DATA SEMANTICS AND THE PRAGMATICS OF INDICATIVE CONDITIONALS

Frank Veltman*)
University of Amsterdam

Some arguments are logically valid but pragmatically incorrect. Other ones, though logically invalid, are pragmatically correct. Grice's *Logic and Conversation*[1] has taught us which distinctions should be drawn, but unfortunately most of us seem to draw them differently. For example, if you believe that indicative conditionals behave like material or strict implications, you will presumably be ready to point out that the intuitively absurd argument:[2]

(1) *If Jones wins the election, Smith will retire to private life*
If Smith dies before the election, Jones will win it

∴ *If Smith dies before the election, he will retire to private life*

is 'just' a pragmatically incorrect instance of the logically valid Hypothetical Syllogism:

(2) *If* B *then* C
If A *then* B

∴ *If* A *then* C

If, on the other hand, your favourite semantic theory attributes the logical properties of variable strict implications to indicative conditionals, in which case you will find the Hypothetical Syllogism logically invalid, you will emphasize that the intuitively sound argument:

(3) *If Jones wins the election, Smith will have to leave the White House*
If Smith goes on antagonizing his supporters, Jones will win the election

∴ *If Smith goes on antagonizing his supporters, he will have to leave the White House*

is 'at best' a pragmatically correct instance of this invalid inference pattern.[3]

The literature on conditionals is full of examples like (1) - put forward by the one author as a clear cut counterexample to a putative logical principle, only to be explained away by the other as an innocent pragmatic exception to an otherwise faultless semantic rule. The strategy described in connection with (3) is less frequently followed. Still, every now and then some author feels called upon to explain why a given inference pattern - in most cases a classical logical principle which 'as recent investigations show' is 'clearly' logically invalid - has for so long kept out of harm's way. Usually, the explanation offered involves a partial rehabilitation of the inference pattern concerned: although not logically valid, most of its instances turn out pragmatically sound.

So we see that what the one logician calls a logically valid argument form with a few pragmatically incorrect instances is for the other a pragmatically correct argument form with a number of logically invalid instances. Unfortunately their disagreement is not just verbal. There may be some overlap, but in general the semantic arguments put forward by the one are quite different in character from the pragmatic arguments put forward by the other. The problem is that in many cases the kinds of semantic arguments put forward by both differ as well. And the same goes for their respective pragmatic arguments. Actually, the pragmatic differences are the least pronounced: most people working on conditionals agree that pragmatic theories begin where semantic theories end and that they should take the form of a theory of conversation à la Grice. But then - and this is typical for the field of conditionals - there is no consensus at all as to what form a semantic theory should take. That of a theory of truth?

According to the majority of logicians, who take the classical standard of logical validity as the starting point of their investigations, yes. (Roughly: an argument is valid iff it is impossible for all its premises to be true while its conclusion is false.) No, answer the Relevance Logicians:[4] truth preservation may be a necessary condition for the logical validity of an argument but it is by no means sufficient. The premises of the argument must in addition be relevant to the conclusion. No, answer Adams *et al.*[5], believing as they do that the proper explication of validity is to be given in terms of probability rather than truth. (Roughly: an argument is valid iff it is impossible for all its premises to be probable while its conclusion is improbable.) And no, I shall answer in this paper; the proper explication of logical validity is this: an argument is valid iff it is impossible for all its premises to be true on the basis of the available evidence while its conclusion is not true on that basis. Consequently, a semantic theory should supply an explication of what it means for a statement to be *true on the basis of the available evidence*.

In the sequel I will sketch such a semantic theory and discuss some of its more salient features. The above remarks should have made clear, however, that there is little sense in discussing a semantic theory - if at least it presents a semantics for conditionals - without paying any attention to its ramifications for pragmatics. Therefore a good deal of this paper is devoted to pragmatic questions: if indicative conditionals do have the semantic properties ascribed to them here, what will their pragmatic properties be? Which logically invalid arguments will become pragmatically correct and which logically valid arguments will on pragmatic grounds become absurd? I hope to show that the answer to this question can be much less arbitrary than the literature suggests. Indeed one might hope that the kind of semantics discussed here allows for a pragmatics in which the dividing line between logical validity and pragmatic correctness is drawn exactly as a criterion of cancellability prescribes.

§ 1 *Information models*

What does it mean for an English sentence, in particular an English conditional sentence, to be true on the basis of the available evidence? Following usual logical practice, I shall not try to answer this question directly but introduce a logical language L, the sentences of which will serve as formal 'translations' of English sentences. L is given by:

(i) a vocabulary consisting of countably many atomic sentences, two parentheses, three one place operators $\neg$, *must*, and *may* and three two place operators $\wedge$, $\vee$, and $\rightarrow$;

(ii) the formation rules that one would expect for a language with such a vocabulary.

As usual the operators $\neg$, $\wedge$ and $\vee$ are meant as formal counterparts of English negation, conjunction and disjunction respectively. If A and B are formal translations of the English sentences A' and B', then $(A \rightarrow B)$ is meant to be a formal translation of the *indicative* conditional with antecedent A' and consequent B'. The operator *may* represents the English expression 'it may be the case that', and the operator *must* the expression 'it must be the case that'. It will appear that the semantic and pragmatic properties of indicative conditionals are closely bound up with the properties of these modal expressions, which is why I have included them in L.

In presenting the semantics for L, I shall again follow usual practice and first specify the admissable *models* for L.

<u>*Definition 1.*</u>[6] An *information model* (for L) is a triple $\langle S,\leqslant,V\rangle$ with the following properties:

(i) $S \neq \emptyset$.

(ii) $\leqslant$ is a partial ordering of S; each maximal chain in $\langle S,\leqslant\rangle$ contains a maximal element.

