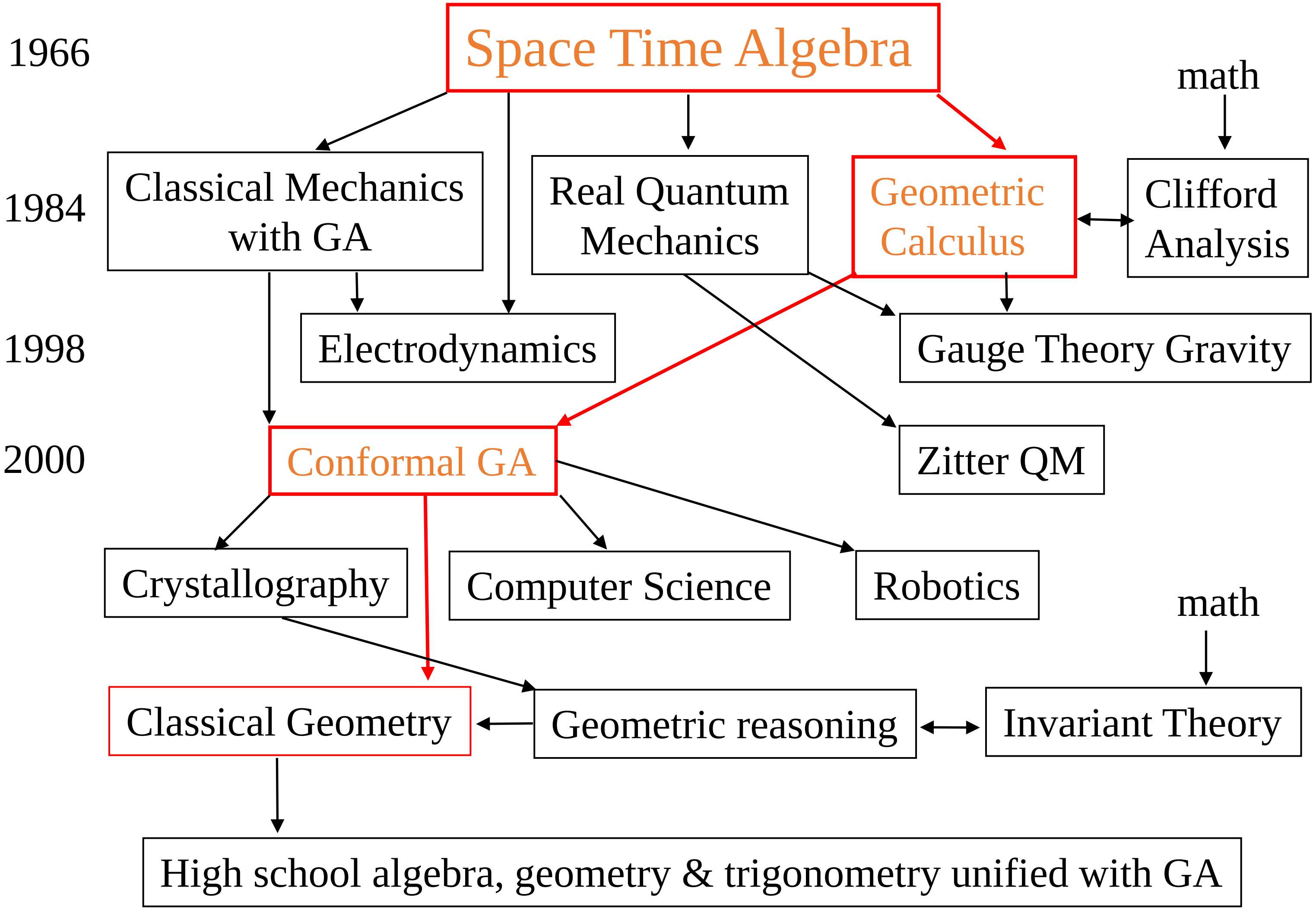


Development of Geometric Algebra & Calculus



Claim: *GA/GC has a wider range of applications than other mathematical system!*

Modeling the **Electron** with **Geometric Algebra**

David Hestenes
Arizona State University

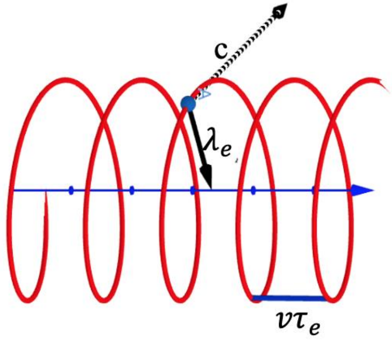
AGACSE 2024

In 1953 when Einstein was asked why he was not excited by new discoveries in high energy physics such as Yukawa's 1949 Nobel Prize, he answered . . .

