Proportionality in Complex Domains

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What are these complex domains?

- **Multiwinner Voting**: A job panel must produce a shortlist of \( k \) candidates to continue to the next interview stage.

- **Participatory Budgeting**: Citizens must decide on the public projects, each coming with a cost, that are to be implemented by the local municipality, subject to a budget.

We look at *other* complex domains.
Talk Outline

★ Proportionality in Multwinner Voting (MWV).
★ MWV with Weighted Seats.
★ Judgment Aggregation.
(Approval-based) MWV Model

★ Candidates $C = \{a, b, c, \ldots\}$.
★ Agents $N = \{1, \ldots, n\}$.
★ Each agent submits an approval ballot $A_i \subseteq C$.
★ Outcome is a committee $W \subseteq C$ of size $k$. 
**Definition (ℓ-cohesiveness)**

For an integer $\ell \in \{1, \ldots, k\}$, a group of agents $N' \subseteq N$ is $\ell$-cohesive if $|N'| \geq n \cdot \frac{\ell}{k}$ and $|\bigcap_{i \in N'} A_i| \geq \ell$.

**Definition (Proportional Justified Representation (PJR))**

A committee $W$ provides PJR if for every $\ell$-cohesive group $N'$, it holds that $|W \cap (\bigcup_{i \in N'} A_i)| \geq \ell$.

**Definition (Extended Justified Representation (EJR))**

A committee $W$ provides EJR if for every $\ell$-cohesive group $N'$, there exists an agent $i \in N'$ such that $|W \cap A_i| \geq \ell$. 
Multiwinner Voting with Weighted Seats

Joint work with Ulle Endriss, Ronald de Haan, Adrian Haret and Jan Maly.
MWV with Weighted Seats

Example
Each seat represents a role and some roles are more valuable than others.
- The committee has 5 seats with the following roles:
  (chair, treasurer, secretary, member, member).

Example
Each seat has an associated budget that is available for the seat’s elected candidate to spend.
- The committee has 5 seats with the following budgets:
  ($3278, $1400, $560, $100, $4).
Model

- Candidates $C = \{a, b, c, \ldots\}$.
- Agents $N = \{1, \ldots, n\}$.
- Each agent submits an approval ballot $A_i \subseteq C$.
- A weight vector $w = (w_1, \ldots, w_k)$ with a weight for each of the $k$ seats.
- $W$ is the sum of all the weights.
- Outcome is a committee $c = (c_1, \ldots, c_k)$.
- For any set of candidates $A \subseteq C$, the satisfaction from a committee $c$ is
  \[ \text{sat}(A, c) = \sum_{j=1}^{k} \mathbb{1}_{c_j \in A} \cdot w_j. \]
Proportionality

For weight vector $\mathbf{w}$, the set of all possible satisfaction values is $\text{SAT}(\mathbf{w})$.

**Example**

If $\mathbf{w} = (5, 3, 1)$, then $\text{SAT}(\mathbf{w}) = \{1, 3, 4, 5, 6, 8, 9\}$.

**Definition ($\ell$-WS-cohesiveness)**

For an integer $\ell \in \text{SAT}(\mathbf{w})$, a group of agents $N'$ is $\ell$-WS-cohesive if $|N'| \geq n \cdot \frac{\ell}{W}$ and there exists a $C' \subseteq \bigcap_{i \in N'} A_i$ with $|C'| = t$ such that there exists a committee $\mathbf{c}$ where $\text{sat}(C', \mathbf{c}) \geq \ell$, and $|N'| \geq n \cdot \frac{t}{k}$.

**Definition ($\ell$-WSJR)**

A committee $\mathbf{c}$ provides $\ell$-WSJR if for every $\ell$-WS-cohesive group $N'$, there exists an agent $i \in N'$ such that $\text{sat}(A_i, \mathbf{c}) \geq \ell$. 
Unfortunately, \(\ell\)-WSJR is not always satisfiable.

**Example**

- Candidates \(C = \{a, b, c\}\).
- Agents \(N = \{1, 2, 3\}\).
- Weight vector \(w = (3, 2, 1)\).
- Approval ballots are \(A_1 = \{a\}\), \(A_2 = \{b\}\) and \(A_3 = \{c\}\).

More negative results:

- It is computationally hard to determine whether such a committee even exists.
- And if such a committee exists, it is computationally hard to compute it.
Weakening $\ell$-WSJR

**Intuition:** some cohesive group member is just one ‘swap’ away from the deserved satisfaction?

$l_c(A)$ is the vector of positions within the committee $c$ of candidates in $A$.

