Proportionality in Multiwinner Voting with Weighted Seats

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Complex Domains

- **Multiwinner Voting:** A job panel must produce a shortlist of k candidates to continue to the next interview stage.
- **Participatory Budgeting:** Citizens must decide on the public projects, each coming with a cost, that are to be implemented by the local municipality, subject to a budget.

We look *another* such complex domain.

Talk Outline

⋆ Standard Multwinner Voting (MWV) Model

⋆ Proportionality in MWV.

⋆ MWV with Weighted Seats.

(Approval-based) MWV Model

- Candidates $C = \{a, b, c, \ldots\}$.
- Agents $N = \{1, \ldots, n\}$.
- Each agent submits an *approval ballot* $A_i \subseteq C$.
- Outcome is a committee $W \subseteq C$ of size k.

Proportionality in MWV

Definition (ℓ-cohesiveness)

For an integer $\ell \in \{1, \dots, k\}$, a group of agents $N' \subseteq N$ is ℓ-cohesive if $|N'| \geqslant n \cdot \frac{\ell}{k}$ and $|\bigcap_{i \in N'} A_i| \geqslant \ell$.

Example

- Candidates $C = \{a, b, c, d\}$ with $k = 3$.
- Agents $N = \{1, 2, 3\}$.
- Approval ballots are $A_1 = \{a, b\}$, $A_2 = \{a, b, c\}$ and $A_3 = \{c, d\}$.
- $\{1, 2\}$ is 2-cohesive.
- $\{2, 3\}$ and $\{3\}$ are 1-cohesive.

Proportionality in MWV

Natural axiom: if a group is ℓ-cohesive then ℓ of their common candidates should be elected to the committee.

Definition (Strong Justified Representation (SJR))

A committee W provides SJR if for every ℓ-cohesive group N', it holds that $|W \cap \bigcap_{i \in N'} A_i| \geqslant \ell$.

However, this requirement is too strong, even when $\ell = 1$.

Example

- Candidates $C = \{a, b, c, d\}$ with $k = 3$.
- Agents $N = \{1, \ldots, 9\}$.
- Suppose 2 agents approve $\{a\}$, another 2 agents approve $\{d\}$, and 1 agent each approves of $\{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, d\}$.
- Each candidate $c \in \{a, b, c, d\}$ must be elected to provide SJR.

Proportionality in MWV

A weaker axiom: if a group is ℓ-cohesive then at least one group member should be represented by ℓ committee members.

Definition (Extended Justified Representation (EJR))

A committee W provides EJR if for every ℓ-cohesive group N', there exists an agent $i \in N'$ such that $|W \cap A_i| \geqslant \ell$.

Multiwinner Voting with Weighted Seats

Joint work with Ulle Endriss, Ronald de Haan, Adrian Haret and Jan Maly.

MWV with Weighted Seats

Example

Each seat represents a role and some roles are more valuable than others.

- The committee has 5 seats with the following roles: (chair, treasurer, secretary, member, member).

Example

Each seat has an associated budget that is available for the seat's elected candidate to spend.

- The committee has 5 seats with the following budgets: (\$3278, \$1400, \$560, \$100, \$4).

Model

- ⋆ Candidates $C = \{a, b, c, \dots\}$.
- ⋆ Agents $N = \{1, \dots, n\}$.
- ⋆ Each agent submits an approval ballot $A_i \subseteq C$.
- ⋆ A weight vector $\boldsymbol{w} = (w_1, \dots, w_k)$ with a weight for each of the k seats.
- ⋆ W is the sum of all the weights.
- ⋆ Outcome is a committee $\boldsymbol{c} = (c_1, \dots, c_k)$.
- ⋆ For any set of candidates $A \subseteq C$, the satisfaction from a committee $\boldsymbol{c}$ is $\mathrm{sat}(A, \boldsymbol{c}) = \sum_{j=1}^{k} \mathbb{1}_{c_j \in A} \cdot w_j$.

Proportionality

For weight vector $\boldsymbol{w}$, the set of all *possible* satisfaction values is $\mathrm{SAT}(\boldsymbol{w})$.

Example

If $\boldsymbol{w} = (5, 3, 1)$, then $\mathrm{SAT}(\boldsymbol{w}) = \{1, 3, 4, 5, 6, 8, 9\}$.

Definition (ℓ-WS-cohesiveness)

For an integer $\ell \in \mathrm{SAT}(\boldsymbol{w})$, a group of agents N' is ℓ-WS-cohesive if $|N'| \geqslant n \cdot \frac{\ell}{W}$ and there exists a $C' \subseteq \bigcap_{i \in N'} A_i$ with $|C'| = t$ such that there exists a committee $\boldsymbol{c}$ where $\mathrm{sat}(C', \boldsymbol{c}) \geqslant \ell$, and $|N'| \geqslant n \cdot \frac{t}{k}$.