"You know, it would be sufficient to really understanderstand the electron!" — *Einstein* (1953)

I claim the key to that understanding is .

Electron **zitter** (zitterbewegung)



which was already discussed by Dirac with his 1933 Nobel prize (shared with Schroedinger)

But **zitter is not recognized** as significant in Standard Quantum Mechanics even today.

I have spent decades trying to understand the significance of zitter
with a definitive conclusion only recently . . .
that is what I will talk about today

Landmarks in Dirac Theory
A personal chronical of my understanding

1933 Matrix Dirac equation Nobel prize **DH birthday** 5/21/1933
after 1932 confirmation of Anderson's discovery of the positron **A miracle**!

Zitterbewegung discovered by Schrödinger and noted in Nobel prize
Dirac always assumed electron is a point charge

1951 **Aether** completes Dirac's proposals for his theory avoiding the the QED wars

1966 STA introduces Real Dirac equation
Introduces **zilch** variable in Dirac and Maxwell theory without relating them
Began study of the geometry of **local observables** to interpret solutions of the Dirac eqn.

1985 Proposed circular zbw as explanation for **interpretation of the Uncertainty Principle**

2001 **Blinder Ansatz** to explain origin of electron mass as energy density in the vacuum.

2021 **London Ansatz** assumes that charge current in electron zitter
is proportional to its vector potential (proposed in the Czech Republic 8/2021)

2022 **Zilch is identified with the Aether** coupling Maxwell with Dirac
(First proposed @ ICACGA 2022 in Denver 8/2022)

2024 Preprint (**Gyromagnetics of the electron clock**) completed and presented @ AGACSE2024
FINI: There are no more degrees of freedom to explain with this theory!

Real Dirac Theory: the geometry of electron motion with de Broglie's **electron clock** in quantum mechanics!

Dirac equation determines a **congruence of streamlines**,
each a potential particle history $x = x(t)$
with particle velocity $\dot{x} = v(t) = Rg_0\tilde{R}$

Spinning frame picture of electron motion

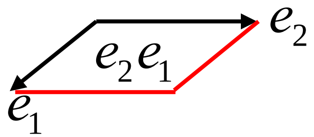
Dirac wave function $\psi = (\rho e^{i\beta})^{1/2}R$ determines

Rotor: $R = R(t) = R[x(t)] = Ve^{-\mathbf{i}\varphi/2}$

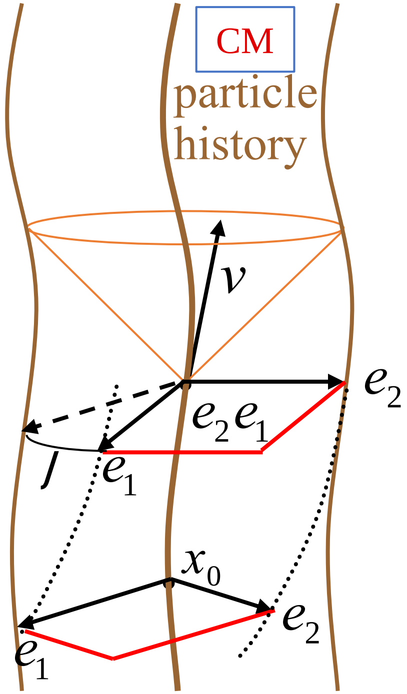
comoving frame: $e_m = Rg_m\tilde{R}$

velocity: $e_0 = Rg_0\tilde{R} = v$

Spin: $S = \frac{\hbar}{2}e_2e_1$



Local Observables



Without loss of generality, the Spin bivector determines a unique *electron rest frame* with $S = \frac{\hbar}{2}\, i\boldsymbol{\sigma}_3 = \boldsymbol{i}s$

$\boldsymbol{i} = i\boldsymbol{\sigma}_3 = \boldsymbol{\sigma}_1\boldsymbol{\sigma}_2 = e_2e_1$

