# Adiabatically Changing the Phase-Space Density of a Trapped Bose Gas

P. W. H. Pinkse, A. Mosk, M. Weidemüller,* M. W. Reynolds, T. W. Hijmans, and J. T. M. Walraven*
*Van der Waals-Zeeman Institute, University of Amsterdam, Valckenierstraat 65/67, 1018 XE Amsterdam, The Netherlands*
(Received 15 October 1996)

We show that the degeneracy parameter of a trapped Bose gas can be changed adiabatically in a reversible way, both in the Boltzmann regime and in the degenerate Bose regime. We have performed measurements on spin-polarized atomic hydrogen in the Boltzmann regime, demonstrating reversible changes of the degeneracy parameter (phase-space density) by more than a factor of 2. This result is in good agreement with theory. By extending our theoretical analysis to the quantum degenerate regime we predict that, starting close enough to the Bose-Einstein phase transition, one can cross the transition by an adiabatic change of the trap shape. [S0031-9007(97)02357-0]

PACS numbers: 03.75.Fi, 67.65.+z, 32.80.Pj

The observation of Bose-Einstein condensation (BEC) in magnetically trapped atomic vapors of rubidium [1], sodium [2], and lithium [3] has opened a new field of study at the intersection of atomic and condensed matter physics. Presently, condensates are produced routinely, and detailed studies of condensate properties, such as collective excitations [4] and interaction of two condensates [5], are being made. The BEC phase transition itself is especially intriguing. Open questions include the kinetics of condensate formation and the effect of interatomic interactions and finite number of particles. Thus far, measurements have relied upon evaporative cooling to prepare the sample at the desired density below the critical temperature. Evaporative cooling, however, is inherently irreversible since it is based on the loss of hot particles from the trap [6–8]. To tackle the above questions it would be extremely valuable to vary the degeneracy of the trapped gas adiabatically in a reversible manner, with a fixed number of particles, especially since nondestructive detection methods have become available [9].

The possibility of increasing phase-space density and reaching BEC by changing the trapping potential was investigated by Ketterle and Pritchard [10]. For a collisionless gas they showed that it is impossible to influence phase-space density by manipulating the trapping potential. An example is cooling by adiabatic expansion in a harmonic trap: One does not lose any atoms, but one does not get closer to BEC either.

In this paper we show that this "no pain, no gain" principle is not true for a collisional gas. We experimentally demonstrate that the degeneracy parameter of a trapped gas can be changed adiabatically (without exchange of heat) and reversibly, without sacrificing atoms, by changing the shape of the trap slowly compared to the internal equilibration time. Our experiments are done in the Boltzmann regime, where the degeneracy parameter $n\Lambda^3$ coincides with phase-space density. Here $n$ denotes the density of the gas at the minimum of the potential, and $\Lambda \equiv (2\pi\hbar^2/mT)^{1/2}$ is the thermal de Broglie wavelength at temperature $T$ (with $m$ the atomic mass and Boltzmann's constant $k_B = 1$). We observed a change of $n\Lambda^3$ by a factor of 2, which agrees well with a quantitative prediction based on statistical thermodynamics applied to a trapped gas. We point out that, given suitable starting conditions, this approach is also suited to cross the BEC phase line. This follows from analytical expressions for heat capacity, entropy, and condensate fraction of a Bose gas as a function of the trap shape.

Our experiment is performed with atomic hydrogen in the cryogenic Ioffe trap described by Van Roijen *et al.* [11]. To determine quantities such as temperature and density we measure the Lyman-$\alpha$ absorption spectrum of the gas and fit calculated spectra to the experimental one (see [12] for details). It takes a measuring time of 40 s to reach a 10% level of accuracy under the present conditions. To minimize heating by photon recoil, we used our Lyman-$\alpha$ source at low intensity (typically $10^6$ photons at the sample per pulse, 20 pulses per second). In order to assure sufficient signal to noise ratio, we replaced the photodiode used in previous experiments [12] by a photomultiplier. This method offers a nondestructive way to follow the evolution of a trapped gas *in situ* under changing conditions.

