OPEN PROBLEMS IN VAN DOUWEN'S PAPERS

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ABSTRACT. Published in
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From the preface:

Eric posed many questions in his papers, some of which were solved. Since I wanted these books to be up to date, I collected then in §4 together with all the *new* information I could gather. Information on a certain problem that can be found in the paper in which the problem appeared or in an addendum by the referee or the editor will not be repeated in §4. Eric worked in the following reasonably well-defined areas:

(1) Cardinal functions
(2) Čech-Stone compactifications
(3) Topological groups
(4) Generalized metrizability
(5) Compact spaces, Boolean algebras and F-spaces
(6) Simultaneous extension of continuous functions
(7) Box products
(8) Measures
(9) Ordered spaces
(10) Miscellaneous.

I sent earlier versions of the first 39 pages of this book to at least two experts per area with the request to comment on my preliminary observations. By what the experts added, I believe that the current information on the problems is fairly accurate and in most cases up to date. The only exception I made was Eric's list of open problems in Boolean Algebras, published together with MONK and RUBIN. I found these volumes not the right the right place for an update of that paper.

It should be recognized that Eric did not pose all the problems published in his papers. Certainly some of the problems were suggested by his co-authors and other problems may simply have been reported by Eric. Therefore the list below shuld not not be called van Douwen's problems; it should be considered as additional information on the material contained in the papers of these books.

Open problems in van Douwen's papers

In this section we refer to the nth paper of van Douwen as [vD n]. The mth paper in the list of additional references to be found at the end (of §5), will be referred to as [m].

1. Problems in papers on cardinal functions

1. Question [vD 21, Question 6.1]. Is a compact Hausdorff space nonhomogeneous if it can be mapped continuously onto $\beta(\omega)$, or onto $\beta(\kappa)$, or onto a Hausdorff space Y with $|Y| > 2^{\pi(Y)}$?

Open.

2. Question [vD 21, Question 6.2]. Is a (compact Hausdorff) space nonhomogeneous if some open subspace can be mapped by an open mapping or a retraction onto $\beta(\omega)$, or onto a Hausdorff space Y with $|Y| > 2^{\pi(Y)}$?

Open for compact Hausdorff spaces only. Uspenskiĭ [67] proved that for every space X there exists a space Y such that $X \times Y$ is homogeneous. So there is a space Y such that the product $\beta\omega \times Y$ is homogeneous.

Later Eric formulated the following problem, which is now known as *van Douwen's Problem*:

3. Question (van Douwen's Problem). Is there a compact homogeneous Hausdorff space of cellularity greater than $\mathfrak{c}$?

There are compact homogeneous spaces with $\mathfrak{c}$-many disjoint open sets (Maurice [44]). This shows the necessity of asking the question in the form "$> \mathfrak{c}$" rather than, say, "$\geq \aleph_1$".

For more information on Questions 1, 2 and 3, see Kunen [40].

4. Question [vD 42, Question 1.5]. If X is an infinite group (or homogeneous space) which is countably compact, is $|X|^\omega = |X|$? Is at least $\mathrm{cf}(|X|) \neq \omega$?

Yes under GCH by [vD 42, Corollary 1.2], but open in ZFC.

5. Question [vD 48, Remark 11.4(a)]. Let X be a compact F-space in which nonempty G_δ-subsets have nonempty interior. Is $c(X)$ not a strong limit with countable cofinality?

Open.

6. Question [vD 48, Question 16.1]. If X is an infinite compact F-space and $\phi \in \{\chi, hL\}$, then is $\phi(X)^\omega = \phi(X)$?

Open.

7. Question [vD 48, Question 16.2]. If X is an infinite compact F-space, is $s(X)^\omega = s(X)$? Is at least $\mathrm{cf}(s(X)) \neq \omega$?

Open.

8. Question [vD 48, Question 16.3]. Does there exist for every $\kappa > 2^\omega$ with $\mathrm{cf}\kappa \neq \omega$ a compact F-space with $\chi(X) = \kappa$? with $hL(X) = \kappa$? with $s(X) = \kappa$? (It would be sufficient to consider κ of the form λ^+ with $\mathrm{cf}\lambda = \omega$.)

Open.

9. Question [vD 48, Question 16.4]. Is $\chi(X) = hL(X) = s(X) = w(X)$ if X is an infinite compact F-space? (This is really 3 questions.)

Open.

10. QUESTION [vD 48, Question 16.5]. Does there exist for every cardinal κ with $\mathrm{cf}\kappa > \omega$ a compact F-space X containing a point p with $\chi(p, X) = \kappa$? (Yes if Yes to the next question.)

Open.

11. QUESTION [vD 48, Question 16.6]. Does there exist for every cardinal κ with $\mathrm{cf}\kappa > \omega$ countably complete filter $\mathcal{F}$ on some set such that $\mathcal{F}$ has κ generators, but does not have a set of fewer than κ generators?

Open.

12. QUESTION [vD 48, Question 16.7]. If X is noncompact, σ-compact and locally compact, and $\phi \in \{c, d\}$, is $\phi(\beta X - X)^\omega = \phi(\beta X - X)$? Is at least $\mathrm{cf}(\phi(\beta X - X)) > \omega$?

Open.

13. QUESTION [vD 48, Question 16.8]. If σ and κ are cardinals with $2^{2^\omega} \cdot 2^\sigma < \kappa < 2^{2^\sigma}$ and $\kappa^\omega = \kappa$ does $\beta\sigma$ have a closed subspace of cardinality κ?

Open.

14. QUESTION [vD 62, Question 6.6]. Is Theorem 6.3 best possible, i.e., is there a compact space of cardinality $2^{\mathfrak{t}}$ which is not sequentially compact?

We quote VAUGHAN [68]:

> Alan Dow has answered the above question in the positive by noting that if X is compact Hausdorff and not sequentially compact, then $\mathfrak{n} \leq |X|$, and by constructing a model (a variation on model V in BALCAR, PELANT and SIMON [3]) where $2^{\mathfrak{t}} < \mathfrak{n}$. Van Douwen's question can be revived by asking: how can the cardinal, which is defined as the smallest cardinality of a compact, non-sequentially compact space, be expressed as a set-theoretically defined cardinal?

15. QUESTION [vD 62, Question 6.7]. Does Corollary 6.4 hold in ZFC?

Open. Corollary 6.4 says that under $2^{\mathfrak{t}} > \mathfrak{c}$ the following conditions on a compact space X are equivalent:

(1) Every countably compact subspace is closed;
(2) X is sequential, i.e., for each $A \subseteq X$ if A is not closed, then some sequence of A converges to a point of $X - A$.

16. QUESTION [vD 62, Question 6.10]. By Theorem 6.1 no product of $\mathfrak{s}$ nondegenerate spaces is sequentially compact, hence $\mathfrak{t} \leq \mathfrak{m}_\mu \leq \mathfrak{s}$, where

$$\mu = \min\{\kappa : \text{ some product of } \kappa \text{ sequentially compact spaces is not sequentially compact}\}.$$

Can μ be expressed as a set theoretically defined cardinal?

We quote VAUGHAN [68]:

> $\mu = \mathfrak{h}$ (NYIKOS, PELANT and SIMON [49] and FRIČ and VOJTÁŠ [31] independently).

17. QUESTION [vD 62, Question 6.11]. Is there is a product of sequentially compact spaces that is not countably compact in ZFC? (There is such a product if $\mathfrak{b} = \mathfrak{c}$, see Example 13.1.)

Open. Solved for Hausdorff spaces by NYIKOS and VAUGHAN [51]. For more information on this question see VAUGHAN [68].

18. QUESTION [vD 62, Question 8.11]. If X is separable metrizable, is $\mathrm{cf}(\mathcal{K}(X)) = k(X) = \mathfrak{d}$ if X is analytic, or at least if X is absolutely Borel?