(iii) V is a function with domain S; (a) for each $s \in S$, V_s is a *partial* function assigning at most one of the values 1 or 0 to the atomic sentences of L; (b) if $s \leqslant s'$, $V_s \subseteq V_{s'}$; (c) if s is a maximal element of $\langle S,\leqslant\rangle$, V_s is *total*.

Amplification. The basic entities of an information model, the elements of S, are called *information states*. All there is to know about any information state s is covered by the relation $\leqslant$ and the partial function V_s. V_s tells for each atomic sentence A whether A is true on the basis of the evidence available at s, in which case $V_s(A) = 1$; or whether A is false on the basis of that evidence, in which case $V_s(A) = 0$; or whether the evidence available at s does not allow for any definite conclusion about the truth value of A, in which case $V_s(A)$ is undefined. The relation $\leqslant$ determines the position of s among the other information states. In this connection it is of particular interest to know what, given the evidence available at s, the outcome of any *further* investigations might be. If $s \leqslant s'$, we say that *it is possible for* s on further investigation *to grow into* s', or alternatively, that *the evidence available at* s' *extends* (or equals) *the evidence available at* s. So understood, it will be clear why $\leqslant$ is taken to be a partial order. If s is a maximal element of $\langle S,\leqslant\rangle$, s is called a *complete* information state. The choice of terminology will do here as an explanation for the assumption that at maximal elements s of $\langle S,\leqslant\rangle$ the function V_s is total; that is, if s is maximal V_s assigns a definite truth value to every atomic sentence. The assumption that each maximal chain in $\langle S,\leqslant\rangle$ contains a maximal element implies that every incomplete information state can be extended to a complete information state. Actually, the assumption is somewhat stronger: it excludes the possibility of there being any sequence of successive information states that does not ultimately end in an information state that is complete.[7] The assumption that $V_s \subseteq V_{s'}$ if $s \leqslant s'$ constrains the semantic properties of atomic sentences considerably: once an atomic sentence A has turned out to be true (or false) on the basis of the evidence, it will remain true (or false) whatever additional data may come to light. As we shall see in due course, not every sentence of L is *stable* in this sense. Notice that it may very well be that $s < s'$, while $V_s = V_{s'}$: accumulation of evidence need not necessarily mean that more atomic sentences turn out true or false. For example, it may very well be that it is possible for s to grow into an information state where both the atomic sentence A and the atomic sentence B will be true on the basis of the available evidence, whereas once s has grown into s' this possibility is excluded. That does not mean, however, that it must be clear on the basis of evidence available at s' which of the atomic sentences A and B is false.

Definition 1 leaves many questions unanswered. For one thing, whenever $V_s(A) = 1$ for a given atomic sentence A and information state s we say 'A is true *on the basis of the evidence available* at s', but the model $\langle S,\leqslant,V\rangle$ does not give us any clue as to what this evidence consists in. Furthermore, apart from the requirement that there be sufficiently complete information states, no constraints at all are placed on the kinds of information states that an information model should contain. Suppose, for example, that the model $\langle S,\leqslant,V\rangle$ contains two information states s and s'; the atomic sentence A is true on the basis of the evidence available at s, and false on the basis of the evidence available at s'. Shouldn't there then be an information state s'' such that $s'' < s$, $s'' < s'$, and $V_{s''}(A)$ is undefined?

The so-called data models introduced in my [1981] do answer these and similar questions. What is here called the evidence available at a given information state, is there identified with a data set, this being a special kind of subset of 'the' *set of possible facts*. What is here taken as the extension relation '$\leqslant$' between information states, there boils down to the subset relation '$\sqsubseteq$' between data sets. Here, a speaker is supposed to be in a given information state; there, he is supposed to be *acquainted* with the facts that constitute a given data set. Within data models atomic sentences are treated as names of possible facts. The atomic sentence A is true on the basis of a given data set iff this set contains the fact named by A. And A is false on the basis of a given data set iff this set contains a fact that is *incompatible* with the fact named by A. Thus, in data models the condition that atomic sentences should remain true (or false) once their truth (or falsity) has been established - which we had to stipulate for information models - is automatically fulfilled.

Data models and information models generate the same logic. In other words, the ontological extras of data models do not yield any logical returns. Actually, from a purely logical point of view information models are much more pleasant to deal with. I would not even have

mentioned data models here if I did not think that there is one particular feature of information models - and an important feature as will appear - that cannot be properly explained without reference to data sets. Notice that the information models $\langle S,\leqslant,V\rangle$ are so defined that it might very well occur that for a given atomic sentence A and an information state s the following holds:

(i) for no $s' \geqslant s$, $V_{s'}(A) = 0$;

(ii) $V_s(A)$ is undefined.

Notice that from (i) it follows that $V_{s'}(A) = 1$ for every *complete* $s' \geqslant s$. So it may very well occur that a certain atomic sentence A is not true on the basis of the evidence available at s while on the other hand it is impossible for s to grow into an information state at which A will turn out false; indeed, s will inevitably grow into an information state at which A is true.

One may wonder whether we should allow this. Wouldn't it be plausible to call A true on the basis of the evidence available at s? Shouldn't we demand that $V_s(A) = 1$ if for no $s' \geqslant s, V_{s'}(A) = 0$?

I do not think so. I think it would blur an important distinction - that between *direct* and *indirect* evidence-if one were to maintain that it is solely on the basis of the evidence available at s that the sentence A is true. In the terminology of data sets: someone in the information state s is not directly acquainted with the fact named by A. His data at best enable him to infer that no fact incompatible with the fact named by A can be added. In other words, his data at best constitute indirect evidence for the truth of A: A *must* be true, all right, but it may take quite some time until this is definitely shown.

