Bose-Einstein Condensation into Nonequilibrium States Studied by Condensate Focusing

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We report the formation of Bose-Einstein condensates into nonequilibrium states. Our condensates are much longer than equilibrium condensates with the same number of atoms, show strong phase fluctuations, and have a dynamical evolution similar to that of quadrupole shape oscillations of regular condensates. The condensates emerge in elongated traps as the result of *local* thermalization when the nucleation time is short compared to the axial oscillation time. We introduce condensate focusing as a new method to extract the phase-coherence length of Bose-Einstein condensates.

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Among the quantum fluids, the quantum gases are especially suited to study the kinetics of Bose-Einstein condensation (BEC). The modest density of the quantum gases makes it possible to stretch the time of condensate formation to values allowing detailed experimental investigation. In external potentials, condensates with repulsive interactions have a characteristic equilibrium shape, known as the Thomas-Fermi shape, that differs markedly from the shape of thermal clouds. Since the pioneering experiments on BEC in the alkali systems [1–3], this feature has been observed by many groups and is used routinely to measure the condensate fraction [4]. Phase coherence is another key property of equilibrium condensates and was established in the first interference experiments [5–8].

Equilibrium condensates [1,2] are produced by quasistatic growth, where heat extraction limits the formation rate. The condensate nucleates as a small feature in the center of the trap and grows as heat is extracted from the sample. To observe the formation kinetics, the gas has to be brought out of equilibrium, in practice by shock cooling. Since the first experiment on condensate growth, by Miesner *et al.* [9], this is done by fast removal of the most energetic atoms from the trap. Starting from a thermal gas just above the phase transition temperature (T_C), the condensate appears as the result of thermalization. Miesner *et al.* [9] observed the growth under adiabatic conditions. Köhl *et al.* [10] continued the extraction of heat and atoms, also during growth. In both experiments, the condensate was observed to grow from the center of the trap, such as in the quasistatic limit.

Kagan *et al.* [11] pointed out that qualitatively different stages have to be distinguished in the formation of equilibrium condensates with a large number of atoms. The early stage (kinetic stage) is governed by Boltzmann kinetic processes and leads to a preferential occupation of the lowest energy levels. Once a substantial fraction of the atoms gathers within an energy band of the order of the chemical potential of the emerging condensate during

formation, the density fluctuations are suppressed in a fast, interaction-dominated regime governed by a non-linear equation for the boson field. The appearing phase-fluctuating condensate then grows and the condensed fraction approaches its equilibrium value. However, the phase fluctuations still persist, giving rise to dynamically evolving flow patterns in search for the true equilibrium state. In elongated 3D trapped gases, the phase fluctuations can be pronounced even under equilibrium conditions as was predicted by Petrov *et al.* [12] and observed experimentally by Dettmer *et al.* [13].

In this Letter, we report the formation of condensates into nonequilibrium states and a new path towards equilibrium in elongated traps. In contrast to the previous experiments, our results were obtained starting from thermal clouds deep in the crossover regime to hydro-dynamic behavior. The condensates are much longer than equilibrium condensates with the same number of atoms. Moreover, they display strong phase fluctuations and a dynamical evolution similar to that of a quadrupole shape oscillation decaying towards equilibrium. We identify $1/\omega_z$ as a characteristic time that should be addressed explicitly for elongated cylindrical harmonic traps, i.e., for traps with $\omega_\rho \gg \omega_z$, where ω_ρ and ω_z are the radial and axial angular frequencies, respectively. We show that these exotic condensates emerge as the result of *local* thermalization when the nucleation time is short as compared to $1/\omega_z$. The dynamical evolution of the condensate in the trap has to be dealt with explicitly to properly interpret time-of-flight absorption images. In this context, we introduce condensate focusing as an alternative to other methods for measuring the phase-coherence length of Bose-Einstein condensates [6,13].