We quote VAUGHAN [68]:

> By [vD 62, 8.10] this question is clearly intended for X that are not σ-compact, and for them $\mathfrak{d} \leq k(X) \leq \mathrm{cf}(\mathcal{K}(X))$. Thus, the question reduces to: is $\mathrm{cf}(\mathcal{K}(X)) \leq \mathfrak{d}$? Here, $\mathrm{cf}(\mathcal{K}(X))$ denotes the smallest cardinality of a family $\mathcal{L}$ of compact subsets of X such that for every compact set $K \subseteq X$, there exists $L \in \mathcal{L}$ with $K \subseteq L$. The answer to the second question is in the affirmative, but the answer to the first question is independent of the axioms of ZFC.
>
> BECKER [6] has constructed a model in which there is an analytic space $X \subset 2^\omega$ with $\mathrm{cf}(\mathcal{K}(X)) > \mathfrak{d}$. On the other hand, under CH, $\mathrm{cf}(\mathcal{K}(X)) = \mathfrak{d} = \omega_1$.
>
> VAN ENGELEN [26] proved that if X is co-analytic, then $\mathrm{cf}(\mathcal{K}(X)) \leq \mathfrak{d}$. The same follows from Fremlin's theory of Tukey's ordering (FREMLIN [29]). Also see FREMLIN [30].

19. QUESTION [vD 62, Question 8.14]. Does 8.13(c) hold if S is absolutely Borel?

Here 8.13(c) states that if S is a subset of a separable metrizable space then if $S \cap \overline{X - S}$ is noncompact and if S is absolutely G_δ, F_σ, or then $\chi(S, X) = \mathfrak{d}$. We quote VAUGHAN [68]:

> Van Engelen and Becker have observed (independently) that the answer is "yes". It follows from van Engelen's result "$\mathrm{cf}(\mathcal{K}(X)) \leq \mathfrak{d}$" and the method of proof of [vD 62, 8.10(c), 8.13(c)].

20. QUESTION [vD 62, Question 8.17]. Is there a (preferably metrizable) nonlocally compact space X with $\mathrm{Exp}_{\mathbb{R}}(X) < \mathrm{Exp}_\omega(X) < \infty$?

Open.

21. QUESTION [vD 62, Question 12.5]. Call a space Ψ-like if it is separable, first countable, locally compact and has a discrete derived set. Say that a subset S of a space X is relatively pseudocompact in X if every continuous real valued function on X is bounded on S, or, equivalently, if every discrete open family $\mathcal{U}$ such that $\forall U \in \mathcal{U}[U \cap S \neq \emptyset]$ is finite[1]. Define

$\mathfrak{a}_p$	$= \min\{\lvert X\rvert :$	X is Ψ-like and pseudocompact$\}$;
$\mathfrak{a}'_p$	$= \min\{\lvert X\rvert :$	X is first countable and pseudocompact but not countably compact$\}$;
$\mathfrak{b}_{rp}$	$= \min\{\lvert X\rvert :$	X is Ψ-like and has a countably infinite closed discrete relatively pseudocompact subset$\}$;
$\mathfrak{b}'_{rp}$	$= \min\{\lvert X\rvert :$	X is first countable and has a countably closed discrete relatively pseudocompact subset$\}$.

¿From Section 11 and Theorem 12.2 we see that $\mathfrak{a}_p = \mathfrak{a}$ and that $\mathfrak{b}_{rp} = \mathfrak{b}'_{rp} = \mathfrak{b}$. (This can be construed as a topological proof that $\mathfrak{a} \geq \mathfrak{b}$ since obviously $\mathfrak{a}_p \geq \mathfrak{b}_{rp}$.) This leaves open the question of what $\mathfrak{a}'_p$ is; we only have the trivial inequalities $\mathfrak{a} \geq \mathfrak{a}'_p \geq \mathfrak{b}$. (In this context we recall that it is unknown if $\mathfrak{a} \neq \mathfrak{b}$ is consistent with ZFC.)

[1] Such an S is also called *bounded* by some authors

Basically solved. Following the terminology of TREE [65], call a first countable pseudocompact $\aleph_1$-compact space that is not countably compact a *splintered* space. NYIKOS constructed a splintered space assuming $\mathfrak{b} = \omega_1$, (see notes to §13 of [vD 62]). Assuming $\diamondsuit$, there is a splintered space that is perfect and locally compact, while assuming MA+¬CH there are no perfect locally compact splintered spaces (TREE [65]). SHELAH [59] produced a model of $\mathfrak{a} > \mathfrak{b} = \omega_1$. NYIKOS' splintered space can be constructed in this model and hence we have Con($\mathfrak{a} > \mathfrak{a}'_p$).

22. QUESTION [vD 62, Question 12.6]. The fact that $\Psi(D, \mathcal{A})$, when pseudocomact, has a closed discrete set of cardinality at least $\mathfrak{a}$ suggests the question of whether there is an honest first countable (preferably separable and locally compact) pseudocompact space that is not countably compact but that has no uncountable closed discrete subset.

Open. See the above information above following the previous question.

23. QUESTION [vD 62, Question 13.6]. Is 13.4 true in ZFC? Is the condition "of cardinality at most $\mathfrak{c}$" essential? How far can "first countable" be weakened?

Here 13.4 is the statement that under $\mathfrak{b} = \mathfrak{c}$ each first countable space of cardinality at most $\mathfrak{c}$ is a quasi-perfect image of some locally compact space.

Open.

24. QUESTION [vD 64, Questions 3.5].

(1) Is $\mu_{cc} = \mathfrak{p}$? Is $\mu_{sc} = \mathfrak{a}$?
(2) Is it consistent that $\mu_{cc} < \mu_{sc}$?

Open. Note that yes to (1) implies yes to (2).

25. QUESTION [vD 67, Page 463]. Let X be a Boolean space. Does the equality $a(X) = pa(X)$ hold? If $\mathcal{B}$ is a BA, is $\mathrm{cf}(B) \leq \omega_1$?

Open.

26. QUESTION [vD 99, Page 510]. Are there for every regular κ two initially κ-compact normal spaces whose product is not initially κ-compact?

Open. YES under GCH [vD 99, Theorem 1.4].

27. QUESTION [vD 99, Page 510]. If initial κ-compactness is productive, does that imply that κ is singular and that $2^\lambda < \kappa$ for all $\lambda < \kappa$?

Open.

28. QUESTION [vD 99, Page 511]. If initial κ-compactness is finitely productive, then is it productive?

Open. YES under GCH [vD 99, Theorem 1.7].

2. PROBLEMS IN PAPERS ON ČECH-STONE COMPACTIFICATIONS

29. QUESTION [vD 13, Question 1]. Is $d(\beta X) = d(X)$ if X is a paracompact p-space or if X is a paracompact absolute G_δ-space?

The first part of the question was answered by BELL [8] in the negative. The second part is still open. Here X is an absolute G_δ-space if X is a G_δ-subset of some (or, equivalently, every) compactification.

30. QUESTION [vD 13, Question 2]. Is $d(\beta X) = c(X)$ if X has a σ-point-finite base?

Open.

31. QUESTION [vD 13, Question 3]. If X is any space, is every point-finite open family in X $c(X)$-centered?

This question was answered in the negative by STEPRĀNS and WATSON [61].

32. QUESTION [vD 23, page 272]. Are there three *infinite* spaces X, Y and Z with $\beta X \times \beta Y \times \beta Z \neq \beta(X \times Y \times Z)$ and $\beta X \times \beta Y \times \beta Z \approx \beta(X \times Y \times Z)$

Open.

33. QUESTION [vD 24, Remark (B)]. Are $T = \beta(\omega \times {}^{\mathfrak{c}}2) - (\omega \times {}^{\mathfrak{c}}2)$ and $\beta\omega - \omega$ homeomorphic under $\neg$CH?

Independent.

They are not homeomorphic under MA+$\neg$CH [vD 24, Remark (B)]. Also, it is consistent with not $\neg$CH that they are homeomorphic. It is an unpublished result of STEPRĀNS (but it shouldn't be published by anyone else) that this holds in the usual Cohen extension.

