# How Occam's razor provides a neat definition of direct causation


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# Outline

1. Woodward's interventionist theory of causation
2. Reconstructing Woodward's theory
3. Result 1: CMC + Min + IE $\Rightarrow$ direct causation Woodward style
4. Result 2: CMC + Min + $\text{IE}_S$ $\Rightarrow$ direct causation Woodward style

# Woodward's interventionist theory of causation

**Definition ($IV_W$)**

$I$ is an intervention variable for $X$ wr.t. $Y$ iff

I1. $I$ causes $X$.

I2. $I$ acts as a switch for all other variables that cause $X$.

I3. Any directed path from $I$ to $Y$ goes through $X$.

I4. $I$ is (statistically) independent of any variable $Z$ that causes $Y$ and that is on a directed path that does not go through $X$.

$I = on$ is an intervention on $X$ w.r.t. $Y$ iff $I$ is an intervention variable for $X$ w.r.t. $Y$ and $I = on$ forces $X$ to take a certain value $x$. (cf. Woodward, 2003, p. 98)

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# Woodward's interventionist theory of causation

## Definition ($DC_W$)

A necessary and sufficient condition for $X$ to be a (type-level) direct cause of $Y$ w.r.t. a variable set $\mathbf{V}$ is that there be a possible intervention on $X$ that will change $Y$ or the probability distribution of $Y$ when one holds fixed at some value all other variables $Z_i$ in $\mathbf{V}$. (Woodward, 2003, p. 59)

Open questions/concerns:

- What is a "possible" intervention?
  - $\Rightarrow$ logically/conceptually possible maybe not restrictive enough...
- Applicable to all kinds of variable sets $\mathbf{V}$?
  - $\Rightarrow$ problems with sets containing variables for which there are no interventions in the sense of ($IV_W$)...

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# Reconstructing Woodward's theory

## Definition (IV)

$I_X \in \mathbf{V}$ is an intervention variable for $X$ w.r.t. $Y$ in $\langle \mathbf{V}, E, P \rangle$ iff

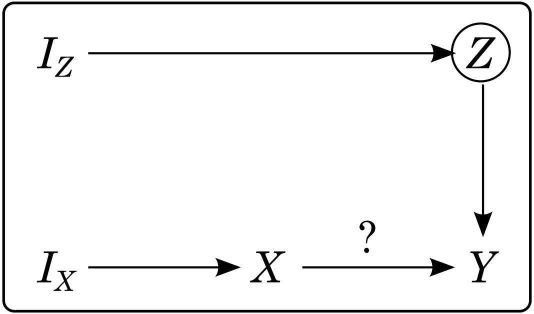
(a) $I_X$ is exogenous and there is a path $\pi : I_X \longrightarrow X$ in $\langle \mathbf{V}, E \rangle$,

(b) for every *on*-value of $I_X$ there is an $X$-value $x$ such that $Dep(x, I_X = on)$ and $P(x|I_X = on) = 1$,

(c) all paths $I_X \longrightarrow ... \longrightarrow Y$ in $\langle \mathbf{V}, E \rangle$ have the form $I_X \longrightarrow ... \longrightarrow X \longrightarrow ... \longrightarrow Y$,

(d) $I_X$ is independent from every variable $C$ (in $\mathbf{V}$ or not in $\mathbf{V}$) which causes $Y$ over a path not going through $X$. (cf. Woodward, 2008)

# Reconstructing Woodward's theory



# Reconstructing Woodward's theory

## Definition (IE)

$M' = \langle \mathbf{V}', E', P' \rangle$ is an *i*-expansion of $M = \langle \mathbf{V}, E, P \rangle$ w.r.t. $Y$ iff

(a) $\mathbf{V}' = \mathbf{V} \dot{\cup} \mathbf{V_I}$, where $\mathbf{V_I}$ contains for every $X \in \mathbf{V}$ different from $Y$ an intervention variable $I$ w.r.t. $Y$ (and nothing else),

(b) for all $Z_i, Z_j \in \mathbf{V}$: $Z_i \longrightarrow Z_j$ in $E'$ iff $Z_i \longrightarrow Z_j$ in $E$,

(c) for every $X$-value $x$ of every $X \in \mathbf{V}$ different from $Y$ there is an *on*-value of the corresponding intervention variable $I_X$ such that $Dep(x, I_X = on)$ and $P'(x|I_X = on) = 1$,

(d) $P'_{\mathbf{I}=\mathbf{off}} \uparrow \mathbf{V} = P$,

(e) $P'(\mathbf{I} = \mathbf{on}), P'(\mathbf{I} = \mathbf{off}) > 0$.