§ 2 *Stability and Instability*

Let $M = \langle S,\leqslant,V\rangle$ be an information model, s an information state in S and A a sentence of L. In the sequel '$M \Vdash_s A$' abbreviates 'A is true (in M) on the basis of the evidence available at s', and '$M \;{}_s\!\dashv\!\mid A$' abbreviates 'A is false (in M) on the basis of the evidence available at s'.

<u>*Definition 2*</u>. Let $M = \langle S,\leqslant,V\rangle$ be a model and s an information state.

- If A is atomic, then

  $M \Vdash_s A$ iff $V_s(A) = 1$

  $M \;{}_s\!\dashv\!\mid A$ iff $V_s(A) = 0$

- $M \Vdash_s \neg A$ iff $M \;{}_s\!\dashv\!\mid A$

  $M \;{}_s\!\dashv\!\mid \neg A$ iff $M \Vdash_s A$

- $M \Vdash_s$ *may* A iff for some information state $s' \geq s$, $M \Vdash_{s'} A$
  $M\ {}_s\!\dashv\!\vert$ *may* A iff for no information state $s' \geq s$, $M \Vdash_{s'} A$
- $M \Vdash_s$ *must* A iff for no information state $s' \geq s$, $M\ {}_{s'}\!\dashv\!\vert\ A$
  $M\ {}_s\!\dashv\!\vert$ *must* A iff for some information state $s' \geq s$, $M \Vdash_{s'} A$
- $M \Vdash_s A \wedge B$ iff $M \Vdash_s A$ and $M \Vdash_s B$
  $M\ {}_s\!\dashv\!\vert\ A \wedge B$ iff $M\ {}_s\!\dashv\!\vert\ A$ or $M\ {}_s\!\dashv\!\vert\ B$
- $M \Vdash_s A \vee B$ iff $M \Vdash_s A$ or $M \Vdash_s B$
  $M\ {}_s\!\dashv\!\vert\ A \vee B$ iff $M\ {}_s\!\dashv\!\vert\ A$ and $M\ {}_s\!\dashv\!\vert\ B$
- $M \Vdash_s A \rightarrow B$ iff for every information state $s' \geq s$, if $M \Vdash_{s'} A$ then $M \Vdash_{s'} B$
  $M\ {}_s\!\dashv\!\vert\ A \rightarrow B$ iff for some information state $s' \geq s$, $M \Vdash_{s'} A$ and $M_{s'}\!\dashv\!\vert\ B$

In discussing this definition I shall often refer to the following information states.

Information state 1. You are presented with two little boxes, box 1 and box 2. Together they contain three marbles, a blue one, a yellow one, and a red one. Each box contains at least one of them. That is all you know.

Information state 2. As 1, except that you now know that the blue marble is in box 1. (Where the other two marbles are remains a secret.)

2.1. *may*

Suppose you are in information state 1. Somebody says: 'Maybe the blue marble is in box 2.' Would you agree?

Suppose you are in information state 2. Somebody says: 'Maybe the blue marble is in box 2.' Would you still agree?

According to definition 2, your answer to the first question should be 'Yes', and to the second question 'No'. Definition 2 says (i) that a sentence of the form *may* A is true on the basis of the evidence available at a given information state s as long as it is possible for s to grow into an information state s' where on the basis of the then available evidence A is true; and (ii) that such a sentence is false on the basis of the evidence available at s as soon as this possibility can be excluded. In information state 1 you must still reckon with the possibility that the blue marble will turn out to be in box 2. Therefore the sentence 'Maybe the blue marble is in box 2' is true on the basis of the evidence available there. In information state 2 you do not have to reckon with this possibility any more. Once you know that the blue marble is in box 1 it is wrong to maintain that it may nevertheless be in

box 2. (You should say that the blue marble *might have been* in box 2.)

Unlike atomic sentences, the truth of sentences of the form *may* A need not be stable. They will often be true on the basis of limited evidence only to become false as soon as new evidence becomes available. Once their falsity has been established, however, it has been established for good. In terms of the following definition: sentences of the form *may* A, though in general not T-stable, are at least F-stable.

Definition 3. Let A be a sentence.
A is *T-stable* iff for every model $M = \langle S, \leq, V \rangle$ and information state $s \in S$, if $M \Vdash_s A$ then $M \Vdash_{s'} A$ for every information state $s' \geq s$;
A is *F-stable* iff for every model $M = \langle S, \leq, V \rangle$ and information state $s \in S$, if $M\ {}_s\!\dashv\!\vert\ A$ then $M\ {}_{s'}\!\dashv\!\vert\ A$ for every information state $s' \geq s$;
A is *stable* iff A is both T-stable and F-stable.

2.2. *must*

I already hinted at the truth condition for the operator *must* near the end of section 1. According to definition 2, a sentence of the form *must* A is true on the basis of the available evidence iff no additional evidence could make A false. (Hence, if one keeps on gathering more information, A must sooner of later turn out true.) As long as A could yet turn out false, *must* A is false.

It is worth noting that in many cases this analysis renders a sentence of the form *must* A weaker than A itself. If an atomic sentence A is true on the basis of the available evidence, then *must* A is true on that basis as well. But *must* A can be true on the basis of the evidence without A being true on that basis. In the latter case the data constitute at best indirect evidence for A, in the first case direct evidence.