34. QUESTION [vD 24, Remark (C)]. Does $\beta\omega - \omega$ have $2^{\mathfrak{c}}$ autohomeomorphisms under $\neg$CH?

No. It is consistent with MA+$\neg$CH that every autohomeomorphism of ω^* trivial, SHELAH and STEPRĀNS [60]. See also the paper of VELICKOVICH [69] where it is shown that under OCA + MA($\aleph_1$), a consequence of PFA, every automorphism of $\mathcal{P}(\omega)/\text{fin}$ is trivial. Van Douwen's question can be revived by asking: what about consistency with $\neg$CH?

35. QUESTION [vD 31, Question P 1066], [vD 51, Page 24]. Let X be a noncompact Lindelöf space without isolated points. Is there an open family $\mathcal{U}$ which has no finite subcover yet for each closed discrete D in X there is a $U \in \mathcal{U}$ with $D \subset U$?[2]

Yes under MA by VAN MILL [48]; open in ZFC.

36. QUESTION [vD 43, Page 50]. Is there a simple nonalgebraic continuum theoretic property that distinguishes between $(\mathbb{H}^n)^*$ and $(\mathbb{R}^n)^*$?

Open.

37. QUESTION [vD 43, Page 50]. Can one see inside $(\mathbb{H}^n)^*$ which points belong to $\text{CL}\partial(\mathbb{H}^n)$?

Open.

38. QUESTION [vD 51, Question 7.9]. Are there noncompact spaces X and Y such that $(X \times Y)^*$ is homeomorphic to $X^* \times Y^*$? Can $X \times Y$ be pseudocompact?

Open.

39. QUESTION [vD 51, Page 7]. If X is not pseudocompact, then is βX extremally disconnected at some point of X^*?

Solved in the negative in [vD 60].

40. QUESTION [vD 51, Page 17]. Is there a space without a dense zero-dimensional subspace?

Open. Solved in the affirmative under CH by CIESIELSKI [17].

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[2]In other words: Does X have a far point?

41. QUESTION [vD 51, Page 18]. Is there a homeomorphism of $\mathbb{R}^*$ that sends a nonremote point to a remote point?

This was solved in the affirmative under CH by YU [75]; for a discussion of this result, see also HART [33]. It was shown in DOW and HART [22] that in Laver's model for the Borel Conjecture the set of far points is topologically invariant.

42. QUESTION [vD 51, Question 7.9]. Is it true that $(\mathbb{Q}^*)^k$ and $(\mathbb{Q}^*)^n$ are homeomorphic (if and) only if $k = n$? Can one at least find for each $k \in \omega$ an $m \in \omega$ such that $(\mathbb{Q}^*)^k$ and $(\mathbb{Q}^*)^n$ are not homeomorphic if $n > m$?

Open. The answer is Yes if $\min\{n, k\} = 1$, see VAN DOUWEN [vD 51, Theorem 7.6].

43. QUESTION [vD 51, Question 13.4]. Let X be a realcompact space which has no nonempty open compact proper subset. (In particular, let X be nowhere locally compact.) Does X have a connected compactification?

Solved in the negative by EMERYK and KULPA [25]. See the conjecture below.

44. QUESTION [vD 51, Conjecture 13.5]. The Sorgenfrey line does not have a connected (Hausdorff!) compactification.

Solved in the affirmative by EMERYK and KULPA [25].

45. QUESTION [vD 51, Remark 17.16]. Is there in ZFC a free ultrafilter $\mathcal{D}$ on ω such that $\mathbb{R} \cup (\mathbb{R}^* - \rho(\mathbb{R}))$ is $\mathcal{D}$-compact?

Open.

46. QUESTION [vD 51, Page 38]. Define a sequence $\langle \mathbb{Q}_n : n \in \omega \rangle$ of spaces by $\mathbb{Q}_0 = \mathbb{Q}$, $\mathbb{Q}_{n+1} = (\mathbb{Q}_n)^*$. Is there an $n \in \omega$ such that $\mathbb{Q}_n$ and $\mathbb{Q}_{n+2}$ are homeomorphic?

Open.

47. QUESTION [vD 51, Question 22.2]. If X is not pseudocompact, does X have a remote point?

Solved in the negative in [vD 60].

48. QUESTION [vD 51, Question 22.4]. Can $\upsilon X - X$ have a remote point of X if X has no isolated points?

No if the cellularity of X is not Ulam-measurable (TERADA [64]). Yes if there is a measurable cardinal (DOW [19]).

49. QUESTION [vD 51, Question 22.6]. If X is nowhere locally compact and non-pseudocompact, and if Y and Z are spaces without isolated points, is it true that X^* and $Y \times Z$ are not homeomorphic?

Open. The answer is Yes in the special case that X has countable π-weight [vD 51, Corollary 7.5].

50. QUESTION [vD 51, Question 22.10]. Does there exist in ZFC a free ultrafilter $\mathcal{D}$ on ω such that for every sequence $\langle q_n : n \in \omega \rangle$ in $\mathbb{Q}$ (equivalently: in $\mathbb{R}$) there is a $D \in \mathcal{D}$ such that $\{q_n : n \in D\}$ is nowhere dense?

Open.

51. QUESTION [vD 60, page 69]. Does every nonpseudocompact separable space have a remote point?

Independent. It follows from a construction in FINE and GILLMAN [28] that the answer is affirmative under CH. DOW [20] constructed a consistent separable space without remote points.

52. QUESTION [vD 60, page 72]. Is there (in ZFC) a compact space which is covered by the collection of its closed nowhere dense P-sets but which has no nonempty nowhere dense P_{ω_2}-set ?

Open.

53. QUESTION [vD 70, Page 469]. Can ω^* have a dense C^*-embedded subspace of cardinality less than $2^{\mathfrak{c}}$, or of cardinality $\mathfrak{c}$?

Yes, under PFA or by adding enough Cohen reals. This is due to DOW and was announced at the Manhattan Kansas Topology Conference in January 1992.

54. QUESTION [vD 70, Page 469]. Can ω^* have two disjoint dense C^*-embedded subspaces?

Yes, under PFA or by adding enough Cohen reals. This is due to DOW and was announced at the Manhattan Kansas Topology Conference in January 1992.

55. QUESTION [vD 74, A.6]. Is there a countable extremally disconnected space E without isolated points such that E^* is normal?

Open. It is known that under CH such an example cannot exist, see WOODS [73].

56. QUESTION [vD 82, Question 6.4]. Does there exist a compact space in which nonempty G_δ's have nonempty interior, but which does not have a base $\mathcal{E}$ such that $\cap\mathcal{W} \neq \emptyset$ for each nonempty countable centered $\mathcal{W} \subseteq \mathcal{E}$?

Open.

57. QUESTION [vD 82, Question 7.4]. If X is noncompact, σ-compact and locally compact, and if X has a countable π-base, is $b(X^*) = b(\mathbb{N}^*)$?

Open.

58. QUESTION [vD 82, Question 8.4]. If X is noncompact and realcompact, and is first countable or has a countable π-base, is $m\chi(X^*) = m\chi(\mathbb{N}^*)$?

Open.

59. QUESTION [vD 82, Page 784]. Is $m\chi(\mathbb{N}^*) \geq \mathfrak{d}$?

Independent. BLASS and SHELAH [12, 13] proved that $\mathfrak{u} = m\chi(\mathbb{N}^*)$ can be smaller than $\mathfrak{d}$ under NCF. Moreover, if one adds enough random reals to a model of CH in the extension one has $\mathfrak{u} = \mathfrak{c} > \omega_1 = \mathfrak{d}$ (this is folklore).

60. QUESTION [vD 82, Question 9.5]. If X is noncompact, σ-compact and locally compact, and has a countable π-base, does X^* have a (κ, λ)-gap iff $\mathbb{N}^*$ has one?

Open.

61. Question [vD 88, Question 8.7]. Do there exist $p, q, r, s \in \mathbb{N}^*$ with $p+q = r \cdot s$?

Open.

Some partial information on this question has appeared in Hindman [37]. A result in that paper which has been since discovered independently by a number of other people is Theorem 5.3: if for infinitely many n the set $\mathbb{N} \cdot n$ is an element of $r \in \mathbb{N}^*$, then for all $p, q \in \mathbb{N}^*$ and $s \in \mathbb{N}^*$, $p + q$ is not equal to $r \cdot s$.

