Bridge between hyperbolic and circular symmetry illuminates spacetime spinors

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Main result:

Rotor representations for the **complete Lorentz group** $\{I, P, T, PT\}$.

$\mathbb{R}^{4,0} \Leftrightarrow \mathbb{R}^{1,3}$

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It is a real pleasure to share my work with you!

A circle is a closed object, where you can return to the starting point after a full 2π rotation. In contrast, a hyperbola is an open object with two separate parts. However, on my poster it is shown that an hyperbola is also a closed object, where you can return to the starting point after a full 2π rotation.

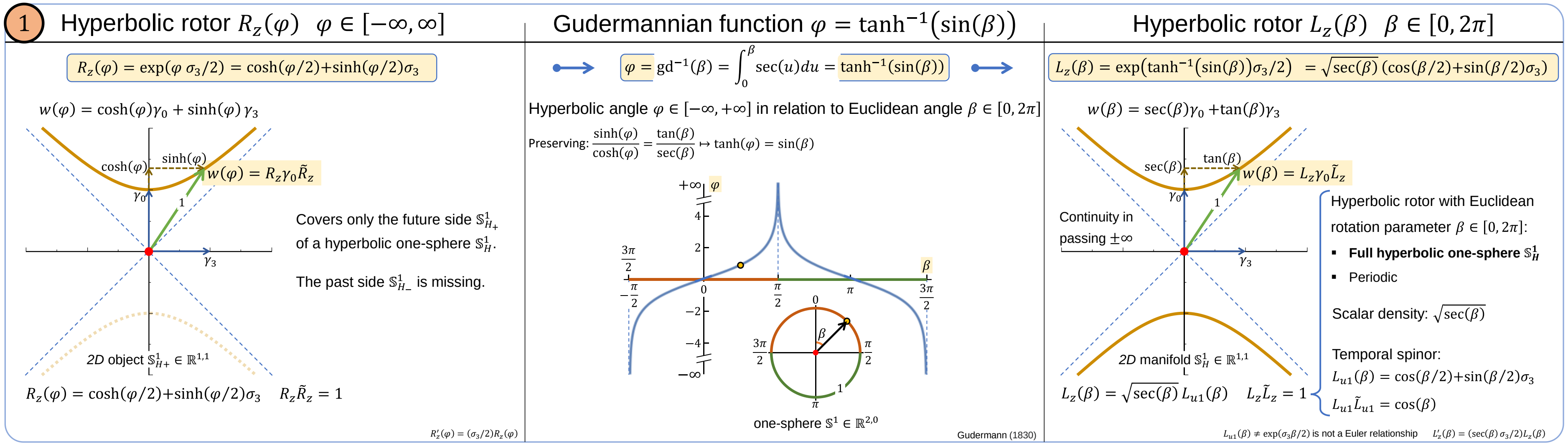
This bridge between hyperbolic and circular symmetry is created by a hyperbolic rotor with a Euclidean rotation parameter β, achieved through the relationship between hyperbolic angle φ and Euclidean angle β as defined by the Gudermannian function (Gudermann 1830)*.

The effect of this bridge is that all spacetime event reflections of the Lorentz group (identity I, parity P, time reversal T and spacetime reversal PT) can be applied through continuous changes in the all-Euclidean set of rotation parameters. So, for each of the spacetime event reflections of the Lorentz group $\{I,\ P,\ T,\ PT\}$, there is a corresponding spacetime rotor and so connected spacetime spinor representation. The fascinating part is:

1) that the members of the Lorentz group are connected to identity.
2) that the spacetime spinors are eigen spinors of the Dirac equation.

A bridge provides connection and gives beautiful views. So, I challenge you to walk across this bridge to explore and gain new perspective …

* Christoph Gudermann (25 March 1798 – 25 September 1852) was a German mathematician noted for introducing the Gudermannian function and the concept of uniform convergence, and for being the teacher of Karl Weierstrass, who was greatly influenced by Gudermann's course on elliptic functions in 1839–1840, the first such course to be taught in any institute (https://en.wikipedia.org/wiki/Christoph_Gudermann).



(1.a) (1.b) (1.c)

(1.a) Hyperbolic rotor $R_z(\varphi) = \cosh(\varphi/2)+\sinh(\varphi/2)\sigma_3$, brings temporal basis vector γ_0 to hyperbolic unit vector $w(\varphi) = R_z\gamma_0\tilde{R}_z = \cosh(\varphi)\gamma_0 + \sinh(\varphi)\,\gamma_3$. However, it covers only the future side of the hyperbolic symmetry. The past side of the symmetry is missing.

(1.b) The gudermannian function relates hyperbolic angle $\varphi \in [-\infty, +\infty]$ to Euclidean angle $\beta \in [0,\ 2\pi]$ by preserving the ratio's $\dfrac{\sinh(\varphi)}{\cosh(\varphi)} = \dfrac{\tan(\beta)}{\sec(\beta)}$. This results in the angle relationship $\tanh(\varphi) = \sin(\beta) \mapsto \varphi = \tanh^{-1}(\sin(\beta))$.

(1.c) Substitution of the Gudermannian function in hyperbolic rotor $R_z(\varphi) = \exp(\varphi\ \sigma_3/2)$ gives the hyperbolic rotor $L_z(\beta) = \exp(\tanh^{-1}(\sin(\beta))\sigma_3/2) = \sqrt{\sec(\beta)}\,L_{u1}(\beta)$ with Euclidean rotation parameter $\beta \in [0,\ 2\pi]$. Here, the hyperbolic rotor $L_z(\beta) = \sqrt{\sec(\beta)}\,(\cos(\beta/2)+\sin(\beta/2)\sigma_3)$:

- Has a clear separation between scalar density $\sqrt{\sec(\beta)}$ and temporal spinor $L_{u1}(\beta) = \cos(\beta/2)+\sin(\beta/2)\sigma_3$, not present in the hyperbolic rotor $R_z(\varphi)$.
- Brings temporal basis vector γ_0 to hyperbolic unit vector $w(\beta) = L_z\gamma_0\tilde{L}_z = \sec(\beta)\gamma_0 + \tan(\beta)\gamma_3$, enabling a full hyperbolic symmetry.

Hence, a full 2π rotation (1.b) offers a full hyperbolic rotation $w(\beta) = \sec(\beta)\gamma_0 + \tan(\beta)\gamma_3 \mapsto \beta \in [0,\ 2\pi]$ (1.c), making the hyperbola a closed object and enabling a continuity in passing through $\pm\infty$ in the hyperbolic symmetry.