# Reconstructing Woodward's theory

We reconstruct Woodward's (2003) definition of direct causation as a partial definition:

## Definition (DC)

If there exist *i*-expansions $\langle \mathbf{V}', E', P' \rangle$ of $\langle \mathbf{V}, E, P \rangle$ w.r.t. $Y$, then: $X \in \mathbf{V}$ is a direct cause of $Y \in \mathbf{V}$ w.r.t. $\mathbf{V}$ iff $Dep(Y, I_X = on | \mathbf{I_Z} = \mathbf{on})$ holds in some *i*-expansions, where $I_X$ is an intervention variable for $X$ w.r.t. $Y$ in $\langle \mathbf{V}', E', P' \rangle$ and $\mathbf{I_Z}$ is the set of all intervention variables in $\langle \mathbf{V}', E', P' \rangle$ different from $I_X$.

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# Result 1: CMC + Min + IE $\Rightarrow$ Woodward

## Definition (CMC)

A causal model $\langle \mathbf{V}, E, P \rangle$ satisfies the causal Markov condition iff every $X \in \mathbf{V}$ is probabilistically independent of all its non-effects conditional on its causal parents (cf. Spirtes et al., 2000, p. 29).

CMC is assumed to hold for causal models whose variable sets are causally sufficient:

## Definition (causal sufficiency)

$\mathbf{V}$ is causally sufficient iff every common cause $C$ of variables in $\mathbf{V}$ is in $\mathbf{V}$ or takes the same value $c$ for all individuals in the domain. (cf. Spirtes et al, 2000, p. 22)

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For acyclic graphs, CMC is equivalent to the *d*-separation criterion (cf. Verma, 1986; Pearl, 1988, p. 119f):

## Definition (d-separation criterion)

$\langle \mathbf{V}, E, P \rangle$ satisfies the d-separation criterion iff the following holds for all $X, Y \in \mathbf{V}$ and $\mathbf{Z} \subseteq \mathbf{V} \backslash \{X, Y\}$: If $X$ and $Y$ are *d*-separated by $\mathbf{Z}$ in $\langle \mathbf{V}, E \rangle$, then $Indep(X, Y|\mathbf{Z})$.

## Definition (*d*-separation, *d*-connection)

$X$ and $Y$ are *d*-separated by $\mathbf{Z}$ in $\langle \mathbf{V}, E \rangle$ iff $X$ and $Y$ are not *d*-connected given $\mathbf{Z}$ in $\langle \mathbf{V}, E \rangle$.
$X$ and $Y$ are *d*-connected given $\mathbf{Z}$ in $\langle \mathbf{V}, E \rangle$ iff $X$ and $Y$ are connected by a causal path $\pi$ in $\langle \mathbf{V}, E \rangle$ such that no non-collider on $\pi$ is in $\mathbf{Z}$, while all colliders on $\pi$ are in $\mathbf{Z}$ or have an effect in $\mathbf{Z}$.

# Result 1: CMC + Min + IE $\Rightarrow$ Woodward

Occam's razor (or the causal minimality/productivity condition):

**Definition (Min)**

If $\langle \mathbf{V}, E, P \rangle$ satisfies CMC, then $\langle \mathbf{V}, E, P \rangle$ satisfies Min iff no submodel $\langle \mathbf{V}, E', P \rangle$ with $E' \subset E$ also satisfies CMC (cf. Spirtes et al., 2000, p. 31).

**Definition (Prod)**

$\langle \mathbf{V}, E, P \rangle$ satisfies Prod iff $Dep(X, Y | Par(Y) \backslash \{X\})$ holds for all $X, Y \in \mathbf{V}$ with $X \longrightarrow Y$ in $\langle \mathbf{V}, E \rangle$. (Schurz and Gebharter, 2014)

**Theorem (1)**

*For acyclic causal models satisfying CMC, Min is equivalent with Prod.*

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# Result 1: CMC + Min + IE $\Rightarrow$ Woodward

## Theorem (2)

*If $\langle \mathbf{V}, E, P \rangle$ is an acyclic causal model and for every $Y \in \mathbf{V}$ there is an i-expansion $\langle \mathbf{V}', E', P' \rangle$ of $\langle \mathbf{V}, E, P \rangle$ w.r.t. $Y$ satisfying CMC and Min, then for all $X, Y \in \mathbf{V}$ (with $X \neq Y$) the following two statements are equivalent:*

*(i)* $X \longrightarrow Y$ *in* $\langle \mathbf{V}, E \rangle$.

*(ii)* $Dep(Y, I_X = on | \mathbf{I_Z} = \mathbf{on})$ *holds in some i-expansions of* $\langle \mathbf{V}, E, P \rangle$ *w.r.t.* $Y$*, where* $I_X$ *is an intervention variable for* $X$ *w.r.t.* $Y$ *in* $\langle \mathbf{V}', E', P' \rangle$ *and* $\mathbf{I_Z}$ *is the set of all intervention variables in* $\langle \mathbf{V}', E', P' \rangle$ *different from* $I_X$.

$\Rightarrow$ Direct causation a la Woodward coincides with the graph theoretical notion of direct causation in systems $\langle \mathbf{V}, E, P \rangle$ with *i*-expansions w.r.t. every $Y \in \mathbf{V}$ satisfying CMC and Min.