62. Question [vD 88, Question 9.9]. Does there exist an embedding $e : \beta\mathbb{N} \to \mathbb{N}^*$ such that e is also a homomorphism from $(\beta\mathbb{N}, +)$ to $(\mathbb{N}^*, +)$?

This question was answered in the negative by Strauss [62].

63. Question [vD 94, Question 1.1]. Let $n \in \mathbb{N}$ Does $\mathbb{Q}^*$ have a crowded point a such that the filter on the set $\mathbb{Q}$ generated by a is the intersection of precisely n ultrafilters?

Open. Yes under MA [vD 94, Theorem 2.1].

64. Question [vD 94, Question 1.2]. Does $\mathbb{Q}$ have a crowded totally nonremote point?

Open. Yes under MA [vD 94, Theorem 2.1].

65. Question [vD 94, Question 1.3]. Is there a noncompact crowded realcompact normal space X such that every crowded point of X^* is totally nonremote?

Open.

66. Question (Woods; [vD 100, Question 1.3]). Does there exist a locally compact extremally disconnected space that is normal but not paracompact?

Open in ZFC. Kunen and Parsons [41] proved that the projective cover of a weakly compact cardinal (viewed as a space of ordinals) is normal, locally compact, extremally disconnected but not paracompact. So, the answer is Yes if there is a weakly compact cardinal.

67. Question [vD 103, Question 1.1]. If X is compact (or normal) and c.d., is X A-w.i.d.? (Without normal the answer is no. With hereditarily normal the answer is yes.)

Open.

68. Question [vD 103, Question 1.2]. Is every closed subspace of a compact (or normal) c.d. space again c.d.? (With hereditarily normal the answer is yes.)

Open.

69. Question (Levy; [vD 105, Page 132]). Does ω^* admit a continuous 2-to-1 map onto a separable space?

Open. But ω^* admits a $\leq$ 2-to-1 separable image, [vD 105].

70. Question (Levy; [vD 105, Page 137]). Does ω^* admit a continuous $\leq$ 2-to-1 map onto a separable space?

Open. This is quite delicate. Levy observed that if ω_1^* or $U(\omega_1)$ admits a $\leq$ 2-to-1 map onto a separable space then $2^{\omega_1} = 2^{\omega}$. Also, if $2^{\omega_1} = 2^{\omega}$ then ω_1^* admits a $\leq$ 4-to-1 map onto a separable space. van Douwen showed that under the same hypothesis ω_1^* even admits a $\leq$ 3-to-1 map onto a separable space. See [vD 105, Page 137] for further information.

3. Problems in papers on topological groups

71. Question [vD 27, page 197]. Let X be a space. Is $|w(x,X)| \leq \mathfrak{c}$ for all $x \in X$ or for at least one $x \in X$)?
Open.

72. Question [vD 27, Question 6.2]. Suppose that for X one of the following conditions is satisfied:

- **(a):** $X = \beta Y$ for some non-pseudocompact Y;
- **(b):** $X = \beta Y - Y$ for some non-pseudocompact Y;
- **(c):** every countable relatively discrete subset of X is C^*-embedded and X has an infinite compact subset.

Then is every power of X nonhomogeneous?
Part (c) of this question is partially solved: Kunen [40] proved that no product of infinite compact F-spaces is homogeneous.

73. Question [vD 41, page 425]. Does ${}^{\mathfrak{c}}2$ have a separable dense countably compact subgroup which has no convergent sequences?
Open. Yes under MA [vD 41, Example 8.1].

74. Question [vD 41, page 425]. Does ${}^{\mathfrak{c}}2$ have a nonseparable dense countably compact subgroup which has no convergent sequences?
Open. Yes under MA [vD 41, Example 8.2].

75. Question [vD 66, Question A]. Which topological groups T are such that every transitive effective subgroup of $H(T)$ is isomorphic to T?
Open.

76. Question [vD 66, Question B]. For which topological groups T is every transitive effective subgroup G of $H(T)$ a topological group, when G is topologized by declaring the evaluation-at-the-identity map $e : G \to T$ to be a homeomorphism?
Open.

77. Question [vD 78, Question 1.2.2]. If G is a countable Abelian group, is BG homeomorphic to the absolute of ${}^{\kappa}2$ for some κ with $\omega \leq \kappa \leq \mathfrak{c}$?
Answered in the affirmative by Balcar and Blaszczyk [2].

78. Question [vD 78, Question 4.9]. Is every subgroup of G $\mathbb{N}$-embedded in $G^{\#}$? 2-embedded in $G^{\#}$? I-embedded in $G^{\#}$?
Open.

79. Question [vD 78, Question 4.10]. Is $G^{\#}$ strongly zero-dimensional? Normal?
The first part of the question was answered in the affirmative by Shakhmatov [58]. The second part of the question was clearly meant for uncountable G since for G countable, $G^{\#}$ is normal being Lindelöf. This part of the problem was answered in the negative recently by Trigos-Arrieta [66].
Van Douwen also asked the following related interesting problem.

80. Question Let G and H be Abelian groups. Are $G^{\#}$ and $H^{\#}$ homeomorphic if and only if G and H have the same cardinality?

81. QUESTION [vD 78, Question 4.12]. Is every (countable) subgroup of $G^{\#}$ a retract of $G^{\#}$?

Open.

82. QUESTION [vD 78, Question 4.13]. Is every countable closed subset of $G^{\#}$ a retract of $G^{\#}$?

Solved in the negative by GLADDINES [32].

83. QUESTION [vD 78, Question 4.14]. Does $G^{\#}$ have a closed discrete subset of cardinality $|G|$?

Answered in the affirmative by HART and VAN MILL [34].

84. QUESTION [vD 78, Question 4.16]. Is $G^{\#}$ realcompact if $|G|$ is not Ulam-measurable, in particular if $|G| = \omega_1$?

Open. The second part of this question was recently answered in the affirmative by COMFORT and TRIGOS-ARRIETA [18] who proved that if G is Abelian and $|G| \leq \mathfrak{c}$ then $G^{\#}$ is hereditarily realcompact.

85. QUESTION [vD 78, Question 4.17]. If $G^{\#}$ is infinite, does $G^{\#}$ have a relatively discrete subset that is not closed?

Open. HART and VAN MILL [34] have shown that if G is Boolean, i.e., every element of G has order at most 2, then $G^{\#}$ does contain a relatively discrete subset that is not closed.

4. PROBLEMS IN PAPERS ON GENERALIZED METRIZABILITY

86. QUESTION [vD 14, page 127]. Are Ω^* and $\Omega^* \times \Omega^*$ homeomorphic?

Open.

87. QUESTION [vD 14, page 129]. Is Ω countably metacompact?

Open.

88. QUESTION [vD 18, page 143]. Under MA+¬CH, is every hereditarily Lindelöf space with a point countable base metrizable?

Yes because under MA+¬CH there are no first countable L-spaces (SZENTMIKLÓSSY [63]).

89. QUESTION [vD 33, page 371]. Is every finite power of the Sorgenfrey line $\mathbb{S}$ a S-space? Is $\mathbb{S}^\omega$ hereditarily a D-space?

Open.

90. QUESTION [vD 33, page 372]. Are there distinct positive integers m and n such that $\mathbb{S}^n$ and $\mathbb{S}^m$ are homeomorphic?

Answered in the negative by BURKE and LUTZER [16].

91. QUESTION [vD 35, page 106]. Does there exist a family $\mathcal{U}$ of open subsets of $\mathbb{Q}$ and a collection $\underline{V}$ of countably infinite families of open subsets of $\mathbb{Q}$, such that

(1) for each closed discrete subset D of $\mathbb{Q}$ there is a $U \in \mathcal{U}$ with $D \subseteq U$;
(2) if $\mathcal{W}$ is any open cover of $\mathbb{Q}$, then there is $\mathcal{V} \in \underline{V}$ which refines $\mathcal{W}$;
(3) for each $U \in \mathcal{U}$ and $\mathcal{V} \in \underline{V}$ there are at most finitely many $V \in \mathcal{V}$ with $V \subset U$.