In the previous experiments on condensate formation, the phase fluctuations were not studied. The results of Miesner *et al.* [9] were compared to an analytical expression for adiabatic growth of a condensate from a thermal cloud, derived in [14]. Although a qualitative agreement between theory and experiment was readily

obtained, it turned out to be impossible to obtain detailed agreement at the quantitative level [15,16]. In the experiment of Köhl et al. [10], quantitative agreement with the quantum kinetic approach (see Refs. [14–16]) was obtained for strong truncation, whereas for weak truncation the observed behavior differed distinctly from theory.

In our experiments, we typically load 4×10^9 atoms of ^{87}Rb in the $|5^2 S_{1/2}, F = 2, m_F = 2\rangle$ state into a horizontal Ioffe-Pritchard quadrupole trap with $\omega_\rho = 2\pi \times 477(2)$ Hz and $\omega_z = 2\pi \times 20.8(1)$ Hz [17]. The trap minimum $B_0 = 88.6(1)\ \mu$T corresponds to a radio frequency of $\nu_0 = 620$ kHz as calibrated against atom laser output coupling [18]. The trap minimum shows a long term drift of 5 kHz/h. At full current thermal drift effects are less than 1 kHz/s.

The gas is prepared by forced RF evaporation at a final rate of $\dot{\nu}_{tr} = -433$ kHz/s to a value of $\nu_{tr,a} = 740$ kHz, followed by 20 ms of plain evaporation at $\nu_{tr,a}$. As

$$|\dot{\nu}_{tr}/(\nu_{tr,a} - \nu_0)| \ll \omega_z, \omega_\rho, \qquad (1)$$

this yields a static, purely thermal cloud of $N_i \approx 5 \times 10^6$ atoms at a temperature $T_0 = 1.3(1)\ \mu$K as determined from the axial $1/e$ size $l_z = [2kT_0/m\omega_z^2]^{1/2}$ of the cloud in the trap, measured by time-of-flight absorption imaging shortly ($\tau < 1/\omega_z$) after release from the trap. We calculate a central density $n_0 \approx 4 \times 10^{14}$ cm^{-3}, corresponding to a mean-free path $\lambda_0 = (2^{1/2} n_0 \sigma)^{-1} \approx 3\ \mu$m and a collision rate $\tau_{col}^{-1} \approx n_0 v_{th} \sigma \approx 5000$ s^{-1} [19]. Radially, we find $\lambda_0/l_\rho \approx 0.5$. Axially, we have $\lambda_0/l_z \approx 0.02$ as compared to $\lambda_0/l_z \approx 0.5$ and 0.3 used in previous experiments [9,10] on condensate formation. Hence, our thermal samples are prepared far deeper in the crossover to the hydrodynamic regime.

The hydrodynamic behavior manifests itself in the damping time τ_Q and a shift of the frequency ω_Q of the quadrupole shape oscillation as well as in an anisotropic expansion of the cloud after release from the trap. We measured $\omega_z \tau_Q = 10(3)$ and $\omega_Q/\omega_z = 1.56(5)$. For $\omega_z \tau_Q = 10(3)$, theory predicts $\omega_Q/\omega_z = 1.57(2)$ [20] and $\omega_Q/\omega_z = \sqrt{12/5} \approx 1.55$ for the hydrodynamic limit in very elongated traps [21,22]. Further, writing $\beta \equiv \omega_z/\omega_\rho$, from the scaling theory [21], we find in this limit $v_z/v_\rho \approx 2.6\beta \approx 0.11$ for the ratio of axial to radial expansion velocity after release from the trap. We measured $v_z/v_\rho = 0.78(2)$ and a final temperature $T_\infty = 0.94(4)\ \mu$K as determined from the axial expansion, which implies that the gas expands hydrodynamically and cools only briefly before the expansion becomes ballistic. For isentropic hydrodynamic expansions, the degeneracy parameter is conserved: $n(\tau)/n_0 = [T(\tau)/T_0]^{3/2}$. Assuming that the expansion is initially hydrodynamic and after a time τ_{freeze} ballistic, the scaling theory [21] gives a relation between T_0/T_∞ and v_z/v_ρ. In elongated traps, for $v_z/v_\rho \gg \beta$, we find