The character of the Ioffe trap can be controlled via the magnetic field at the trap center, $B_0$. For $\mu_B B_0 \gg T$ (with $\mu_B$ the Bohr magneton), the trap is essentially harmonic. For $\mu_B B_0 \ll T$, the trap is close to linear in the radial direction and nearly harmonic axially. We selected two magnetic field configurations, A and B, shown in Fig. 1, that maximize the change in phase-space density within the constraints of our apparatus.

After loading the trap from a cryogenic dissociator, the sample is evaporatively cooled by ramping down the magnetic barrier at one of the longitudinal ends of the trap (at $z = 5$ cm, see Fig. 1). Evaporation is then stopped by raising this barrier. Hereafter we let the sample decay and equilibrate for about 1000 s to prepare a thermal sample that is better than 95% doubly spin polarized with typical densities at about $10^{11}$ atoms/cm$^3$ [11]. This density was selected to assure negligible sample

 0031-9007/97/78(6)/990(4)$10.00 

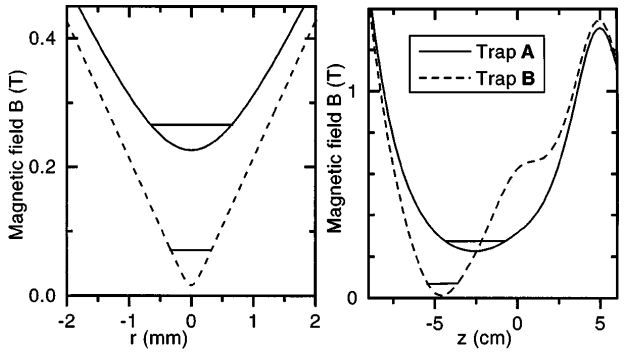


FIG. 1. Magnetic field profile of traps A and B radially cut through the field minimum (left) and cut along the principal axis of the trap (right). The horizontal lines indicate the measured thermal energy $T$. Trap A has a depth of 0.72 K and a field minimum $B_0$ of 226 mT. Trap B has a depth of 0.85 K and a field minimum $B_0$ of 12 mT.

loss during measurement of the spectrum. We gradually change from trap A to trap B or vice versa in 60 s, which is much slower than the average elastic collision time of 5 s at this density. After each change of trap, a spectrum is measured to determine the number of atoms and the degeneracy parameter.

Changing the trap configuration from A to B, the temperature increased reversibly from $T_A \approx 46$ mK to $T_B \approx 60$ mK. Both $T_A$ and $T_B$ showed an upward drift of $\sim 10$ mK to the final values mentioned above, reached after 5000 s. In Fig. 2 we plot the degeneracy parameter $n\Lambda^3$ versus time for a sample cycled between trap A and trap B. It can be seen that the phase-space density differs by a constant factor $2.05 \pm 0.13$ between trap A and trap B. The number of trapped atoms $N$ versus time is plotted in the lower graph of Fig. 2. Although $T$ and $n$ differ considerably between the two traps, the measured value for $N$ is seen to be conserved within experimental error. The slow decay of $N$ was found to be described by $N(t) = N(0)/[1 + N(0)Gt]$ as one would expect for a second-order decay process. The effective rate constant $G$ is an average of the (field-dependent) intrinsic rate constant $g$ (defined such that $g = -\dot{n}/n^2$ for a uniform gas) times a trap- and temperature-dependent factor [11]. Inverting this average is impracticable. However, the theoretical decay [13] under the conditions of trap A, $g = 4.1 \times 10^{-15}$ cm$^3$ s$^{-1}$, would give $G = 1.1(3) \times 10^{-14}$ s$^{-1}$ for this trap. For trap B, $g = 2.7 \times 10^{-15}$ cm$^3$ s$^{-1}$ which would give $G = 1.3(5) \times 10^{-14}$ s$^{-1}$. Hence the observed $G = 1.34(5) \times 10^{-14}$ s$^{-1}$, for a trap alternating between A and B, is consistent with intrinsic relaxation.