That *must* A is weaker than A on many occasions has been noticed by a number of authors. Karttunen (1972) illustrates this with the following examples:

(a) *John must have left*
(b) *John has left*

His informal explanation fits in neatly with my formal analysis: 'Intuitively, (a) makes a weaker claim than (b). In general, one would use (a), the epistemic *must*, only in circumstances where it is not yet an established fact that John has left. In (a), the speaker indicates that he has no first hand evidence about John's departure, and neither has it been reported to him by trustworthy sources. Instead (a) seems to say that the truth of *John* *has left* in some way logically follows from other facts the speaker knows and some reasonable assumptions that he is willing to entertain. A man who has actually seen John leave or has read about it in the newspaper would not ordinarily assert (a), since he is in the position to make the stronger claim in (b).' Similar remarks can be found in Groenendijk & Stokhof (1975) and in Lyons (1977). Despite this unanimity no formal theory has so far been proposed which actually predicts that on many occasions *must* A is a logical consequence of A. Most theories treat *may* and *must* as epistemic modalities and depending on whether the underlying epistemic notion is knowledge or belief *must* A turns out to be either stronger than A or independent of it.

Notice that sentences of the form *must* A are T-stable though they are not in general F-stable. Consider for example the sentence

(c) *Either the yellow or the red marble must be in box 2*

For all you know in information state 1 it may very well be that the blue marble is in box 2 while both the yellow and the red marble are in box 2. Hence it is not the case that either the yellow or the red one must be in box 2. But as soon as you are told that the blue marble is in box 1 this is different. At least one of the marbles is in box 2 and it cannot be the blue one. So it must be the yellow one or the red one.

2.3. *if*

According to definition 2, a sentence of the form (A → B) is true on the basis of the evidence available at a given information state s iff s cannot grow into an information state s' at which A is true on the basis of the available evidence and B is not true. If, by any chance, further investigations should reveal that A is true they will reveal that B is true too. Furthermore, it is stated that (A → B) is false on the basis of the evidence available at a certain information state s iff it is still possible for s to grow into an information state at which A is true and B false on the basis of the available evidence.

It will be clear that on this account a sentence of the form (A → B) is not necessarily F-stable. Consider the sentence

(d) *If the yellow marble is in box 1, the red one is in box 2*

Again, the evidence available in information state 1 allows for the possibility that both the yellow and the red marble are in box 1. So on the basis of the limited evidence available in information state 1, (d) is false: it is not the case that if the yellow marble is in box 1, the red one is in box 2. In information state 2, however, the sentence is not false any more. Once you know that the blue marble is in box 1, you can be sure that if the

yellow marble is in box 1 then the red one will turn out to be in box 2.

Now consider the negation of (d):

(e) *It is not the case that if the yellow marble is in box 1, the red one is in box 2*

This sentence is true on the basis of the evidence available at information state 1 - at least if we apply definition 2 to it. Suppose you are in information state 1. Somebody says (d): 'If the yellow marble is in box 1, the red marble is in box 2'. Would it be correct, then, to reply like this: 'No, you are wrong, it may very well be that both the yellow and the red marble are in box 1. So it is not the case that if the yellow marble is in box 1, the red one is in box 2'? I think that such a reply would only under very special circumstances be correct. Only when you know for certain that the speaker in question is *not better informed than* yourself. Because only then can you be sure that he is mistaken. Usually you will not know whether or not the speaker is better informed than yourself. Perhaps he is. Perhaps he is telling you something about the marbles you did not yet know. Certainly, for all *you* know in information state 1, sentence (d) is false and sentence (e) is true, but unfortunately sentence (d) is not F-stable, and sentence (e) is not T-stable. If by any chance the blue marble should be in box 1 and if the speaker happens to know this, then he is right after all. So perhaps he knows where the blue marble is. Why not ask him whether he means to say that the blue marble is in box 1, rather than question the truth of what he is saying? In normal conversation every statement is meant to convey some new information and only when this new information is incompatible with any T-stable sentence that is true on the basis of the evidence gathered may one raise doubts about it. Like when you are in information state 2 and somebody says 'If the yellow marble is in box 1, the red marble is in box 1 too'. But even then it would be incorrect to reply with a simple denial: 'No, that is not so, it *may* very well be that the yellow marble is in box 1 and the red one in box 2'. Again, these sentences are not T-stable, in most cases they owe their truth just to a lack of information on the part of the speaker - that is certainly what the addressee will think. So what you will have to reply is something much stronger: 'No, it cannot be that the yellow and the red marble are both in box 1. If the yellow one is in box 1, the red one will be in box 2'.

These considerations may help us to understand some of the pecularities of negated conditionals. For one thing, they explain why a conditional statement (A → B) is so often refuted with a counter conditional (A → ¬B) rather than a negated conditional ¬(A → B). But they do so without equating sentences of the form (A → ¬B) with sentences of the form ¬(A → B): on the present account the sentence

It is not the case that if Jones wins the election, Smith will retire to private life

is not equivalent [8] to

If Jones wins the election, Smith will not retire to private life

but to

It may very well be that Jones wins the election and that Smith will not retire to private life

(Suppose Smith dies before the election. And then suppose he doesn't.)

Let A be F-stable and suppose that A is false on the basis of the available evidence. Then according to definition 2, $(A \rightarrow B)$ is true on the basis of the evidence for *any* sentence B. Similarly, if B is stable and true on the basis of the available evidence then $(A \rightarrow B)$ is true on the basis of the evidence for *any* sentence A. In other words: the present treatment of conditionals does not meet the requirement that a sentence of the form $(A \rightarrow B)$ should *never* be true unless the antecedent A is somehow 'relevant' to the consequent B.