Zitter Solutions of the Dirac equation

The conservation law: $\Box \cdot (\rho u) = 0$

for the Dirac current: $\psi \gamma_0 \tilde{\psi} = \rho R \gamma_0 \tilde{R} = \rho u$

implies: Dirac *streamlines* ≈ *particle paths*

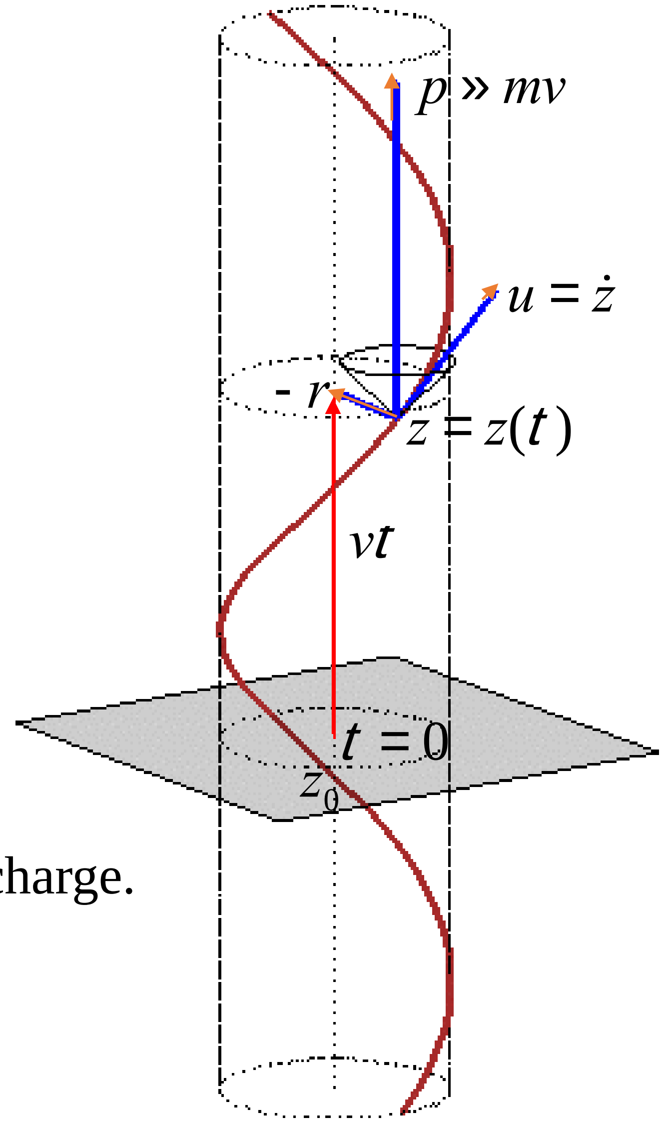
$$z = z(t) \qquad u = \dot{z} \qquad u^2 = \dot{z}^2 = 0$$

Rotor $R = e^{-Ij} V$ with $u = v + e_2$ so $u^2 = 0$

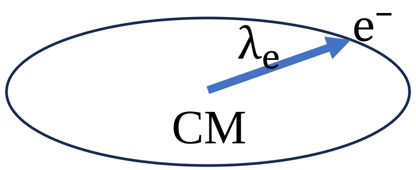
determines a *lightlike helical path* with

Zitter Radius

$$l_e = \frac{c}{w_e} = \frac{\hbar}{2m_e c} = 1.93079 ´ \ 10^{-3} \text{ Å} = \frac{l_C}{4p}$$



Dirac assumed that the electron is a point particle with charge.



Real Dirac Electron Theory

Chiral Forms: $\Psi_{\pm} = (\rho e^{i\beta})^{1/2} R_{\pm}$ (2+6=8) $R_{\pm} = U_n \Lambda_{\pm} U_l U_m$ $\Lambda_{\pm} = (1 \pm \boldsymbol{\sigma}_2)$

Canonical Dirac: $(\partial_t + c\nabla)\Psi \boldsymbol{i}\hbar = m_e c^2 \Psi^* + e(A_0 - \mathbf{A}))\Psi$

$(\partial_t + c\nabla)\Psi \boldsymbol{i}\hbar = m_e c^2 e^{-i\beta} \Psi + e(A_0 - \mathbf{A}))\Psi$ $\Psi^* = e^{-i\beta} \Psi$

Zitter-Dirac: $\dot{\Psi} \boldsymbol{i}\hbar = m_e c^2 \Psi^* + e(A_0 - \mathbf{A}))\Psi$