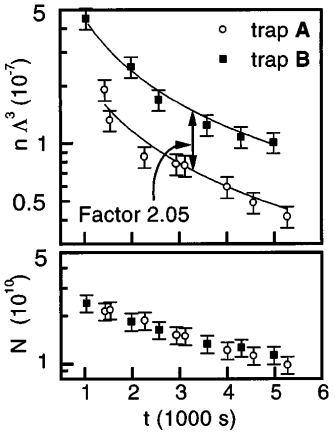


FIG. 2. Experimentally determined phase-space densities of trapped atomic hydrogen (upper graph), and number of atoms $N$ in the trap (lower graph) as a function of time, while the trapping potential was alternated between traps A and B. The curves are fits to a second-order decay, with the constant ratio $2.05 \pm 0.13$ of phase-space density between the two traps.

Our results can be understood within a dilute gas model for $N$ atoms at temperature $T$ trapped in a deep external potential, so that evaporation is negligible. Collisions keep the gas in internal thermal equilibrium. The number of particles in the gas is sufficient to enable a thermodynamic description. Since we change the trap potential slowly compared to the thermalization time, thermodynamic processes proceed reversibly. Since there is no exchange of heat or particles with the environment, the thermodynamic entropy $S$ of the gas is constant. In our experiment, the influence of quantum statistics is small and interactions between atoms do not influence thermodynamics as the mean-field interaction energy is much smaller than $T$. The degeneracy parameter can be expressed in terms of the single-particle partition function $Z_1$ and the total number of particles $N$ as $n\Lambda^3 = N/Z_1$ [7].

The internal energy $E$ can be calculated from the partition function to give [8]

$$E = (\tfrac{3}{2} + \gamma)NT\,, \tag{1}$$

where $\gamma = (T/V_e)\partial V_e/\partial T$, with $V_e \equiv N/n = Z_1\Lambda^3$ the effective volume. Here $\gamma T$ is the average potential energy per particle. For many trapping potentials $\gamma$ is a constant independent of $T$ and $V_e$ scales like $T^\gamma$. For example, for a box $\gamma = 0$, for a harmonic trap $\gamma = 3/2$, and for a spherical quadrupole trap $\gamma = 3$. The canonical partition function $Z_1^N/N!$ can be written as the exponential of $-(E - TS)/T$ (see, e.g., [14]) to arrive at the following expression for the degeneracy parameter:

$$n\Lambda^3 = \exp\left(\frac{5}{2} + \gamma - \frac{S}{N}\right). \tag{2}$$

Since in an adiabatic process $S$ and $N$ are constant, the phase-space density does not change unless $\gamma$ changes. It immediately follows that many of the most obvious ways of changing the trapping potential, such as isotropic or anisotropic scaling of a harmonic trap, do *not* influence phase-space density. If we change $\gamma$ adiabatically (e.g., experimentally, by changing $B_0$ in an Ioffe trap), $n\Lambda^3$ will change as $e^\gamma$. In an extreme case, by slowly changing the trap shape from square well to spherical quadrupole,

the phase-space density in the center of the trap can be increased by as much as a factor $e^3 \approx 20$.

For the two traps used in our experiment the value of $\gamma$ is weakly temperature dependent. For the measured temperatures we find $\gamma_A \approx 1.79$ for trap A and $\gamma_B \approx 2.53$ for trap B, the difference $\Delta\gamma = \gamma_B - \gamma_A$ after changing trap shape always being $0.74 \pm 0.01$. This implies a change in phase-space density by a factor $\exp[\Delta\gamma] = 2.10 \pm 0.02$, in good agreement with the measured value of $2.05 \pm 0.13$.

