Homework Stochastic Simulation (2017) - Third set

The deadline of this homework set is Friday December 1 at 15:00. For the programming exercises, you are free to use any programming language or mathematical software package (Maple/Matlab/Mathematica/R/anything) as long as you

- include all code in your deliverables, and
- for random number generation purposes, you use standard built-in methods to generate standard uniform samples. For generating samples from any other distribution, you'll have to construct them yourself using the standard uniform samples.

Please compile answers and code in a single PDF document, and send it by e-mail to Jan-Pieter Dorsman (j.l.dorsman 'at' uva...).

Should you have any questions about the homework exercises, please reach out to one of the lecturers during the lecture.

Exercise 1 Make exercise V.1.4 of [AG].

Exercise 2 Make exercise V.1.5 of [AG].

Exercise 3 Suppose that Z is normal random variable with unit mean and unit variance. Moreover, let X be a random variable which again has a normal distribution, but now with variance parameter 10 and a stochastic mean parameter Z. In other words, conditional on $Z = z$, X has a normal distribution with mean z and variance 10. In this exercise we are interested in the parameter $\theta = \mathbb{P}(X \geq 1)$.

1. What is the theoretical value of θ? Below, we'll act as if we don't know the value, and we wish to simulate this value.
2. Explain how the crude Monte Carlo estimator is used to simulate the value of θ.
3. Show how the conditional Monte Carlo method can be used to improve the crude Monte Carlo Estimator.
4. Show how the method of antithetic sampling can be used to improve the crude Monte Carlo Estimator.
5. Show how the method of control variables can be used to improve the crude Monte Carlo Estimator.
6. Implement each of these methods, and check the variance that each estimator entails. Which of the four estimators is best?

Exercise 4 Make exercise V.5.1 of [AG].

Exercise 5 Make exercise V.7.1 of [AG], by interpreting the expression as $\mathbb{E}[h(X)]$ for some function h, and some random variable X. Suggest at least three methods, of which one is based on control variates and one is based on stratification (the third is completely up to you!). Explain how of each of the methods work, and explain why variance reduction is to be expected in each of them.