Open. A positive answer to this question would yield a ZFC example of a σ-discrete CWH Moore space which is not pseudonormal [vD 35, Lemma 3.2].

92. Question [vD 44, page 77]. Can the main positive result in Section 3 be [vD 44] proved for semistratifiable spaces or for semi-metric spaces?

Open. The main positive result in Section 3 is that any σ-space is the union of a collection of $\mathfrak{c}$ closed metrizable subspaces.

93. Question [vD 44, page 77]. Is it true that every regular space with a point-countable base can be written as the union of continuum many (closed) metrizable subspaces?

Open. Yes for T_1-spaces with a σ-point-finite base [vD 44, Corollary 3.3].

94. Question [vD 46, Remark 2.4]. Are $D(\kappa)$ and $T(\kappa)$ equivalent for all κ?

Open.

95. Question [vD 46, Remark 2.8]. Does $P(\kappa)$ imply $D(\kappa)$ for all κ? Does MA imply $D(\mathfrak{c})$?

Open.

96. Question [vD 46, Question 6.1]. Can every first countable compact Hausdorff space be embedded in a separable first countable space which is Hausdorff? is regular? is compact Hausdorff?

Partly open. Bell [10] constructed a first countable compact Hausdorff space which consistently cannot be embedded in a separable first countable compact Hausdorff space (because it is not continuous image of ω^*).

97. Question [vD 46, Question 6.2]. Can a Moore space with weight $\leq \mathfrak{c}$ be embedded in a separable first countable (developable?) Hausdorff (regular?) space provided it is locally compact, or has a point-countable base, or is metacompact?

Open.

98. Question [vD 46, Question 6.3]. Can a first countable Hausdorff or regular space X with weight $\leq \mathfrak{c}$ be embedded in a separable first countable Hausdorff (regular? — only if X is of course) space provided X is locally compact, or has a point-countable base, or has a σ-point-finite base, or is metacompact?

Partly open. Bell'S space mentioned above has weight $\mathfrak{c}$ but cannot be embedded in a separable first countable completely regular space.

99. Question [vD 91, Page 95]. Does there exist a 3-separable Moore space which is not 2-separable?

Answered in the negative by Reed; see Heath and Reed [35] for details.

100. Question [vD 91, Question 3.2.3.6]. Must $\mathcal{M}(X)$ be a strongly 2-star-Lindelöf Moore space if X is a regular, first countable strongly 2-star-Lindelöf space?

Open.

101. Question [vD 113, Question 9]. Are there characterizations of when $\langle X, \tau \vee \mu \rangle$ is normal? is perfect?

Open.

5. Problems in papers on compact spaces, Boolean Algebras and F-spaces

102. Question [vD 40, Question 1]. Does there exist for each n with $3 \leq n < \omega$ a nonseparable $(n+1)$-supercompact Hausdorff space whose topology is σ-n-linked? Or at least a compact Hausdorff space whose topology is σ-n-linked but not σ-$(n+1)$-linked?

Solved in the affirmative by Bell [9].

103. Question [vD 40, Question 2]. Are the following conditions equivalent on a Hausdorff continuous image X of a κ-supercompact Hausdorff space?

(1) X is separable;
(2) the topology of X is σ-centered; and
(3) X has a σ-κ-nlinked base (or π-base).

Open.

104. Question [vD 45, page 122]. Is CH equivalent to the statement that every compact zero-dimensional F-space of weight $\leq \mathfrak{c}$ can be embedded in an extremally disconnected space?

No. This is due to Frankiewicz, Zbierski and Dow [24].

105. Question [vD 52, Question 6.7]. If X is an infinite compact space such that X and ${}^2X$ are homeomorphic, or such that ${}^2X$ can be embedded into X, then X cannot be a $\beta\omega$-space. Is there a compact space satisfying either condition that has no nontrivial convergent sequences?

Open.

106. Question [vD 52, Page 16]. Is the condition that $\kappa, \lambda \leq \omega$ essential in Corollary 7.4?

Open. Here Corollary 7.4 says that if X is nonpseudocompact and has no isolated points, and if κ and λ are cardinals with $q \leq \kappa, \lambda \leq \omega$, then $\kappa = \lambda$ iff ${}^\kappa\beta X$ and ${}^\lambda\beta X$ are homeomorphic.

107. Question [vD 52, Page 16]. Are there noncompact spaces X and Y such that $(X \times Y)^*$ is homeomorphic to $X^* \times Y^*$? Can $(X \times Y)$ be pseudocompact?

Open.

This is the same as Question 38.

108. Question [vD 52, Conjecture 8.3]. Let X be ω^* or $\beta\omega$. For every autohomeomorphism h of ${}^2X$ there is a disjoint open cover $\mathcal{U}$ of X such that $h|U \times V$ is elementary for all $U, V \in \mathcal{U}$.

Open.

109. Question [vD 52, Conjecture 8.4]. Let X be ω^* or $\beta\omega$. If ϕ is any continuous binary operation on X, then there is a disjoint open cover $\mathcal{U}$ of X such that $\pi|U \times V$ depends on one coordinate for every $U, V \in \mathcal{U}$.

Open.

110. Question [vD 52, Question 9.6]. Does there exist an infinite compact space with a mean that has no nontrivial convergent sequences?

Open.

111. QUESTION [vD 52, Question 10.3]. Let X be pseudocompact and let μ be a continuous binary operation on X such that the equations $\mu(x,a) = b$ and $\mu(a,x) = b$ each have exactly one solution for $a, b \in X$. Is it possible to extend μ to a continuous binary operation on $\beta\omega$?

Open. It is known that if X is not pseudocompact then this is not possible (vD 52, Theorem 2.7]).

112. QUESTION ([dD 52, Remark 17.2]). If X is a space with ${}^2X \approx X$, does there exist a compactification γX of X with ${}^2\gamma X \approx \gamma X$?

Open. Yes if X is locally compact [vD 52, Theorem 17.1].

113. QUESTION [vD 52, Page 30]. Which spaces have an antiselective mean?

Open.

114. QUESTION [vD 58, Question 1.6]. Let Y be a Hausdorff continuous image of supercompacta Hausdorff space (or just a supercompact Hausdorff space). If K is a countable subet of Y, then is *every* cluster point of K the limit of a nontrivial convergent sequence in Y?

Solved in the affirmative by YANG [74].

115. QUESTION [vD 58, Question 1.7]. Is a retract of a supercompact Hausdorff space again supercompact? Are the factors of a supercompact product supercompact? Is every dyadic space supercompact?

The first two questions are still open. It is known however that a retract of a Cantor cube, i.e., a 0-dimensional Dugundji space, is supercompact; this is an unpublished result of KOPPELBERG. It also follows from the main theorem of HEINDORF [36]. BELL [11] has given an example of a 0-dimensional dyadic space which is not supercompact.

116. QUESTION [vD 76, Page 50]. Let A be a BA. Which of the implications of the following do not reverse:

$$\begin{aligned} A\ \sigma\text{-centered} \quad &\Rightarrow \quad (\forall f \in {}^{\mathbb{N}}\mathbb{N})[A \text{ is } f\text{-linked} \Rightarrow A \text{ is } \langle n\rangle_{n\in\mathbb{N}}\text{-linked}] \\ &\Rightarrow \quad (\forall f \in\uparrow)[A \text{ is } f\text{-linked}] \Rightarrow (\forall k \in \mathbb{N})[A \text{ is } \sigma\text{-}k\text{-linked}] \end{aligned}$$

Open. On this problem, KOPPELBERG notices the following. All of the conditions mentioned imply that A satisfies the ccc and has size at most $\mathfrak{c}$. Now if MA holds, A satisfies the ccc and the size of A is less than $\mathfrak{c}$, then A is σ-centered (see e.g. WEISS [71, Theorem 4.5]). Thus in this case all conditions are equivalent.