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# Result 2: CMC + Min + $IE_S \Rightarrow$ Woodward

**Definition ($IV_S$)**

$I_X \in \mathbf{V}$ is a stochastic intervention variable for $X$ w.r.t. $Y$ in $\langle \mathbf{V}, E, P \rangle$ iff

(a) $I_X$ is exogenous and there is a path $\pi : I_X \longrightarrow X$ in $\langle \mathbf{V}, E \rangle$,

(b) for every *on*-value of $I_X$ there is an $X$-value $x$ such that $Dep(x, I_X = on)$,

(c) all paths $I_X \longrightarrow ... \longrightarrow Y$ in $\langle \mathbf{V}, E \rangle$ have the form $I_X \longrightarrow ... \longrightarrow X \longrightarrow ... \longrightarrow Y$,

(d) $I_X$ is independent from every variable $C$ (in $\mathbf{V}$ or not in $\mathbf{V}$) which causes $Y$ over a path not going through $X$.

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# Result 2: CMC + Min + $\text{IE}_S \Rightarrow$ Woodward

**Definition ($\text{IE}_S$)**

$M' = \langle \mathbf{V}', E', P' \rangle$ is a stochastic $i$-expansion of $M = \langle \mathbf{V}, E, P \rangle$ for $X$ w.r.t. $Y$ iff

(a) $\mathbf{V}' = \mathbf{V} \dot{\cup} \mathbf{V}_{I_X}$, where $\mathbf{V}_{I_X}$ contains a stochastic intervention variable $I_X$ for $X$ w.r.t. $Y$ (and nothing else),

(b) for all $Z_i, Z_j \in \mathbf{V}$: $Z_i \longrightarrow Z_j$ in $E'$ iff $Z_i \longrightarrow Z_j$ in $E$,

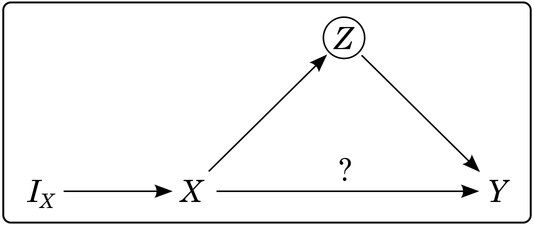
(c) for every $X$-value $x$ of every $X \in \mathbf{V}$ different from $Y$ there is an *on*-value of the corresponding intervention variable $I_X$ such that $Dep(x, I_X = on)$,

(d) $P'_{I_X = off} \uparrow \mathbf{V} = P$,

(e) $P'(I_X = on), P'(I_X = off) > 0$.

# Result 2: CMC + Min + $IE_S \Rightarrow$ Woodward

- To establish a direct causal relationship $X \longrightarrow Y$, Woodward (2003) needs to block probability propagation between $X$ and $Y$ over indirect paths.
- Alternatively one can block all indirect paths between $X$ and $Y$ by conditionalizing on $Par(Y)\backslash\{X\}$.



# Result 2: CMC + Min + $IE_S \Rightarrow$ Woodward

We reconstruct the stochastic variant of Woodward's (2003) definition of direct causation as a partial definition:

## Definition ($DC_S$)

If there exist stochastic *i*-expansions $\langle \mathbf{V}', E', P' \rangle$ of $\langle \mathbf{V}, E, P \rangle$ for $X$ w.r.t. $Y$, then: $X \in \mathbf{V}$ is a direct cause of $Y \in \mathbf{V}$ w.r.t. $\mathbf{V}$ iff $Dep(Y, I_X = on | Par(Y) \backslash \{X\})$ holds in some stochastic *i*-expansions, where $I_X$ is an intervention variable for $X$ w.r.t. $Y$ in $\langle \mathbf{V}', E', P' \rangle$.

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# Result 2: CMC + Min + $\text{IE}_S \Rightarrow$ Woodward

**Theorem (3)**

*If $\langle \mathbf{V}, E, P \rangle$ is an acyclic causal model and for all $X, Y \in \mathbf{V}$ (with $X \neq Y$) there is a stochastic i-expansion $\langle \mathbf{V}', E', P' \rangle$ of $\langle \mathbf{V}, E, P \rangle$ for $X$ w.r.t. $Y$ satisfying CMC and Min, then for all $X, Y \in \mathbf{V}$ (with $X \neq Y$) the following two statements are equivalent:*

*(i)* $X \longrightarrow Y$ *in* $\langle \mathbf{V}, E \rangle$.

*(ii)* $Dep(Y, I_X = on | Par(Y) \backslash \{X\})$ *holds in some stochastic i-expansions of* $\langle \mathbf{V}, E, P \rangle$ *for* $X$ *w.r.t.* $Y$.

$\Rightarrow$ The stochastic version of direct causation a la Woodward coincides with the graph theoretical notion of direct causation in systems $\langle \mathbf{V}, E, P \rangle$ with stochastic *i*-expansions for every $X \in \mathbf{V}$ w.r.t. every $Y \in \mathbf{V}$ (with $X \neq Y$) satisfying CMC and Min.

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# Conclusion

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Many thanks!

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