CM path $\Psi = \Psi(z(\tau)) = \Psi(z(t), t)$ $(\partial_t + c\nabla)\Psi = \dot{\Psi}$

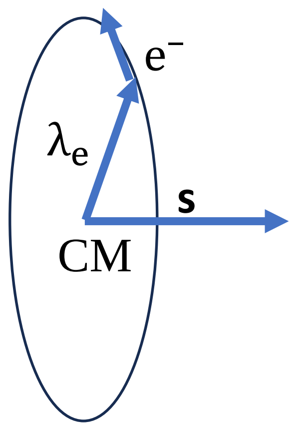
Chiral Local Observables

Spin density: $\frac{\hbar}{2}\Psi i\gamma_3 \tilde{\Psi} \boldsymbol{v} = \rho i s \boldsymbol{v} = \rho i s$ *for* $v = \gamma_0$

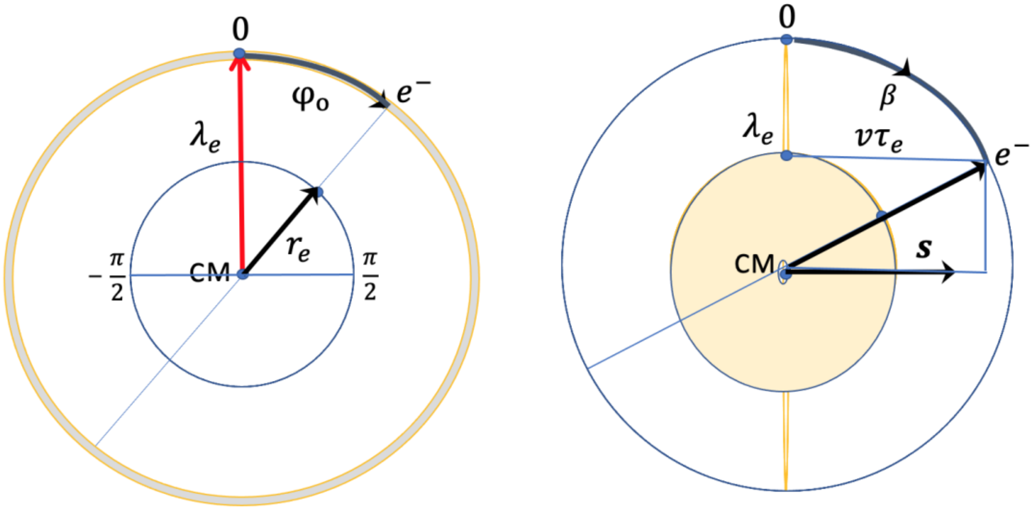
Spindle: $\rho S = \frac{\hbar}{2}\Psi_+ \gamma_2 \gamma_1 \tilde{\Psi}_+ = \frac{\hbar}{2}\Psi \Sigma_+ \tilde{\Psi}$ $RS\tilde{R} = e^{i\beta} S$

Spindle: **S** = **h** + i**s** $\mathbf{S}^2 = 0$

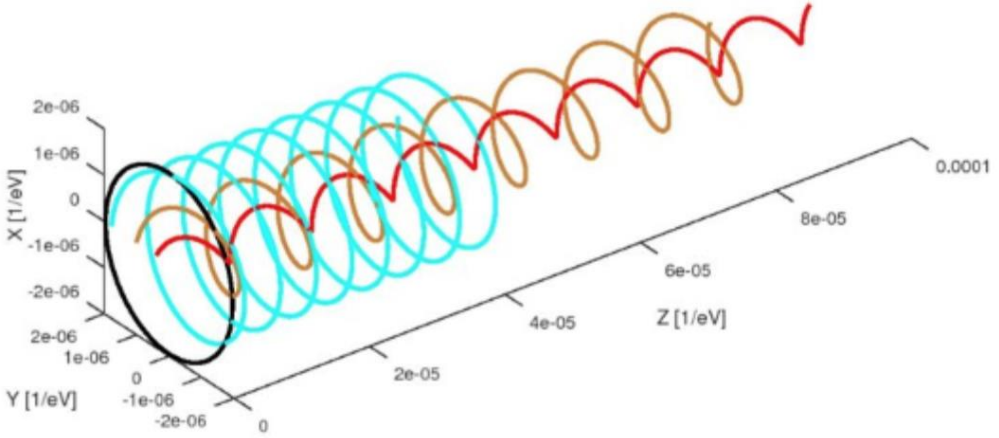
Spinette: $\mathbf{h} = \frac{\hbar}{2}\hat{\mathbf{h}}$ (clock hand)