117. QUESTION [vD 80, Question 4.1]. Let $\kappa > \omega$ Is the statement $\mathrm{Aut}(\mathcal{Q}(\kappa)) = T^*_\kappa$ true? Consistent? False?

Open.

118. QUESTION [vD 80, Question 6.3]. Let $\kappa > \omega$. Is $\mathrm{Aut}(\mathcal{Q}(\kappa))$ simple?

Yes. The BA $\mathcal{Q}(\kappa)$ is σ-complete and homogeneous and the automorphism group of every homogeneous σ-complete BA is algebraically simple, a result essentially shown by ANDERSON. For details, see ŠTĚPÁNEK and RUBIN [70, Theorem 5.9b].

119. QUESTION [vD 87, Question 1.7]. When does $\mathcal{L}(\kappa)$ embed into $\mathcal{L}(\lambda)$?

Open. See VAN MILL [46] for information on the case that $\kappa = \omega$.

120. QUESTION [vD 87, Question 1.8]. Which BAs embed into $\mathcal{L}(\lambda)$?

Open.

121. QUESTION [vD 87, Question 1.13]. Is it true that

$$(\forall\lambda)(\forall\kappa)[\kappa < \lambda \text{ and } \mathrm{cf}(\kappa) = \mathrm{cf}(\lambda) \Rightarrow \mathcal{L}(\kappa) \leq \mathcal{L}(\lambda)]?$$

Open.

122. QUESTION [vD 87, Question 1a, page 41]. Is

$$(\forall\kappa \leq \lambda)[\mathrm{cf}(\kappa) = \mathrm{cf}(\lambda) \Rightarrow \mathcal{L}(\kappa) \leq \mathcal{L}(\lambda)]$$

true?

Open.

123. QUESTION [vD 87, Question 2a, page 42]. Is the statement

$$(\forall B)[|B| \leq \lambda^+ \Rightarrow B \leq \mathcal{L}(\lambda)]$$

equivalent with $\lambda = 2^{<\mathrm{cf}(\lambda)}$? At least consistent? False?

Solved. Let $(*)$ be the statement: Every Boolean Algebra of cardinality ω_2 embeds (mod countable) into the power set of ω_1. VAN DOUWEN [vD 87, Theorem 1.17] proved that (*) implies CH. So for the case $\lambda = \omega$, this question boils down to whether CH implies $(*)$. DOW recently proved the following:

(1) $\diamondsuit$ implies $(*)$
(2) $(*) + 2^{\omega_1} = \omega_2$ implies $\diamondsuit$,
(3) $(*) + \neg\diamondsuit$ is consistent.

124. QUESTION [vD 109, Question 1.2]. Does every subalgebra of an interval algebra have an nA generating set?

This question is still open; it is also contained in KOPPELBERG and MONK [38]. There are two partial results — the answer is yes in each of the following cases.

(a): The subalgebra is superatomic (BONET [14, Main Theorem].
(b):). The subalgebra is generated by intervals (KOPPELBERG and MONK [38, 3.2(b)]).

6. PROBLEMS IN PAPERS ON SIMULTANEOUS EXTENSION OF CONTINUOUS FUNCTIONS

125. QUESTION [vD 6, page 4]. Is there a K_1-space not having property D_1^* or the monotone extension property?

Open.

126. QUESTION [vD 6, page 9]. Is there a K_1-space which is not K_0?

Yes. An example of such a space was constructed by M. E. RUDIN [56].

127. QUESTION [vD 6, page 13]. Let m and n be integers with $m \geq n$. Is every M_n-space an M_n-space? Is every M_n-space with $n \geq 1$ a K_0-space?

Open.

128. QUESTION [vD 6, page 15]. Is every monotonically normal space K_0?

No. A counterexample was constructed by M. E. RUDIN [56].

129. QUESTION [vD 6, page 31]. Must a monotonically normal space have property $D^*(\mathbb{R}; +; \text{closed convex hull})$?

Open.

130. QUESTION [vD 6, page 37]. Does the monotone extension property imply property D_1^* or $D^*(\mathbb{R};+;\text{closed convex hull})$? Does property D_1^* imply property $D^*(\mathbb{R};+;\text{closed convex hull})$ or the monotone extension property?

Open.

131. QUESTION [vD 6, page 41]. Must a K_1-space have property D_1^* or the monotone extension property?

Open.

132. QUESTION [vD 6, page 42]. Are the properties $D^*(\mathbb{R};+;\text{closed convex hull})$ and D_1^* and the monotone extension property hereditary?

Open.

133. QUESTION [vD 6, page 45]. Does Theorem 4.2.2 hold for property D_c^*, if $c > 1$? Must a space have property D_c^* if it is the union of two closed subspaces with propertyD_c^*, for $c > 1$?

Open.

134. QUESTION [vD 6, page 83]. Does the Sorgenfrey line $\mathbb{S}$ have property

$$D^*(\mathbb{R};+;\text{closed convex hull}) \text{ or } D(\mathbb{R};+;\text{closed convex hull})?$$

Open.

135. QUESTION [vD 6, page 89]. Is there for every real number $\lambda > 1$ a (compact) space which has property D_c^* for $c = \lambda$ but not for any $c < \lambda$?

Open.

136. QUESTION [vD 7, §6(1)]. Must a K_1-space have property D_1^*?

Open. This is the same as Question 125.

137. QUESTION [vD 7, §6(2)]. Is property D_1^* hereditary? Is property D_c^* hereditary in hereditarily normal spaces?

Open.

138. QUESTION [vD 7, §6(3)]. Must a space which has any property D_c^* be collectionwise normal? Or is Corollary 3.3 best possible?

Open. Here Corollary 3.3 states that if X has property D_c^* and n is the smallest integer $> \frac{1}{2}(c-1)$ then for every discrete collection $\mathcal{A}$ of closed sets in X there is an open collection $\{\kappa(A) : A \in \mathcal{A}\}$ such that

(1) $A \subset \kappa(A)$, for $A \in \mathcal{A}$;
(2) $\kappa(A) \cap B = \emptyset$, for $A, B \in \mathcal{A}, A \neq B$;
(3) any point of X belongs to at most n sets $\kappa(A)$.

139. QUESTION [vD 7, §6(4)]. Does there exist for each real number $\lambda > 1$ a space which has property D_λ^* but not property D_c^* for any $c < \lambda$?

Open. This is the same as Question 135.

140. QUESTION [vD 16, §5(a)]. Let X be a normal space having a closed subset A. Does there exist an extender $\eta : C^*(A) \to C^*(X)$ such that $\|\eta(f) - \eta(g)\| \leq 2\|f - g\|$?

This question was answered in the negative by PELANT [52] who proved the there is no uniformly continuous extender from $C(\beta\omega - \omega)$ to $C(\beta\omega)$. See also Question 141 below.

141. QUESTION [vD 16, §5(b)]. Let X be a normal space having a closed subspace A. Does there exist a uniformly continuous extender from $C^*(A)$ to $C^*(X)$?

This question was answered in the negative by PELANT [52] who proved that there is no uniformly continuous extender from $C(\beta\omega - \omega)$ to $C(\beta\omega)$.

142. QUESTION [vD 16, §5(c)]. Suppose that for each closed subset A of X there is a linear isometric extender $\eta : C^*(A) \to C^*(X)$. Let Y be a subspace of X. Is it true that for each relatively closed $B \subseteq Y$ there is a linear isometric extender from $C^*(B)$ to $C^*(Y)$?

Open.

7. PROBLEMS IN PAPERS ON BOX PRODUCTS

143. QUESTION [vD 4, page 129]. For what kind of metrizable spaces X is the box product of countably many copies of X normal or paracompact?

Open. There are a lot of partial results. See for example KUNEN [39], LAWRENCE [42] and RUDIN [55]. See also WINGERS [72].

144. QUESTION [vD 4, page 130]. If $\{X_\alpha : \alpha \in A\}$ is a family of stratifiable spaces, is Ξ_p stratifiable for each $p \in \Box_{\alpha\in A} X_\alpha$?

Answered in the affirmative by BORGES [15].