I do not think that this matters too much: pragmatic constraints ensure that an indicative conditional $(A \rightarrow B)$ will not normally be uttered in circumstances where so much is known about the truth or falsity of A and B. As will be shown in section 4, an indicative conditional $(A \rightarrow B)$ is most naturally asserted in a context in which neither the truth nor the falsity of the antecedent A or consequent B is definitely established: *may* A, *may* $\neg$A, *may* B, *may* $\neg$B are all 'normal' implicatures of $(A \rightarrow B)$. In fact, the present truth condition by itself guarantees that a sentence of the form $(A \rightarrow B)$, when uttered in such a context, cannot be true unless A is highly relevant to B: $(A \rightarrow B)$ is true on the basis of the available evidence iff no additional evidence can establish the truth of A without establishing that of B as well. Clearly, there must be some positive connection between A and B if the two are to behave in this way.

2.4. *and, or* and *not*

I trust that the truth and falsity conditions for sentences of the form A, $(A \wedge B)$, and $(A \vee B)$ do not need any further explanation. [9] In the sequel I shall sometimes discriminate between the sentences in which $\neg$, $\wedge$ and $\vee$ are the only occurring operators and the other ones by calling

the former *descriptive* and the latter *nondescriptive*. All descriptive sentences are stable, most nondescriptive sentences are not. Intuitively, the difference between these two kinds of sentences amounts to this: by uttering a descriptive sentence a speaker only informs his audience of the evidence he already has. By uttering a nondescriptive sentence he also expresses his expectations about the outcome of further investigations.

Notice that at *complete* information states s the following holds:

$M \Vdash_s A$ or $M \,_s\!\dashv\!\!| \; A$

$M \Vdash_s A \rightarrow B$ iff $M \nVdash_s A$ or $M \Vdash_s B$

$M \Vdash_s$ *may* A iff $M \Vdash_s A$

$M \Vdash_s$ *must* A iff $M \Vdash_s A$

In other words, it does not make much sense to use the phrases 'if...then', 'must' and 'may' in a context where the information is complete. 'If...then' gets the meaning of the material implication while the meaning of both 'must' and 'may' boils down to that of the empty operator. However, in such a context there is no need to use nondescriptive sentences - the information is *complete*; so what could possibly be the good of speculations on the outcome of further investigations?

§ 3 *Logical Validity and Invalidity*

Definition 4. Let A be a sentence and Δ a set of sentences.
$\Delta \Vdash A$ iff there is no model $M = \langle S, \leqslant, V\rangle$ such that for some $s \in S$, $M \Vdash_s B$ for every $B \in \Delta$ while $M \nVdash_s A$.

'$\Delta \Vdash A$' abbreviates 'the argument Δ/A (i.e. the argument with premises Δ and conclusion A) is valid'. I shall write '$\Vdash A$' instead of '$\emptyset \Vdash A$' and '$\Delta, B_1, \ldots, B_n \Vdash A$' instead of '$\Delta \cup \{B_1, \ldots, B_n\} \Vdash A$'. Read '$\Vdash A$' as 'A is logically valid'.

The logic generated by the above definition differs in some important respects from any in the literature. Most of the following observations have to do with these differences. For a more systematic account the reader is refered to my (forthcoming).

3.1. *and, or* and *not*

As far as descriptive sentences are concerned, the departure from classical logic is not too drastic. Many classical principles concerning $\wedge$, $\vee$ and $\neg$ are valid in the sense of definition 4 as well:

- If $\Delta \Vdash A$ or $\Delta \Vdash B$, then $\Delta \Vdash A \vee B$
- If $\Delta \Vdash A \vee B$ and $\Delta, A \Vdash C$ and $\Delta, B \Vdash C$, then $\Delta \Vdash C$
- If $\Delta \Vdash A$ and $\Delta \Vdash B$, then $\Delta \Vdash A \wedge B$
- If $\Delta \Vdash A \wedge B$, then $\Delta \Vdash A$ and $\Delta \Vdash B$
- If $\Delta \Vdash A$ and $\Delta \Vdash \neg A$, then $\Delta \Vdash C$

The reader will notice that this list is made up entirely of principles which underly the classical system of natural deduction for $\wedge$, $\vee$ and $\neg$. Actually only one of the principles underlying that system is missing. Within the present context proofs by *Reductio ad Absurdum* are not in general valid. It is not generally so that

- If $\Delta, \neg A \Vdash B$ and $\Delta, \neg A \Vdash \neg B$ then $\Delta \Vdash A$

The closest approximation available is this:

- If $\Delta, \neg A \Vdash B$, and $\Delta, \neg A \Vdash \neg B$, and each C in Δ is T-stable, then $\Delta \Vdash$ *must* A

Since descriptive sentences are all T-stable, this implies the following proposition:

Proposition. Let $\wedge$, $\vee$ and $\neg$ be the only operators occuring in the sentences of Δ/A. Suppose Δ/A is classically valid. Then $\Delta \Vdash$ *must* A.

In other words, if by the standards of classical logic, the descriptive sentence A must follow from the descriptive sentences Δ then *at least* 'it must be the case that A' follows from Δ by the standards set here.

Illustration. Let A be descriptive. Then $\not\Vdash A \vee \neg A$. In fact there are no valid descriptive sentences at all.
It is easy to check, however, that $A \Vdash$ *must* $(A \vee \neg A)$ for any sentence A
In this connection it is worth noting that also $\Vdash$ *must* A $\vee \neg$ *must* A.
Another related principle is this: $\Vdash$ *must* A $\vee$ *must* $\neg$A $\vee$ (*may* A $\wedge$ *may* $\neg$A).

3.2. *must* and *may*

The weakened version of *Reductio ad Absurdum* mentioned above does not hold if the requirement that each C in Δ be T-stable is dropped.

Illustration. Let A be an atomic sentence. Then we find *may* A, $\neg A \Vdash B \wedge \neg B$ whereas on the other hand *may* A $\not\Vdash$ *must* A.

