Homework Stochastic Simulation (2017) - Fourth set

The deadline of this homework set is Friday December 15 at 09:00. For the programming exercises, you are free to use any programming language or mathematical software package (Maple/Matlab/Mathematica/R/anything) as long as you

- include all code in your deliverables, and
- for random number generation purposes, you use standard built-in methods to generate standard uniform samples. For generating samples from any other distribution, you'll have to construct them yourself using the standard uniform samples.

Please compile answers and code in a single PDF document, and send it by e-mail to Jan-Pieter Dorsman (j.l.dorsman 'at' uva...).

Should you have any questions about the homework exercises, please reach out to one of the lecturers during the lecture.

Exercise 1 Let X be a random variable which is Weibull distributed with parameters 2 and λ. In other words, its density function is given by

$$f_X(x) = 2\lambda^2 x e^{-\lambda^2 x^2} \mathbb{1}_{\{x>0\}}.$$

Furthermore, let $Z = h(X)$, where $h(x) = 2^x$ and $\theta > 0$.

a) Explain briefly how one can estimate $\frac{d}{d\lambda}\mathbb{E}[Z]\big|_{\lambda=1}$ using the method of finite differences, while not using common random numbers.

b) Explain briefly how one can estimate $\frac{d}{d\lambda}\mathbb{E}[Z]\big|_{\lambda=1}$ using the method of finite differences and using common random numbers.

c) Explain briefly how one can estimate $\frac{d}{d\lambda}\mathbb{E}[Z]\big|_{\lambda=1}$ using the method of infinitesimal perturbation analysis.

d) Explain briefly how one can estimate $\frac{d}{d\lambda}\mathbb{E}[Z]\big|_{\lambda=1}$ using the likelihood ratio method.

e) Implement each of these methods in a simulation program. Then, compare how well each of these methods works in this case (e.g. by creating confidence intervals). How do the methods rank in order of estimator quality? Report and argue your findings.

Exercise 2 Consider the problem of estimating $z(x) = \mathbb{P}(\tau(x) < \infty)$ for $x > 0$, where $\tau(x) = \inf\{n : S_n > x\}$, $S_n = \sum_{i=1}^n X_i$ and the X_i are i.i.d. copies of a random variable X. Assume that X has a a non-lattice, light-tailed distribution with a negative mean. Assume moreover that X can take positive values.

Using Siegmund's algorithm, what is the expected number of replicates of $Z(x)$ one requires as $x \to \infty$ to obtain a confidence interval with a confidence level of 95%, of which the width does not exceed 20% of the value of $z(x)$? Give an expression in terms of $\mathbb{E}_\gamma\left[e^{-\gamma\xi(\infty)}\right]$.