We emphasize that it should be possible to achieve considerable changes in the degeneracy parameter also around the BEC transition. For this purpose we extend our theoretical consideration to the case of a noninteracting Bose gas in the degenerate regime. This is a good approximation also for a weakly interacting Bose gas as long as the gas parameter $na^3 \ll 1$, where $a$ is the scattering length. Below the critical temperature we have the additional condition that $n_0\tilde{U} \ll T$, with $\tilde{U} = 4\pi\hbar^2 a/m$ the scattering strength and $n_0$ is the condensate density. Expressions obtained for the entropy of an ideal Bose gas are still good approximations around and above the critical temperature of a nonideal Bose gas. Interactions will change the shape of the condensate, but its entropy will always be zero. Because the influence of the interactions on the entropy of the above-condensate particles is negligible in the binary collision regime, the interactions only become important when the condensate fraction becomes so large that the mean-field energy of the condensate changes the effective potential for the above-condensate particles. Gases used in current investigations, such as Rb, Li, Na, and H are sufficiently close to this ideal gas limit that thermodynamics are essentially unaffected by interactions, unless the condensate fraction becomes appreciable.

In the degenerate regime we continue to assume quasiclassical motion of the atoms. For clarity we restrict ourselves to the case of a power-law potential, although our expressions can easily be generalized to include the Ioffe trap. A power-law trap is characterized by a density of states of the type $\rho(\epsilon) = A\epsilon^{1/2+\delta}$ (see [15] and [16] for details). The scaling parameter $A$ determines the size of the trap. E.g., for a harmonic trap $A = \frac{1}{2}(\hbar\omega)^{-3}$, where $\omega$ is the trap frequency. The parameter $\delta$ governs the shape of the trap. It can be shown that, in the Boltzmann regime, $\gamma = \delta$ independent of $T$ for all power-law traps.

The internal energy of a Bose gas in a power-law trap above and below the critical temperature $T_c$ [17] is given by

$$E = NT\left(\frac{3}{2} + \delta\right)\frac{g_{5/2+\delta}(z)}{g_{3/2+\delta}(z)}, \qquad T \geq T_c, \tag{3a}$$

$$E = NT\left(\frac{3}{2} + \delta\right)\frac{g_{5/2+\delta}(1)}{g_{3/2+\delta}(1)}\left(\frac{T}{T_c}\right)^{3/2+\delta}, \qquad T \leq T_c, \tag{3b}$$

where the Bose-Einstein integrals are expressed in polylogarithms $g_\alpha(x) = \sum_{l=1}^{\infty} x^l l^{-\alpha}$. The fugacity $z = \exp(\mu/T)$ with $\mu$ the chemical potential [18]. Note that as we have $T_c^{3/2+\delta} \propto N$, Eq. (3b) is in fact independent of the number of atoms. Above $T_c$ the fugacity is given implicitly by

$$N = AT^{3/2+\delta}\Gamma(\tfrac{3}{2} + \delta)g_{3/2+\delta}(z), \tag{4}$$

with $\Gamma(x)$ the Euler gamma function. Below $T_c$, $z = 1$ and the number of atoms in the condensate, $N_0$, is given by $N_0/N = 1 - (T/T_c)^{3/2+\delta}$. In the high temperature (Boltzmann) limit the parameter $n\Lambda^3$ introduced earlier reduces to the fugacity $z$. The specific heat at constant particle number and constant trap potential can now be found by taking the derivative of $E$ with respect to temperature. We obtain

$$C = N\left(\frac{3}{2} + \delta\right)[f_{5/2+\delta}(z) - f_{3/2+\delta}(z)], \qquad T > T_c, \tag{5a}$$

$$C = N\left(\frac{3}{2} + \delta\right)f_{5/2+\delta}(1)\left(\frac{T}{T_c}\right)^{3/2+\delta}, \qquad T < T_c, \tag{5b}$$