$$T_0/T_\infty \approx [1 + 2(v_\rho/v_z)^2]/3. \qquad (2)$$

The observed ratio v_z/v_ρ and temperature T_0 lead to $T_\infty = 0.91(7)\ \mu$K, which coincides with T_∞ measured from the axial expansion. From the obtained value of $n(\tau_{freeze})/n_0$, we calculate $\tau_{freeze} \approx 0.3$ ms, i.e., a couple of collision times. This is consistent with the ratio $\lambda_0/l_\rho \approx 0.5$ mentioned above and confirms the picture that we are operating near the onset of hydrodynamic behavior in the radial direction. We verified that the expansion becomes isotropic, $v_z/v_\rho = 1.02(4)$, when the atom number is reduced by a factor of 30.

Once the thermal cloud is prepared, we distinguish three distinct stages. First, in the truncation stage, the radio frequency is set to the value $\nu_{tr,b} = 660$ kHz. This stage has a duration $t_{tr} = 1$ ms, which is chosen to be long enough ($t_{tr} > 1/\omega_\rho$) to allow atoms with radial energy ε_ρ larger than the RF truncation energy ε_{tr} to escape from the trap, yet is short enough to disallow evaporative cooling. We found that in this stage 50% of the atoms are removed. Notice that due to the finite radial escape horizon ($\lambda_0/l_\rho \approx 0.4$) the ejection is not expected to be complete. Futhermore, the escape efficiency is anisotropic as a result of gravitational sag. The truncation energy ε_{tr} covers the range 3 μK–5 μK depending on the position of the truncation edge in the gravity field and is lowered by an additional 1 μK due to RF dressing (Rabi frequency $\Omega_{rf} \approx 2\pi \times 14$ kHz). At the start of the second stage, the thermalization stage, the radio frequency is stepped back up for a time t_{th} to the frequency $\nu_{tr,a}$ to allow the gas to thermalize under formation of a condensate. The value $\nu_{tr,a}$ is chosen to eliminate any appreciable evaporative cooling. The third stage, the expansion stage, starts by switching off the trap and covers the time-of-flight τ after which the sample is absorption imaged on the $|5^2 S_{1/2}, F = 2\rangle \rightarrow |5^2 P_{3/2}, F = 1, 2, \text{ or } 3\rangle$ transition.

To follow the evolution of the trapped gas after the truncation, we took time-of-flight absorption images for a range of evolution times $t \equiv t_{tr} + t_{th}$ and a fixed expansion time τ. The images show a bimodal distribution, indicating that the truncation procedure results in BEC. The condensate fraction grows to a final value of 6% with a characteristic time of 6 ms. This corresponds to $30\tau_{col}$, in accordance with previous experiments.

Rather than discussing the details of the growth kinetics, we emphasize that our condensates nucleate into nonequilibrium states. In Fig. 1, we plot the Thomas-Fermi half length L_z obtained with the standard fitting procedure of a bimodal distribution to our data [4]. For the shortest expansion time, $\tau = 2.8$ ms, the axial size of the condensate image equals to good approximation ($\tau \ll 1/\omega_z$) the axial size of the condensate in the trap. We see that $L_z(t)$ is initially oversized by a factor $L_z(0)/L_z(\infty) = 2.2(3)$ and rapidly decreases to reach its equilibrium size after roughly one strongly damped shape oscillation (see open triangles in Fig. 1). Hence, the condensate is clearly not in equilibrium [23].
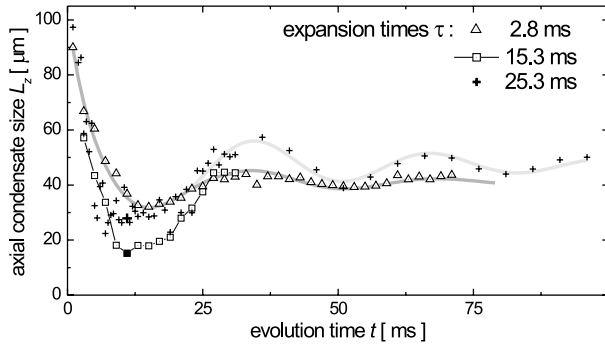
FIG. 1.   Condensate half length $L_z(t, \tau)$ versus evolution time t for three different expansion times τ. The dark grey line is a guide to the eye. The light grey line represents a fit to a damped quadrupole shape oscillation.