145. QUESTION [vD 4, page 131]. Is Ξ_p (hereditarily) normal if $\{X_n : n < \omega\}$ is a countable family of spaces such that all finite subproducts are (hereditarily) normal? Is Ξ_p paracompact if$\{X_\alpha : \alpha \in A\}$ is an uncountable family of spaces such that all finite subproducts are paracompact Hausdorff?

The first part of this question is still open while the second part was answered recently in the affirmative by NYIKOS and PIATKEWICZ [50].

146. QUESTION [vD 12, page 72]. Is it consistent that the box product of countably many copies of 2^{ω_1} is not normal?

Open.

147. QUESTION [vD 12, page 74]. Are $\Pi^\kappa_{\alpha\in\kappa^+} D_\alpha(\mu)$ and $\Pi^\kappa_{\alpha\in\kappa} D_\alpha(\mu)$ normal forsingular κ, and μ an arbitrary cardinal ≥ 2?

Open.

148. QUESTION [vD 38, Page 87]. Can $\Box^\omega({}^{\omega_1}2)$ be paracompact or normal if CH faiis?

Open. See also Question 4.7.

149. QUESTION [vD 38, Questions 10.16].

(1) Is $\Box_n X_n$ paracompact in ZFC if each X_n is compact and either first countable or has $w(X_n) \leq \omega_1$?
(2) Is it consistent that a countable $\Box$-product of infinite compact spaces never is countably orthocompact?

Open.

150. QUESTION [vD 38, Page 98: Q4]. Is it consistent that $\nabla^\omega(\omega + 1)$ is paracompact but not hereditarily paracompact?

Open.

151. QUESTION [vD 38, Page 118]. Is $\kappa^{\smile^{\kappa}}$ necessary for $\Delta(\kappa)$ to be true? Is $\Delta(X)$ true in ZFC for all n, or at least for regular κ?

Open.

152. QUESTION [vD 38, Question 13.13]. Is it true that ${}^{\omega}\omega \times \square^{\omega}(\omega+1)$ is not countably orthocompact, and that $\square^{\omega}(\omega+1)$ is not hereditarily countably orthocompact? Or is a (countable) $\square$-product of metrizable spaces always metacompact (or submetacompact)?

Open.

153. QUESTION [vD 38, Remark 15.5]. Are $\mathbb{E}(\nu,\kappa)$ and $\mathbb{E}(\nu,\kappa^{+})$ equivalent for all ν,κ? $\mathbb{E}(\upsilon,2^{\nu})$ independent from ZFC for $\nu > \omega$?

Open.

8. PROBLEMS IN PAPERS ON MEASURES

154. QUESTION [vD 61, Question 11.1]. Does there exist an infinite compact connected group such that

- **(a):** every autohomeomorphism preserves Haar measure; or
- **(b):** every autohomeomorphism of G that leaves the identity fixed is either an isomorphism or an anti-isomorphism; or
- **(c):** if $*$ is any group operation on the underlying space of G which also makes G a topological group, and which has the same identity element, then either $x * y = xy$ $(x, y \in G)$ or $x * y = yx$ $(x, y \in G)$.

Solved in the negative by W. RUDIN [57].

155. QUESTION [vD 61, Question 11.2]. Does every nondiscrete second countable locally compact abelian group have a stiff (tiny fat) subgroup?

Open.

156. QUESTION [vD 61, Question 11.3]. Does every homogeneous zero-dimensional separable metrizable space admit the structure of a topological group?

Answered in the negative by VAN DOUWEN but never published; for a proof see VAN ENGELEN and VAN MILL [27].

157. QUESTION [vD 61, Question 11.4]. Is there a satisfactory characterization of nonshrinking groups?

Open.

158. QUESTION [vD 77, Question 1.4]. If H is any countably infinite amenable group, then is there an almost free transitive action of F_2 on H such that every left-invariant measure on H is F_2-invariant?

Open.

159. QUESTION [vD 97, Question 1.6]. Is each stretchable measure thinnable?

Open.

160. QUESTION [vD 97, Question 1.9].

- **(i):** Is there a $[1,\infty)$-scale-invariant measure that is not shift-invariant?
- **(ii):** Is there an $\mathbb{N}$-scale-invariant measure that is not $\{r\}$-scale invariant for each noninteger $r \in [1,\infty)$?

Open.

161. Question [vD 97, Question 1.15]. What are the extreme points in the (closed convex) set of stretchable measures? Or the set of elastic measures (of Section 5)? (Those are the nicest measures I can currently think of.)
Open.

162. Question [vD 97, Question 5.5]. Is there a nonelastic stretchable and thinnable measure on $\mathbb{N}$?.
Open.

163. Question [vD 97, Question 5A.2]. Let G be a group, let S be a set and let $\pi : G \times S \to S$ be attraction from G on S. Suppose that S has a G-invariant measure μ such that $\mu(\{x \in S : g(x) \neq x\}) = 1$ for each $g \in G \setminus \{1\}$. Is it true that the action π has trivial kernel?
Open.

164. Question [vD 97, Question 5A.3]. Let D be the group of all distortions of $\mathbb{N}$, i.e., all permutations f of $\mathbb{N}$ such that $\langle f_n - n \rangle_{n \in \mathbb{N}}$ is bounded. Does D have a left-invariant measure? Does the free group on two generators embed in D?
Open.

165. Question [vD 97, Question 6.2]. Does (B) hold without the restriction that $M \subseteq \mathbb{N}$?
Open.

166. Question [vD 97, Question 7A.1]. Let $\mathcal{C}$ be the set of all measures μ on $\mathbb{N}$ such that

$$\mu(K) \leq \limsup_{n\to\infty} \frac{1}{n} |K \cap [1, n]|,$$

and let $\mathcal{D}$ be the set of all measures on $\mathbb{N}$ which extend density. So $\mathcal{A}' \subseteq \mathcal{C} \subseteq \mathcal{D}$. Is $\mathcal{A}' = \mathcal{C}$ or not? Is $\mathcal{C} = \mathcal{D}$ or not?
Open.

167. Question [vD 97, Question 9.1]. Is there an $\mathbb{N}$-scale-invariant measured μ on $\mathbb{N}$ such that for each $q \in \mathbb{Q}^{\geq 1} \setminus \mathbb{N}$ there is $K \in \mathcal{P}(\mathbb{N})$ with $\mu([q \cdot K]) \neq q^{-1} \cdot \mu(K)$? Can one have K independent of q?
Open.

168. Question [vD 97, Question 11.1]. Is $\sigma(N)$ defined for each $\mathcal{E}$-sizable set N?
Open.

9. Problems in papers on ordered spaces

169. Question [vD 32, Question 8.1]. Give necessary and sufficient condition on two stationary subsets S and T of κ (or even of ω_1) so that T is a continuous image of S (a homeomorph of S)
Open.

170. Question [vD 32, Question 8.2]. Let D be κ with the discrete topology. For each $A \in S(\kappa)$ let

$$\overline{A} = \cap\{\mathrm{Cl}_{\beta D}(A \cap C) : C \in \mathrm{cub}(\kappa)\}.$$

Given that $\overline{A}$ and $\overline{B}$ are homeomorphic does it follow that $A = B$?
Open.

171. QUESTION, [vD 32, Question 8.3]. In this question we are asking for topological properties which rather strongly distinguish different types of bistationary subsets of ω_1. Are there "nice" topological properties $\mathcal{A}$ and $\mathcal{B}$ such that

(1) there is a bistationary set $A \subset \omega_1$, such that A has $\mathcal{A}$ and $\omega - A$ has $\mathcal{B}$;
(2) given any two stationary sets S and T in ω_1, if S has $\mathcal{A}$ and T has $\mathcal{B}$ then $S \cap T$ is not stationary.

Open.

172. QUESTION [vD 63, page 101]. Is the statement that every orderable space is normal properly weaker than the Axiom of Choice?
Open.

173. QUESTION [vD 63, page 105]. Does the statement that every orderable space is normal imply that every orderable space is monotonically normal?
Open.