where we have introduced $f_\kappa(z) \equiv \kappa g_\kappa(z)/g_{\kappa-1}(z)$. For $\delta = 0$, $\frac{3}{2}$, and 3 we find $f_{5/2+\delta}(1) = 1.284$, 3.602, and 5.346, respectively. In Eq. (5a) the second term gives rise to a discontinuity in the heat capacity at $T_c$ as already found by Bagnato, Pritchard, and Kleppner [16], which appears only for $\delta > 1/2$ because for $\delta < 1/2$ the function $g_{1/2+\delta}$ diverges, and hence $f_{3/2+\delta}$ tends to zero, as $T \to T_c$. This jump in the heat capacity was recently observed by the JILA group [19]. The entropy $S$ obeys $dS/dT = C/T$, and hence has a kink at $T_c$ for $\delta > 1/2$. We find that

$$S = N[f_{5/2+\delta}(z) - \ln(z)], \qquad T \geq T_c, \tag{6a}$$

$$S = Nf_{5/2+\delta}(1)\left(\frac{T}{T_c}\right)^{3/2+\delta}, \qquad T \leq T_c. \tag{6b}$$

For the homogeneous case ($\delta = 0$) these expressions can be found in standard statistical mechanics textbooks (e.g., [18]). The entropy as a function of $T/T_c$ is plotted in Fig. 3.

It is noteworthy that the ideal Bose gas in a power-law trap in three space dimensions is isomorphic to the uniform Bose gas in $2\delta + 3$ dimensions. There is no heat capacity jump when the effective dimension is less than four. Remarkably, the processes which change the degeneracy parameter are just those which correspond to adiabatic changes of dimensionality.

If one moves along an isentropic line in Fig. 3 by increasing $\delta$, one can take a dilute gas close to BEC through the transition. For example, by varying $\delta$ from 0 to 3 it is possible to Bose condense a gas that had initially a temperature 12 times higher than the critical temperature for that trap and given $N$. Equations (6a) and (6b) allow

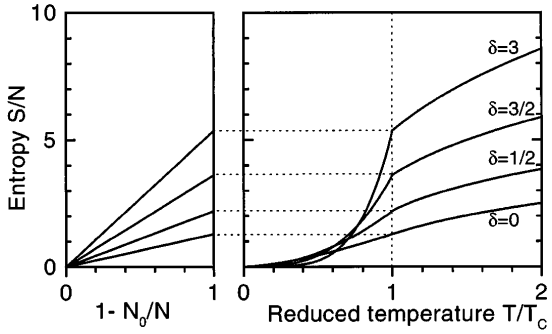


FIG. 3. Entropy of an ideal Bose gas in power-law traps as a function of $T/T_c$ (right) and, below $T_c$, also as a function of the above-condensate fraction (left).

one to calculate the fraction of condensate particles $N_0/N$. Starting with a Bose gas in an ideal harmonic potential at $T = T_c$, and reversibly changing the trapping potential to an ideal spherical quadrupole, we would arrive below $T_c$ with a condensate fraction of 0.33. For the Ioffe trap, starting at $T = T_c$ at the limit of high $B_0$, we expect a condensate fraction of 0.25 in the trap at the limit of low $B_0$. Of course, in these cases, $T/T_c$ may already be so low that the influence of interactions can no longer be neglected.

Our method allows one to control the degeneracy of a Bose gas and gently pull it across $T_c$ and back. Cycling times are only limited by the elastic collision rate which for the alkali atoms has been demonstrated to be much faster than for hydrogen. Combined with nondestructive diagnostics [9] this yields the unique possibility to study both condensate formation and destruction and to establish the presence or absence of asymmetries and hysteresis.

This work is part of a research program of the Stichting voor Fundamenteel Onderzoek der Materie (FOM), which is a subsidiary of the Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO). M. W. acknowledges a TMR grant by the European Commission. The research of M. W. R. is supported by the Royal Netherlands Academy of Arts and Sciences (KNAW).