What is important here is no so much the invalidity of *may* A / *must* A as the validity of *may* A, $\neg$A / B $\wedge \neg$B. Actually, any conclusion can be drawn

from the premises *may* A and $\neg$A as long as $\neg$A is F-stable. In other words, according to this theory the sentence

(a) *The blue marble may be in box 1 and it isn't*

is just as contradictory as

(b) *The blue marble is in box 1 and it isn't*

This example shows that the present theory of 'may' differs widely from those developed within the framework of possible worlds semantics. According to the latter a sentence like (a) can be perfectly true although no one can ever sincerely mean it: (a) is held to be a pragmatic absurdity and not a logical contradiction.

Is there any evidence in favour of this claim, that sentences like (a) are pragmatically rather than logically absurd? I am afraid not. The only empirical support which it could conceivably get would consist in an informal example which shows that the apparent inconsistency of sentences of the form (*may* A $\wedge \neg$A) can sometimes be cancelled. I am pretty sure, however, that no such example will ever be found. Anyone asserting a sentence like (a) fails to fulfill the conversational *maxim of quality* as for example Groenendijk & Stokhof (1975) are ready to explain. (Roughly: by asserting the left hand conjunct 'the blue marble *may be* in box 1', the speaker indicates that the sentence 'the blue marble *is* in box 1' is consistent with everything he believes. But according to the maxim of quality he is not allowed to assert the right hand conjunct if he does not believe that the blue marble *is not* in box 1.) So if there is any example showing that the apparent inconsistency of these sentences can really be cancelled, it must be one in which the speaker indicates (either explicitly or implicitly, but at least in a way clear enough to the hearer) (i) that he is stating something he does not himself believe, but (ii) that he is doing so for some good reason - i.e. one which can be reconciled with the overall Cooperative Principle. I am afraid that no hearer will ever be found who is able to detect what good reason that might be.

That it is impossible to flout the maxim of quality and yet observe the overall Cooperative Principle has been noticed before. For example, Gazdar (1975) notices that an implicature arising from the maxim of quality 'differs from those arising from the other maxims because it cannot be intelligibly cancelled'.[10)] Yet the only conclusion which is usually drawn is that the maxim of quality has a privileged position among the other maxims. Everybody seems to accept, if reluctantly, that the criterion of cancellability offers at best a sufficient condition for calling something pragmatic instead of logical.

The one argument I have to offer in favour of the position that sentences of the form (may A $\wedge \neg$A) are logically rather pragmatically absurd is highly *theoretical*, if not metatheoretical. Consider the following (re)formulation of the maxim of quality: *Do not assert a sentence* A *unless* A *is true on the basis of the evidence at your disposal.* Notice that every argument which owes its pragmatic correctness *exclusively* to this maxim is also logically valid in the sense discussed here. Likewise, any sentence which owes its pragmatic absurdity simply and solely to the fact that it cannot be asserted without violating this maxim is a data semantical contradiction. Hence the question of cancellability need not arise. By doing data semantics instead of the usual truth-conditional semantics, we have so to speak annexed part of what was always called pragmatics. As a consequence the border between logical and pragmatic-but-not-logical validity and that between logical and pragmatic-but-not-logical inconsistency has been redrawn. Actually it seems that now cancellability *can* serve as a condition which an argument *must* satisfy in order to be classified as pragmatically but not logically valid - an almost natural border it would seem.

Let us return to the logic of *may* and *must*. We have already discussed the fact that on many occasions *must* A is weaker than A: if A is T-stable, then A $\Vdash$ *must* A. Notice that for T-unstable A this is often just the other way around: *may* A $\nVdash$ *must may* A; instead, *must may* A $\Vdash$ *may* A.

Except for the above, *may* and *must* behave almost like ordinary modal operators. So we find:

- If $\Vdash$ A, then $\Vdash$ *must* A
- *may* A is equivalent to $\neg$*must* $\neg$A
- $\Vdash$ *must* (A $\to$ B) $\to$ (*must* A $\to$ *must* B), provided that B is F-stable.
- $\Vdash$ *must* A $\to$ *may* A
- $\Vdash$ *must* A $\to$ *must must* A
- $\nVdash$ *must* A $\to$ A

So, if we did not already know better, it would seem that *must* and *may* behave like the obligation and permission operators of some system of *deontic* logic.

3.3. *if*

One of the most striking features of data logic is that the *Principle of Substitution* does not generally hold. It may very well be that a given

argument is valid while uniform substitution of a complex sentence for an atomic sentence occurring in the argument concerned turns it in an invalid argument. In general only uniform substitution of a *stable* sentence for an atomic sentence will transform a valid argument into a valid one.

Example. Let A be atomic. It is easy to check that A $\Vdash$ B $\to$ A. But if we substitute *may* A for A we get an invalid argument: *may* A $\nVdash$ B $\to$ *may* A.

Compare

(a) *The blue marble is in box 1*

∴ *If the blue marble is in box 2, it is in box 1*

and

(b) *The blue marble may be in box 1*

∴ *If the blue marble is in box 2, it may be in box 2*

Perhaps the reader finds it difficult to accept the validity of (a). If so, he is invited to read the conclusion once more without losing sight of the premise. The conclusion does not say that the blue marble *would have been* in box 2 if it *had been* in box 2. Or perhaps it helps to compare (a) with (c):

(c) *The blue marble is in box 1*

∴ *The blue marble is in box 1, if it is in any box at all*

If this does not help either, the reader is referred to section 4.2 where I shall argue in more detail that (a) is pragmatically incorrect rather than logically absurd.

From the above, it will be clear that the logic of $\to$ is - at least in some respects - rather strong. Actually, in many respects $\to$ behaves like *intuitionistic* implication.