The formation of oversized condensates follows from the local formation concept underlying Ref. [11]. Starting with a thermal cloud of $N_i = 5 \times 10^6$ atoms in an 11.5 mK deep harmonic trap at a temperature $T_0 = 1.3$ μK, we calculate that 55% of the atoms remain trapped after all atoms with energy $\varepsilon > \varepsilon_{tr} = 3.4$ μK are removed. The gas will rethermalize well within a time short compared to $1/\omega_z$. Hence, the resulting temperature varies along the trap axis. In this respect, the thermalization is a *local* phenomenon. The local T_C is given by [24]

$$kT_C(z) \approx 1.28\hbar\omega_\rho[n_{1D}(z)r_\rho]^{2/5}, \qquad (3)$$

where $r_\rho = [\hbar/m\omega_\rho]^{1/2}$ is the radial oscillator length and $n_{1D}(z)$ the atom number per unit length at position z. We find that the local temperature $T(z)$ is lower than the local $T_C(z)$ over a length of order of the thermal length l_z. In view of the simplicity of this model, we consider this as good qualitative agreement with experiment.

To further investigate the formation process, we introduce condensate focusing. In our case, one-dimensional focusing results from axial contraction of the expanding cloud when the gas is released from the trap during the inward phase of a shape oscillation. The focus is best demonstrated by plotting the Thomas-Fermi axial size $L_z(t, \tau)$ of the condensate as a function of expansion time τ after a fixed evolution of $t = 11$ ms in the trap (see Fig. 2). The axial size first decreases, reaches a focus, and then increases again [25].

We elucidate the focusing in relation to the low-frequency branch of the $m = 0$ quadrupole shape oscillation of the Thomas-Fermi condensate. In the limit of linear response, the scaling parameter b_z for the axial size of the condensate in the trap can be written as $b_z(t) \equiv L_z(t)/L_z(0) = 1 + a_z \cos\omega_Q t$, with $a_z \ll 1$ the scaled axial amplitude and t the evolution time. Releasing the gas at time t and observing the cloud after an expansion time $\tau \gg 1/\omega_\rho$, the scaling theory [21,26] offers an approximate expression for the scaled axial size:
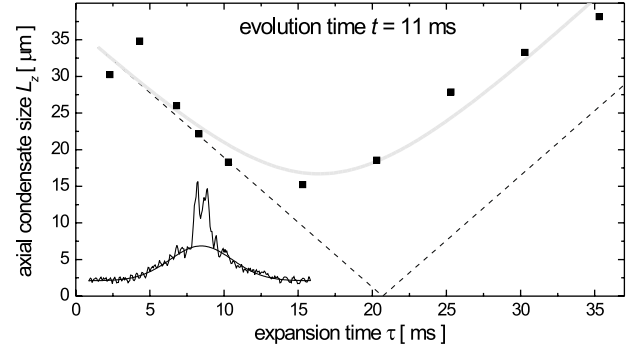

FIG. 2.   Condensate half length $L_z(t, \tau)$ versus expansion time τ after 11 ms of evolution (ten point averages; see [25]). The grey (dashed) line is a fit to the scaling equations including (excluding) phase fluctuation broadening. Inset: Axial density profile showing phase fluctuation stripes ($\tau = 6.8$ ms).

$$b_z(t, \tau) \approx 1 + (\pi\beta\omega_z/2 - a_z\omega_Q \sin\omega_Q t)\tau. \qquad (4)$$

The first term in brackets corresponds to the axial expansion kick caused by the declining chemical potential at trap release. The second term represents the scaled axial dilatation velocity in the trap at the moment of release. From Eq. (4), we see that the axial dilatation field reaches a real (virtual) focus for positive (negative) values of the expansion time $\tau_{focus} = (a_z\omega_Q \sin\omega_Q t - \frac{\pi}{2}\beta\omega_z)^{-1}$. A real focus can be obtained even for small amplitudes, $a_z > \beta$.