*Present address: FOM Institute for Atomic and Molecular Physics (AMOLF), Kruislaan 407, 1098 SJ Amsterdam, The Netherlands.

[1] M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Science **269**, 198 (1995).

[2] K. B. Davis, M.-O. Mewes, M. R. Andrews, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle, Phys. Rev. Lett. **75**, 3969 (1995).

[3] C. C. Bradley, C. A. Sackett, and R. G. Hulet, Phys. Rev. Lett. **78**, 985 (1997); see also C. C. Bradley, C. A. Sackett, J. J. Tollet, and R. G. Hulet, Phys. Rev. Lett. **75**, 1687 (1995).

[4] D. S. Jin, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. **77**, 420 (1996); M.-O. Mewes, M. R. Andrews, N. J. van Druten, D. M. Kurn, D. S. Durfee, C. G. Townsend, and W. Ketterle, *ibid.*, 988 (1996).

[5] C. G. Townsend, *et al.*, in Proceedings of the 15th International Conference on Atomic Physics, Amsterdam, 1996 (to be published).

[6] H. F. Hess, Phys. Rev. B **34**, 3476 (1986).

[7] J. T. M. Walraven, in *Proceedings of the SUSSP 44 Conference On Quantum Dynamics of Simple Systems*, edited by G.-L. Oppo, S. M. Barnett, E. Riis, and M. Wilkinson (Institute of Physics Publishing, Bristol, 1996); W. Ketterle and N. J. van Druten, in *Advances in Atomic, Molecular and Optical Physics*, edited by B. Bederson and H. Walther (Academic Press, San Diego, 1996), Vol. 37, p. 181.

[8] O. J. Luiten, M. W. Reynolds, and J. T. M. Walraven, Phys. Rev. A **53**, 381 (1996).

[9] M. R. Andrews, M.-O. Mewes, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle, Science **273**, 84 (1996).

[10] W. Ketterle and D. E. Pritchard, Phys. Rev. A **46**, 4051 (1992).

[11] R. van Roijen, J. J. Berkhout, S. Jaakkola, and J. T. M. Walraven, Phys. Rev. Lett. **61**, 931 (1988).

[12] O. J. Luiten, H. G. C. Werij, I. D. Setija, M. W. Reynolds, T. W. Hijmans, and J. T. M. Walraven, Phys. Rev. Lett. **70**, 544 (1993); Appl. Phys. B **59**, 311 (1994).

[13] A. Lagendijk, I. F. Silvera, and B. J. Verhaar, Phys. Rev. B **33**, 626 (1986); H. T. C. Stoof, J. M. V. A. Koelman, and B. J. Verhaar, Phys. Rev. B **38**, 4688 (1988).

[14] F. Reif, *Fundamentals of Statistical and Thermal Physics* (McGraw-Hill, New York, 1965).

[15] This includes the important examples of square ($\delta = 0$), spherical-quadrupole ($\delta = 3$), and harmonic traps ($\delta = 3/2$), the low and high $B_0$ limits of the Ioffe trap [8], and all traps with potentials that can be written as $U(x, y, z) = a|x|^{1/\delta_1} + b|y|^{1/\delta_2} + c|z|^{1/\delta_3}$ with $\delta = \sum_i \delta_i$.

[16] V. Bagnato, D. E. Pritchard, and D. Kleppner, Phys. Rev. A **35**, 4354 (1987).

[17] T. W. Hijmans, Yu. Kagan, G. Shlyapnikov, and J. T. M. Walraven, Phys. Rev. B **48**, 12 886 (1993).

[18] K. Huang, *Statistical Mechanics* (Wiley, New York, 1963).

[19] C. E. Wieman, E. A. Cornell, D. Jin, J. Ensher, M. Matthews, C. Myatt, E. Burt, and R. Ghrist, in Proceedings of the 15th International Conference on Atomic Physics, Amsterdam, The Netherlands, 1996 (to be published).