Proposition. Suppose that $\wedge$, $\vee$ and $\to$ are the only operators occurring in the sentences of the argument Δ/A. Then, $\Delta \Vdash$ A iff Δ/A is intuitionistically valid.

That we cannot replace 'intuitionistically' by 'classically' here, may be clear from the fact that

$(A \wedge B) \to C \nVdash (A \to C) \vee (B \to C)$

Informally:

If both the blue and the yellow marble are in box 1, the red one is in box 2

∴ *Either if the blue marble is in box 1 the red one is in box 2, or, if the yellow marble is in box 1 the red one is in box 2*

Or, since the reader will by now have lost all his intuitions about the marbles, perhaps the next example taken from Adams (1975) is more convincing:

If switches A and B are thrown the motor will start

$\therefore$ *Either if switch A is thrown the motor will start, or, if switch B is thrown the motor will start*

The above proposition does not hold if we permit other connectives to occur in the sentences of an argument. In some respects the resulting logic is much stronger than intuitionistic logic. For example, every sentence of the form $\neg\neg A \rightarrow A$ is valid in the sense of definition 4, but a sentence of that form is not in general intuitionistically valid. In other respects, however, the resulting logic is much weaker than intuitionistic logic. For one thing, the deduction theorem fails. It is not generally so that

if $\Delta, A \Vdash B$ then $\Delta \Vdash A \rightarrow B$

We only have

if $\Delta, A \Vdash B$ and each $C \in \Delta$ is T-stable then $\Delta \Vdash A \rightarrow B$.

In other words, if you want to prove $A \rightarrow B$ from the premises Δ, you cannot just add A to the premises and try to prove B from Δ together with A. By making an assumption A - suppose that A will turn out true - you may rule out some of the possibilities explicitly left open by the premises - like *may* $\neg A$ for example. A could interfere with the T-unstable sentences in Δ. So if you want to prove B from the assumption A, you may only use the T-stable premises. (We found that *may* $A, \neg A \Vdash (B \wedge \neg B)$. This does not mean, however, that *may* $A \Vdash \neg A \rightarrow (B \wedge \neg B)$.)

In one important respect the behaviour of $\rightarrow$ matches with the behaviour of the *strict implication* occurring in the *Lewis Systems*.[11]
As we saw in section 2.3, $\neg(A \rightarrow B) \Vdash$ *may* $(A \wedge \neg B)$ and *may* $(A \wedge \neg B) \Vdash \neg(A \rightarrow B)$, which is exactly what one would find if $\rightarrow$ were the implication and *may* the possibility operator of another extension of S 0.5.

Yet, also this resemblance is misleading. In some respects the logic of $\rightarrow$ is weaker than any logic for 'if...then' proposed so far. The principle of *Modus Tollens*, which holds both in intuitionistic logic and in the systems of strict implication (and also in the system of variable strict implication proposed by Stalnaker, even in such a weak system as the system of *Relevance Logic*), fails here. It is not generally so that $A \rightarrow B, \neg B \Vdash \neg A$. The closest approximation available is this: if B is F-stable then $A \rightarrow B, \neg B \Vdash$ *must* $\neg A$. If B is not F-stable, even this weakened version of *Modus Tollens* fails. Consider for example the premises $A \rightarrow (B \rightarrow C)$ and $\neg(B \rightarrow C)$, where A, B and C are three distinct atomic sentences. Neither $\neg A$ nor *must* $\neg A$ follow from these principles, we only have that $A \rightarrow (B \rightarrow C), \neg(B \rightarrow C) \Vdash$ *may* $\neg A$.

Example. Suppose you are in information state 1. Then for all you know it may very well be that neither the yellow nor the blue marble is in box 2. So *it is not the case that if the yellow marble isn't in box 2, the blue one is*. However, if the red marble happens to be in box 1 things are different. Indeed *if the red marble is in box 1, then if the yellow marble isn't in box 2 the blue one is*. Now by an application of *Modus Tollens*, it would follow from the italicized sentences that the red marble isn't in box 1, but clearly it may very well be there.

By now it will be clear that the logical system divised here cannot easily be fitted into the spectrum formed by the other theories which have been devised for indicative conditionals. In certain respects it is weaker than the weakest system proposed so far - *Modus Tollens* is not always valid. In other respects it is at least as strong as any of the others: happily enough we have $A \vee B \Vdash \neg A \rightarrow B$, at least for T-stable A. In yet other respects it lies somewhere inbetween: we find that $\neg(A \rightarrow B)$ is equivalent to *may* $(A \wedge \neg B)$. So I am expecting criticism from all corners at once. I flatter myself with the idea that the arguments which I have dubbed invalid cannot without any very good reasons be explained away as 'just' pragmatically unsound. Notoriously difficult (for those who, as Grice does, believe that indicative conditionals behave like material implications) are for example the schemes $\neg(A \rightarrow B)$ / A and $(A \wedge B) \rightarrow C$ / $(A \rightarrow B) \vee (A \rightarrow C)$. So far no one has given a good pragmatic explanation for what is wrong with either of these.[12]

On the other hand, those who think that my theory is too strong, that I have dubbed too many of the wrong arguments valid, can produce a lot of counterexamples to make their point. Here I am the one who has to produce the good reasons for saying that these are pragmatically unsound instances of valid argument forms. I shall turn to this in 4.2.

§ 4 *Pragmatic Correctness and Incorrectness*

In a previous section (2.3) I raised the possibility that definition 2 does not meet one requirement which might reasonably be placed on a truth definition for conditionals, namely that they be true only if the antecedent is relevant to the consequent. There I stated that those contexts in which definition 2 makes a conditional sentence true without the one being relevant to the other are contexts in which so much is known about the truth and falsity of the antecedent and consequent that it cannot properly be asserted. Pragmatic constraints would result in the antecedent of a true conditional being relevant to the consequent. I did not yet say which pragmatic constraints they would be.