For $\tau \gg 1/\omega_\rho$, the *radial* expansion is described by $b_\rho(t, \tau) \approx \omega_\rho\tau(1 - \frac{1}{4}a_z\cos\omega_Q t)$; i.e., it shows no focus. As the radial size remains finite, the chemical potential builds up near the focus until the compression is balanced and the axial size starts to expand again. The scaling equations [26,21] predict a maximum axial compression of a factor of $2\beta^2$.

Our data match a scaled focal size $b_z(t, \tau_{focus}) = 0.49(6)$; i.e., the focus is strongly broadened as compared to the minimum scaled size $2\beta^2$. The broadening is attributed to local variations in expansion velocity caused by the presence of phase fluctuations in our condensates. After some expansion these variations give rise to irregular stripes (see the inset of Fig. 2) as previously observed in Hannover [13]. At the focus, the axial distribution maps linearly onto the momentum distribution due to the phase fluctuations in the original condensate. The scaled focal size is given by $b_z(t, \tau_{focus}) \sim (\hbar/mL_\phi)\tau_{focus}/L_z$, where L_ϕ is the phase-coherence length and $(\hbar/mL_\phi)$ characterizes the expansion velocity due to the phase fluctuations. For equilibrium phase fluctuations close to T_C, we estimate $L_z/L_\phi = 7(4)$ (see [27]) and $b_z \sim 0.05$. As the observed focal size is larger by an order of magnitude, the phase fluctuations have a nonequilibrium origin with a phase-coherence length of only $L_\phi \sim 1$ μm. The decay of the phase fluctuations as a function of evolution time is a subject of further investigation in our group.

We point out that for a *thermal* cloud driven on the low-frequency quadrupole mode the phenomenon of focusing is absent except deeply in the hydrodynamic regime. In the latter case, the ratio T_∞/T_0 is small and the scaled focal size will be given by $b_z(t, \tau_{focus}) \approx v_{th}(T_\infty)\tau_{focus}/l_z(T_0) \approx (T_\infty/T_0)^{1/2}\omega_z\tau_{focus} < 1$.

In Fig. 1, we also show the oscillation in the axial size of the condensate as observed for 15.3 ms (open squares) and 25.3 ms (crosses) of expansion [28]. Because of enhancement by focusing the amplitude of the oscillation has increased as compared to the 2.8 ms results. For $\tau = 25.3$ ms, the shape oscillation is seen to exceed the noise for at least 100 ms. This oscillation can be described by a linear response expression for evolution times $t \gtrsim 20$ ms where $a_z \le 0.2$. We measure a damping time of $\tau_Q = 50(9)$ ms and a frequency ratio $\omega_Q/\omega_z = 1.54(4)$. The latter is slightly lower than the frequency expected for a quadrupole shape oscillation of a pure condensate in very elongated traps, $\omega_Q/\omega_z \approx \sqrt{5/2} \approx 1.58$ [21,26]. A 5% negative frequency shift was observed for the quadrupole mode in Na condensates just below T_C [29] and is consistent with theory [30].

We conclude that BEC in elongated traps should be regarded as a local phenomenon for nucleation times short compared to the axial oscillation time. In our experiment, this was realized by preparing samples at high densities, reaching the crossover to classical hydrodynamic behavior. To analyze the nonequilibrium state, we introduced condensate focusing as a new method to investigate degenerate Bose gases. This method enhances the visibility of the shape oscillations and enables the observation of small condensate fractions close to T_C. The focal size provides information about the phase-coherence length in the evolving condensate.

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