174. QUESTION [vD 108, Question 8.2]. Is there a noncompact nonorderable space which is homeomorphic to each of its cubs? Is there such a space which is a bear?
Open.

175. QUESTION [vD 108, Page 170]. Is a locally countable ω-bounded space without disjoint cubs a continuous image of ω_1?
Open. No under $\clubsuit^2$ [vD 108, Example 9.2].

10. Problems in miscellaneous papers

176. QUESTION [vD 3, page 438]. If X and Y are metrizable spaces, is it true that $\mathrm{ind}(X \times Y) \leq \mathrm{ind}X + \mathrm{ind}Y$?
Open.

177. QUESTION [vD 43, Question 1]. Let X be a continuum. Is the space $C(X)$ zero-dimensional closed set aposyndetic?
Open.

178. QUESTION [vD 43, Question 2]. Let X be a continuum. Is 2^X countable closed set aposyndetic? zero-dimensional closed set aposyndetic?
Open.

179. QUESTION [vD 43, Question 3]. Is countable closed set aposyndesis a Whitney property? Is zero-dimensional closed set aposyndesis a Whitney property?
Open.

180. QUESTION [vD 43, Question 4]. If X is a one-dimensional, acyclic continuum such that for each (some) Whitney map A for $C(X)$ and for each $t \in (0, \mu(X))$, $\mu^{-1}(t)$ is aposyndetic, then must X be aposyndetic? In fact, must X be locally connected? What if "acyclic" is replaced by "unicoherent"?
Open.

181. QUESTION [vD 49, page 424]. Is every compact Hausdorff Fréchet locally pathwise connected space path-determined?
Open.

182. QUESTION [vD 57, page 149]. Does MA+¬CH imply that whenever $x_\alpha \in \mathcal{P}(\omega)$ for $\alpha < \omega_1$, there is an uncountable $I \subseteq \omega_1$, such that either

(1) $\forall \alpha, \beta \in I(\alpha \neq \beta \Rightarrow x_\alpha \nsubseteq x_\beta \ \&\ x_\beta \nsubseteq x_\alpha)$ or
(2) $\forall \alpha, \beta \in I(x_\alpha \subseteq x_\beta$ or $x_\beta \subseteq x_\alpha)$.

BAUMGARTNER [4] proved that this is *consistent* with MA+¬CH. ABRAHAM and SHELAH have shown that it does not follow from MA+¬CH alone. For information, see BAUMGARTNER [5, The end of §6].

183. QUESTION [vD 81, Page 112]. Are there $\mathfrak{c}$ many isomorphism classes of "nice" spreads?

Open.

184. QUESTION [vD 81, Page 112]. How many finitary-isomorphism classes of spreads are there?

Open.

185. QUESTION [vD 83, Page 282]. Does DFA imply that there is a set $\{(U_\alpha, V_\alpha) : \alpha < \omega_1\}$ of pairs of disjoint subsets of 2^ω so that for every disjoint sequence, $\{B_n : n \in \omega\}$ of clopen subsets of 2^ω, there is α so that

$$|\{n : B_n \subseteq U_n\}| = \omega = |\{n : B_n \subseteq V_\alpha\}|.$$

Open.

186. QUESTION [vD 83, Page 288]. Is wDFA equivalent to the apparently weaker statement that every weakly definable ccc poset of size $\leq 2^{\aleph_0}$ is $\aleph_1$-centered?

Open.

187. QUESTION [vD 83, Page 289]. Does wDFA imply that for every ccc σ-ideal in the Borel sets there exists a Luzin set of size $\aleph_1$?

Open.

188. QUESTION [vD 85, Page 262]. Let $\mathcal{L}$ demote the set of all Lebesgue measurable real-valued functions defined on $\mathbb{R}$ endowed with the topology of close approximation. Is $\mathcal{L}$ normal?

No. The closed subset of $\mathcal{L}$ consisting of all functions which are zero off of the (measure zero) Cantor set is homeomorphic to the box product of continuum many lines, which is non-normal by the result of LAWRENCE [43].

189. QUESTION [vD 85, Page 262 and 268]. Let $\mathcal{L}$ be as in Question 4.10. Is the subspace of $\mathcal{L}$ consisting of all surjective measurable functions from $\mathbb{R}$ to $\mathbb{R}$ a G_δ-subset of $\mathcal{L}$? How about the set $\mathcal{C}_1$ consisting of measurable functions that are continuous in at least one point?

Open.

190. QUESTION [vD 85, Page 270]. Let $\mathcal{L}$ be as in Question 4.10. Are the bijective measurable functions dense in the surjective measurable functions (with the respective subspace topologies inherited from $\mathcal{L}$)?

Open.

191. QUESTION [vD 86, Remark 2.5.7]. If D is uncountable and disrete, what can be said about the normality or paracompactness of $C_m(D)$?

Since $C_m(D)$ is the box product of $|D|$ many copies of $\mathbb{R}$, this question is answered by the result of LAWRENCE [43].

192. QUESTION [vD 86, Remark 5.14]. Are there spaces X and Y such that neither $C_m(X)$ nor $C_m(Y)$ is paracompact but $C_m(X \times Y)$ is paracompact?
Open.

193. QUESTION [vD 98, Question 2.7]. We do not know whether the hypothesis "open and z-closed" can be reduced to "z-open" in 2.6 or whether 2.6 remains true in the fibers of f are nearly realcompact. We also do nt know whether the countable union of nearly realcompact z-embedded subsets is nearly realcompact (see 2.2).

Open. Here 2.6 says that if $f : X \to Y$ is a continuous open z-closed surjection such that $f^{-1}\{y\}$ is realcompact and z-embedded in X for each $y \in Y$ and if Y is nearly realcompact then so is X.

194. QUESTION [vD 101, Page 7]. Let X be a metrizable Suslinean continuum. Is it true that

$$\sup\{nc(Y, y) : y \in Y\} < \omega_1$$

for all $Y \subseteq X$?
Open.

195. QUESTION [vD 102, Page 2]. Let X be a separable, completely metrizable space, and let K be compact. Then $K \times {}^{\omega}2$ embeds into X if (and only if) X has an uncountable pairwise disjoint family of subspaces each homeomorphic to K. Is it possible to weaken the condition on X in this result?

If $\mathrm{Det}(\Pi^1_1)$ holds then every coanalytic space containing uncountably many copies of a compact space K contains a copy of $K \times {}^{\omega}2$. Also, there is an example of a σ-compact space containing uncountably many pairwise disjoint copies of the circle S^1 but not $S^1 \times {}^{\omega}2$, granting the existence of an uncountable coanalytic space without perfect subsets. Even for σ-compact metrizable spaces the answer to this question is independent of the usual axioms of set theory. For details, see BECKER, VAN ENGELEN and VAN MILL [7].

196. QUESTION [vD 107, Page 149]. Is there a compact countable antiHausdorff USL homogeneous Fréchet space in ZFC?
Open.

197. QUESTION [vD 107, Question 6.4]. Let $\mathcal{A}$ be κ-MAD. Is $\mathcal{A}$ γ-sliceable for $\gamma = tk(\mathcal{A})$? $\gamma = \mathfrak{a}_\kappa$? $\gamma = \kappa^+$? $\gamma = \kappa$? $\gamma = \omega$? $\gamma = 2$?
Open.

198. QUESTION [vD 107, Question 6.5]. Suppose there is an AD subfamily of $[\kappa]^\kappa$ of size α. Is there a γ-sliceable κ-MAD for $\gamma = \alpha$? $\gamma = \mathfrak{a}_\kappa$? $\gamma = \kappa^+$? $\gamma = \kappa$? $\gamma = \omega$? $\gamma = 2$?

199. QUESTION [vD 110, Conjecture 1.2]. If $2 \le k \le \aleph_0$ then the open balls and closed balls in $\mathbb{R}^n$ are not k-divisible.
Open.

200. QUESTION [vD 110, Conjecture 1.3]. If $n \ge 2$ are open balls and closed balls in $\mathbb{R}^n$ not $\aleph_0$-divisible.
Open.

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