Definition (ℓ-WSJR)

A committee $\boldsymbol{c}$ provides ℓ-WSJR if for every ℓ-WS-cohesive group N', there exists an agent $i \in N'$ such that $\mathrm{sat}(A_i, \boldsymbol{c}) \geqslant \ell$.

ℓ-WSJR

Unfortunately, ℓ-WSJR is not always satisfiable.

Example

- Candidates $C = \{a, b, c\}$.
- Agents $N = \{1, 2, 3\}$.
- Weight vector $\boldsymbol{w} = (3, 2, 1)$.
- Approval ballots are $A_1 = \{a\}$, $A_2 = \{b\}$ and $A_3 = \{c\}$.

Also, even if such a committee exists, it is computationally hard to compute it.

What now? Weaken the axiom.

Weakening ℓ-WSJR: Part 1

Intuition: some cohesive group member is just one 'swap' away from the deserved satisfaction?

$I_{\boldsymbol{c}}(A)$ is the vector of positions within the committee $\boldsymbol{c}$ of candidates in A.

Definition (ℓ-WSJR-1)

A committee $\boldsymbol{c}$ provides ℓ-WSJR-1 if for every ℓ-WS-cohesive group N', there exists an agent $i \in N'$ and some $j \in I_{\boldsymbol{c}}(C \setminus A_i)$ such that either (i), we have $w_j + \mathrm{sat}(A_i, \boldsymbol{c}) \geqslant \ell$ if there exists some candidate $c \in A_i$ with $c \notin \boldsymbol{c}$, or (ii), for some $h \in I_{\boldsymbol{c}}(A_i)$, it holds that $w_j + \mathrm{sat}(A_i, \boldsymbol{c}) - w_h \geqslant \ell$.

Can ℓ-WSJR-1 always be satisfied?

w-MES

Inspired by the Method of Equal Shares (MES) rule in standard MWV.

The rule works in k rounds where agents pay to assign candidates to weights from $\boldsymbol{w} = (w_1, \ldots, w_k)$:

- ⋆ In round $r \in \{1, \ldots, k\}$, agents consider assignments to weight w_r.
- ⋆ $b_i(r)$ is agent i's budget to start round r, and in round 1, we set $b_i(1) = \frac{W}{n}$.
- ⋆ In round r, we say a pair (c, w_r) is *q-affordable* for some $q \in \mathbb{R}_{\geqslant 0}$, with c currently unelected, if:

$$\sum_{i \in N : c \in A_i} \min(q, b_i(r)) \geqslant w_r.$$

- ⋆ If no pair is q-affordable then go to the next round, otherwise, for a q-affordable pair (c, w_r) for a minimum q, assign c to w_r and continue to the next round.

w-MES and ℓ-WSJR-1

Good news in the following *restricted setting*.

Party-list elections: An election where for every pair of agents $i, j \in N$, it holds that either $A_i = A_j$, or $A_i \cap A_j = \emptyset$, and for every agent i, we have $|A_i| \geqslant k$.

Theorem

w-MES satisfies ℓ-WSJR-1 on party-list elections.

Weakening ℓ-WSJR: Part 2

Use $\textsc{LowSAT}(\boldsymbol{w}) = (\ell_1, \ell_2, \ldots, \ell_k)$ where $\ell_t = \sum_{j=1}^{t} w_j$.

Example

If $\boldsymbol{w} = (5, 3, 3, 1)$, then $\textsc{LowSAT}(\boldsymbol{w}) = (1, 4, 7, 12)$.

Definition (Lower ℓ-WS-cohesiveness)

For an integer $\ell \in \textsc{LowSAT}(\boldsymbol{w})$, a group of agents N' is *lower* ℓ-WS-cohesive if $|N'| \geqslant n \cdot \frac{\ell}{W}$ and there exists a $C' \subseteq \bigcap_{i \in N'} A_i$ with $|C'| = t$ such that there exists a committee $\boldsymbol{c}$ where $\mathrm{sat}(C', \boldsymbol{c}) \geqslant \ell$, and $|N'| \geqslant n \cdot \frac{t}{k}$.

Definition (Lower ℓ-WSJR)

A committee $\boldsymbol{c}$ provides *lower* ℓ-WSJR if for every *lower* ℓ-WS-cohesive group N', there exists an agent $i \in N'$ such that $\mathrm{sat}(A_i, \boldsymbol{c}) \geqslant \ell$.

Lower ℓ-WSJR

Bad news! w-MES does not satisfy *lower* ℓ-WSJR.

Is *lower* ℓ-WSJR is always satisfiable? Yes, use MES as in standard MWV.

- ⋆ Treat all seats as having weight 1.
- ⋆ Run MES where each agent i has initial budget $b_i(1) = \frac{k}{n}$ instead of $\frac{W}{n}$.
- ⋆ When a seat is bought for a candidate c, assign c to some weight.
- ⋆ MES ensures that cohesive groups get the seats that they deserve.

Future Work

- ⋆ Test more rules.
- ⋆ Define other fairness notions.
- ⋆ More axioms for the setting.