**Definition ($\ell$-WSJR-1)**

A committee $c$ provides $\ell$-WSJR-1 if for every $\ell$-WS-cohesive group $N'$, there exists an agent $i \in N'$ and some $j \in l_c(C \setminus A_i)$ such that either (i), we have $w_j + \text{sat}(A_i, c) \geq \ell$ if there exists some candidate $c \in A_i$ with $c \notin c$, or (ii), for some $h \in l_c(A_i)$, it holds that $w_j + \text{sat}(A_i, c) - w_h \geq \ell$.

Can $\ell$-WSJR-1 always be satisfied?
The rule works in $k$ rounds where agents pay to assign candidates to weights from $\mathbf{w} = (w_1, \ldots, w_k)$:

- In round $r \in \{1, \ldots, k\}$, agents consider assignments to weight $w_r$.
- $b_i(r)$ is agent $i$’s budget to start round $r$, and in round 1, we set $b_i(1) = \frac{W}{n}$.
- In round $r$, we say a pair $(c, w_r)$ is q-affordable for some $q \in \mathbb{R}_{\geq 0}$, with $c$ currently unelected, if:
  \[ \sum_{i \in N(c)} \min(q, b_i(r)) \geq w_r. \]
- If no pair is q-affordable then go to the next round, otherwise, for a q-affordable pair $(c, w_r)$ for a minimum $q$, assign $c$ to $w_r$ and continue to the next round.
Good news in the following restricted setting.

**Party-list elections**: An election where for every pair of agents \(i, j \in N\), it holds that either \(A_i = A_j\), or \(A_i \cap A_j = \emptyset\), and for every agent \(i\), we have \(|A_i| \geq k\).

**Theorem**

\(w\)-MES satisfies \(\ell\)-WSJR-1 on party-list elections.
Future Work

- Test more rules.
- Define other fairness notions.
- More axioms for the setting.
Judgment Aggregation

Joint work with Ulle Endriss and Ronald de Haan.
Judgment Aggregation (JA)

Work done in the general JA framework.


Interpretation: MWV with a variable number of winners (VMWV), and with logical constraints.

Example

- The candidates are \( \{a, b, c, d, e\} \).
- A constraint may be: \( \neg (a \land b \land c) \land (d \rightarrow \neg e) \).
Model (VMWV with logical constraints)

- Candidates $C = \{a, b, c, \ldots\}$.
- Agents $N = \{1, \ldots, n\}$.
- A logical constraint $\Gamma$.
- Each agent submits an approval ballot $A_i \subseteq C$ that respects $\Gamma$.
- $\text{Mod}(\Gamma)$ is the set of all committees respecting $\Gamma$.
- Outcome is a committee $W \in \text{Mod}(\Gamma)$. 
Proportionality

**Definition ((\(W, \Gamma, \ell\))-cohesiveness)**

For an integer \(\ell \in \{1, \ldots, |W|\}\) for a committee \(W\), we say a group of agents \(N'\) is \((W, \Gamma, \ell)\)-cohesive if
\[
|N'| \geq n \cdot \frac{\ell}{|W|}
\]
and
\[
|\{c \in \bigcap_{i \in N'} A_i \mid c \text{ is logically independent of } C \setminus \{c\}\}| \geq \ell.
\]

Adapt PJR instead of EJR.

**Definition (\(\ell\)-JA-PJR)**

Given a constraint \(\Gamma\), we say that a committee \(W\) provides \(\ell\)-JA-PJR, if for every \((W, \Gamma, \ell)\)-cohesive group of agents \(N'\), it is the case that
\[
|W \cap (\bigcup_{i \in N'} A_i)| \geq \ell.
\]

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Aggregation Rules

* Use scoring functions $a$ and $d$, for approvals and disapprovals (with $a(0) = d(0) = 0$).

$$\arg\max_{W \in \text{Mod}(\Gamma)} \sum_{i \in N} a(|W \cap A_i|) - d(|W \cap C \setminus A_i|)$$

**Definition (PAV-JA)**

PAV-JA uses $a(t) = t$ and $d(t) = \sum_{j=m}^{t} \frac{1}{j}$.

**Definition (CC-JA)**

CC-JA uses $a(t) = 1$ when $t \geq 1$, and $d(t) = 1$ if $t \geq \left\lceil \frac{m}{2} \right\rceil + 1$, otherwise, $d(t) = 0$. 
Rules and $\ell$-JA-PJR

**Theorem**

PAV-JA satisfies $\ell$-JA-PJR for every value $\ell \geq \frac{|W|}{m - |W| + 1}$.

**Theorem**

Assuming logical independence between all candidates, CC-JA satisfies $\ell$-JA-PJR for $\ell = 1$ and fails it for every $\ell > 1$. 
Future Work

⋆ Test more rules.
⋆ Adapt axioms to deal better with constraints.
⋆ Proportionality with standard interpretation of Judgment Aggregation.