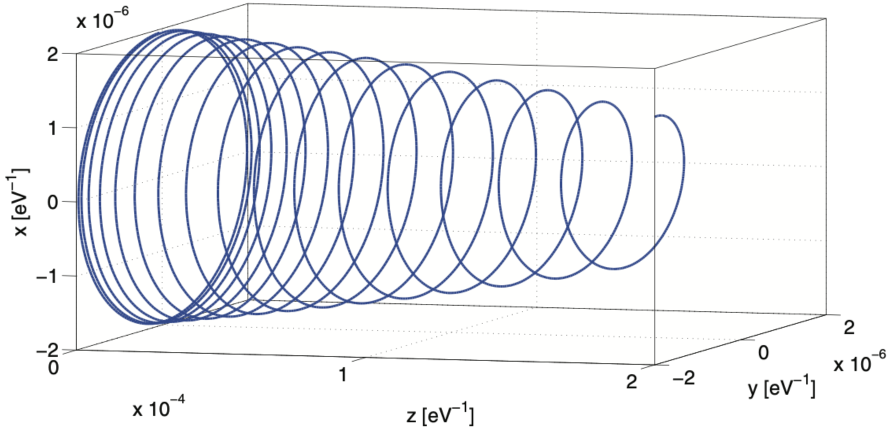
The electron **Energy Shell** is a sphere of **zitter radius** $\lambda_e = \hbar / 2m_e c$



(a) (b)



Zitter motion with constant speeds along spin ***s.***



Zitter path during acceleration along spin ***s.***

The **electron Spindle** frames a sphere with two orthogonal cross sections:

(a) The Spindle ring has a fixed pole and electron position on an Energy Bubble generated by accelerating the electron.

(b) The tilt angle β measures energy in the bubble and its propagation along the spin **s**.

Spin — Spindle — Spinet

What is an electron?!

"It is a delusion to think of electrons and fields as two physically different, independent entities. Since neither can exist without the other, there is only *one* reality to be described, which happens to have two different aspects; and the theory ought to recognize this from the outset instead of doing things twice!" – *Einstein*

Field and particle are **all ready unified** *in the Dirac equation!!*

Zitter-Dirac	$\dot{\Psi}\boldsymbol{i}\hbar = m_e c^2\, \Psi^* + e(A_0 - \mathbf{A}))\Psi$	$\mathbf{A} = \mathbf{A}(\mathbf{x}, t)$
CM path	$\Psi = \Psi(z(\tau)) = \Psi(z(t), t)$	$(\partial_t + c\nabla)\Psi = \dot{\Psi}$

$$\rho = \rho(\mathbf{x}, t) = \boldsymbol{e}^{-\lambda/r}$$ *Blinder function!*

P Electrons are elementary singularities in the vacuum!

P All elementary particles are topological defects in the vacuum!

What flows in solutions of the Dirac equation?

$r v =$ ***probability current.*** **Born–Dirac Theory**

$e r u =$ ***charge current.*** **Maxwell–Dirac Theory**

Giving the electron a charge requires a particle path

Blinder mass density: $\rho = e^{-\lambda_c / r} = 0 \xrightarrow{u=\dot{z}(\tau)}$ path

Consistent solution with Blinder ρ **requires** two phases with

lightlike helical path (zitter), and

magnetic moment with $g = 2$

Rotor Solution: $R_{\pm} = U_n \Lambda_{\pm} U_l U_m$ $\Lambda_{\pm} = (1 \pm \boldsymbol{\sigma}_2)$

Coulomb Magnetic

Electron vector potential: $A_e = A_C + A_M$

The singular electron: a classical – QM synthesis

Momentum: $p = P - \frac{e}{c}A = m_e c u \qquad u^2 = 0$

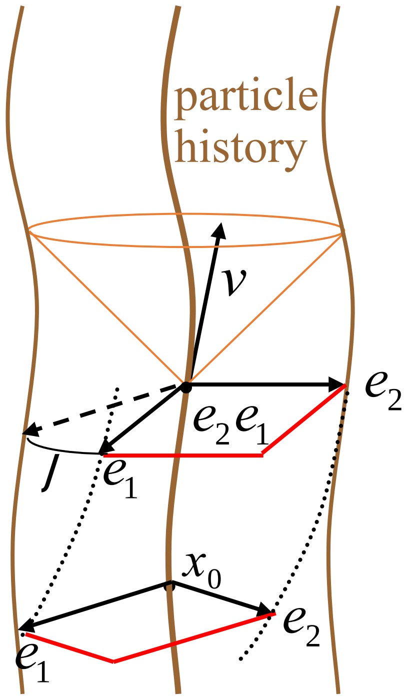
CM velocity: $v = \bar{u} = v(\tau) = \dot{z}(\tau)$

Principal rotor: $R = U_e \Lambda_\pm U_m$

Electric phase: $U_e = e^{-I(\omega_e \tau + \varphi)}$

Canonical Momentum: $P = \frac{\hbar}{2} R \gamma_0 \tilde{R}$

Gen. Lorentz Force: $f = \dot{p} = \frac{e}{c} F \cdot v + v(v \cdot \dot{p})$



The term $v \cdot \dot{p}$ describes the transfer of energy (mass) from particle to field.
Two ways to do that: *Production of light and magnetic field*.

ELF: Extended Lorentz Force:

$$f = \dot{p} = \frac{e}{c} F \cdot v + v(v \cdot \dot{q})$$

Spacetime split with respect to the particle velocity v *separates force from energy*:

$$\dot{p} \wedge v = e[\boldsymbol{E} + \mathbf{v} \times \mathbf{B}/c] = \mathbf{f} \qquad v(v \cdot \dot{q}) = \dot{q} \qquad \boldsymbol{q} = -\boldsymbol{s}\dot{\beta}$$

zilch flux

Explains **atomic stability** and **synchrotron radiation!**

Standard circuit theory: $\mathcal{E}mf = \oint \mathbf{f} \cdot d\,\boldsymbol{r} = e \oint \boldsymbol{E} \cdot d\,\boldsymbol{r}$ misses Zilch flux

$\mathcal{E}mf = \oint \mathbf{f} \cdot d\,\boldsymbol{r} = e \oint [\boldsymbol{E} + \mathbf{v} \times \mathbf{B}/c] \cdot d\,\boldsymbol{r}$ around a closed curve $\mathcal{C}$

Magnetic flux across through a closed curve $\mathcal{C}$ $\quad \Phi_B = \int_{\partial\mathcal{C}} \mathbf{B} \cdot d^2\,\boldsymbol{r}$

$$\mathcal{E}mf = -\frac{d\Phi_B}{dt} = \frac{d}{dt}\int_{\partial\mathcal{C}} \mathbf{B} \cdot d^2\,\boldsymbol{r} = e \oint [\boldsymbol{E} \pm \mathbf{v} \times \mathbf{B}/c] \cdot d\,\boldsymbol{r}$$

Consequently it overlooked a major source of energy in the EM vacuum!
Stay tuned for consequences!

Electromagnetic Field Theory

EM Field: F = **E** + *i***B** = F(x) | ε(x) = vacuum permittivity

Field Intensity: G = **D** + *i***H** = G(x) | $c^2 = 1/\varepsilon\mu$

Maxwell's Eqn: $\nabla G = J_e$ | G(x) = ε(x)F(x)

$$\nabla G = \nabla \cdot G + \nabla \wedge G$$

$\nabla \wedge G = 0$ Faraday's Law

$\nabla \cdot G = J_e$ Ampere's Law

Blinder ansatz: energy density ∝ charge density $\rho = 1/\varepsilon$

$\rho = \rho(r) = e^{-\lambda_c/r}$

$\lambda_c = \alpha_e\, \lambda_e = e^2/2m_e c^2$

Ties charge to mass

$\Longrightarrow$ $W_e = \frac{1}{2}\int \boldsymbol{E}\cdot\boldsymbol{D}\, d^3r = m_e c^2$

$r = (x - z(\tau))\cdot v = |\mathbf{r}|$

$r = (x - z(\tau))\wedge v = |\mathbf{r}|$

Zilch Invariance

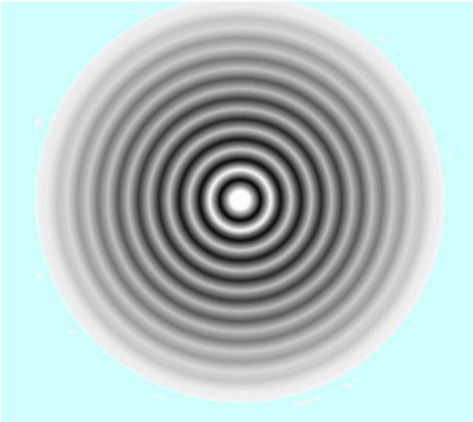
$\mathrm{F} = \mathbf{f}e^{i\beta} \Longrightarrow$

Poynting Energy flux: $T(n) = \frac{1}{2}\tilde{\mathrm{F}}n\mathrm{F} = \frac{1}{2}\tilde{\mathbf{f}}e^{i\beta}ne^{i\beta}\mathbf{f} = \frac{1}{2}\tilde{\mathbf{f}}n\mathbf{f}$

Ties field to source

The missing **magnetic field:**

J. J. Thompson 1899: **electron mass**
Uhlenbeck and Goudsmit 1925: **electron spin**



Ampere's Law: $\nabla \times \mathbf{H} = \frac{1}{c}\mathbf{J}_e = \frac{e}{c}\dot{\mathbf{r}}(\tau)$

Blinder Ansatz: $r = r(x) = e^{-l_c/r} = \mathbf{B} / \mathbf{H}$

London Ansatz: $\mathbf{J}_e(\tau) = e\mathbf{A}(\tau)$

Beltrami eqn: $\Longrightarrow$ $\nabla \times \mathbf{A} = \rho(r)\,\mathbf{A}(r,\tau) = \mathbf{B}$

Solution: **Hopf fibration** space-filling curves (Magnetic force lines)

$\mathbf{B} = \nabla \times \mathbf{A} = \lambda^2\,\mathbf{n}(r)$ scaling factor

Rotor generator: $\mathbf{n}(r) = \mathrm{U}\boldsymbol{\sigma}\mathrm{U}^{-1}$ $\lambda = 1/(1+r^2)$

$\dot{\mathbf{A}}(\tau) = \frac{\mathbf{v}}{c} \times \mathbf{E} = \mathbf{B}$ **Biot – Savart Equation**

Maxwell-Dirac QED:

A complete and consistent *classical theory* of

Dirac Particles $\Leftrightarrow$ $\boldsymbol{Maxwell\ Fields}$

Invariant formulation with (Real) Space-Time Algebra (STA)
Including Space-Time Splits for Inertial systems

Structured by Conservation Laws: for particle paths embedded in fields
Removes ambiguities in Dirac current interpretation:
conservation of: charge q vs. mass m vs. probability!

The REAL hidden variable in QM is the Zilch angle $\beta = \beta(x)$, or just **Zilch!**

It provides the missing link between particles and fields: Dirac $\longleftrightarrow$ $Zilch$ $\longleftrightarrow$ Maxwell

And the embedding of particle paths in fields: Blinder $\Leftrightarrow$ $London$

Without requiring renormalization

The problem of mass: Higgs vs. Zilch is resolved by:

$$m_e c^2 cos\beta = Vacuum\ Energy\ density\ from\ an\ electron$$

The main problem with standard QM and QED:

They don’t know Zilch!

Math language: Complex matrix algebra vs. **GA & GC: Algebra + Geometry**

Why are observables represented by Hermitian matrices? Math vs. Physics

Because of a non–geometric degree of freedom

Feynman’s tirade: Axiomatic Field Theory
Foldy–Wouthuysen transformation hides $\boldsymbol{\beta}$ Cut his feet off!

Excessive use of Fourier analysis: Misses: **\beta =** $\boldsymbol{\beta} = \boldsymbol{Zilch}$

Superposition principle: from linearity of Dirac equation
not from probability!

Fini: A **Zitter-Zilch mantra: Spin — Spinet — Spindle**

quod erat demonstrandum

THE WORD IS OUT!

A challenge to the physics community!

Critically examine the following claims:

- GA provides a unified language for the whole of physics that is conceptually and computationally superior to alternative math systems in every application domain.
- GA can enhance student understanding and accelerate student learning of physics.
- GA is ready to incorporate into the physics curriculum.
- GA provides new insight into the structure and interpretation of quantum mechanics and relativity theory
- Research on the design and use of mathematical tools is equally important for instruction and for theoretical physics.