You will recall that the scheme *must* B ∨ *must* ¬B ∨ (*may* B ∧ *may* ¬B) is logically valid. This means that the possible contexts in which a conditional with antecedent A and consequent C could be uttered all fall into the following nine categories.

1. *must* A *must* C	2. *must* A *may* C *may* ¬C	3. *must* A *must* ¬C
4. *may* A *may* ¬A *must* C	5. *may* A *may* ¬A *may* C *may* ¬C	6. *may* A *may* ¬A *must* ¬C
7. *must* ¬A *must* C	8. *must* ¬A *may* C *may* ¬C	9. *must* ¬A *must* ¬C

table 1

(Read this as follows: in category 1 *must* A is true on the basis of the evidence available to the speaker and *must* C too. Etc.)

The middle category, 5, was the one in which definition 2 has the effect that a conditional cannot be true without the antecedent being relevant to the consequent. I believe that the contexts in category 5 are the only ones in which the Maxims of Quality and Quantity can both be observed.[13)] In any case as far as conditionals with descriptive antecedent and consequent are concerned.

For the contexts fitting into 2, 3 or 6 this should be fairly obvious. In these cases A → C is simply false on the basis of the evidence available to the speaker. So anyone who says A → C in one of these contexts could be lying, in violation of the Maxim of Quality (the one formulated in section 3.2).

Anyone who knows - or at least could have known - that *must* ¬A and who therefore 'falls within' one of the categories 7, 8 or 9 could truthfully say that A → C. But if he did so he would be sinning against the Maxim of Quantity. *Must* ¬A is stronger than A → C. So if he said

must ¬A (or perhaps even ¬A) then he would be being more helpful.

The only remaining categories are 1 and 2, in both of which the speaker knows *must* C. In those cases in which he knows just a bit more than this, namely not only *must* C but also C itself, his statement A → C is once again pragmatically unacceptable according to the Maxim of Quantity. He is not telling us all he knows.

I am not quite sure as yet how the remaining cases, where the speaker knows that *must* C but no more than this, can best be explained. Given definition 2 *must* C is stronger than A → *must* C but it is not stronger than A → C. For this reason the maxims only tell us that *must* C is, pragmatically speaking, more correct than A → *must* C. We would have to blur the distinction between *must* C and C in order to conclude that A → C is also incorrect.

The problem here is closely tied up with the distinction between direct and indirect evidence. In my (forthcoming) it will receive closer attention.

4.1. *Odd Conditionals*

Hopefully the above has been enough to establish that the proper place for a conditional is in the middle of table 1, in category 5. This is of course not to say that A → C should never be said in other circumstances only because this violates one or the other conversational maxim. There are plenty of good occasions for doing just this, only it must be obvious that a maxim has been overruled and why.

Contexts fitting into 2, 3 and 6 are not among these occasions. There the conditional is false on the basis of the evidence available to the speaker and as we noticed in section 3.2 there is something selfdefeating about saying untrue things if the hearer knows that the Maxim of Quality is being violated.

The literature is full of if's and then's with the most excentric things inbetween and all those I know fit quite neatly in that part of table 1 formed by the categories 1, 4, 7, 8, and 9.

All of the examples which go '*if* A *then I'll eat my hat*' fit into category 9: one is clearly not intending to eat ones hat and the hearer is expected to complete the (weakened version of) Modus Tollens for himself. (Why say 'If A I'll eat my hat' instead of 'it cannot be that A'? Surely in order to make the claim that A is as definitely false as the applied Modus

Tollens is valid.)

There are also plenty of examples of which the antecedent is trivially true and the hearer is supposed to apply Modus Ponens:

She is on the wrong side of thirty if she is a day

or

If there is one thing I cannot stand it is getting caught in the rushhour traffic

Since these examples pragmatically imply the truth of both the antecedent and the consequent they belong to category 1.

Another example - one which could under some circumstances be interpreted as a normal conditional (category 5) - is:

There is coffee in the pot, if you want some

But they would be very special circumstances, perhaps in a fairy story (if you'd wanted it, then there would have been distilled sunlight in the pot instead). Conditionals like this one fall more naturally into category 4, along with things like

This is the best book of the month, if not the year

The consequent of each of these two conditionals is a conversational implicature of the whole sentence. In the first example the hearer knows that the speaker is not in a position to know whether the antecedent is true or not. Hence, since this is not a fairy-tale world, the whole sentence would be false if its consequent is undecided. The hearer supposes the speaker to observe the Maxim of Quality and is thus able to conclude that its consequent must be true: there is coffee in the pot.

To what good purpose if any does the speaker prefer the 'if' form to the statement that *there is coffee in the pot*? I think that the speaker in simply asserting this would run the risk of defying the Maxim of Relevance, by saying something which does not interest the hearer at all. With the antecedent the speaker indicates that he is well aware of this: it provides a condition under which the consequent will be interesting. (The antecedent introduces as it were a new topic, c.f. *If I may change the subject...*)

The second example works differently. The speaker supposes that the hearer is well aware of the trivial truth that the best book of the year is certainly the best book of the month. In formulas, the hearer is supposed to know that $\neg A \rightarrow C$. From this together with what the speaker tells him, $A \rightarrow C$, he could conclude *must* C: This must be the best book of the month. Just as in some of the other examples the speaker intends the hearer to draw this conclusion. He uses a conditional for purely rethorical reasons.

Yet another conditional which breaches the conversational maxims to good effect is

If it does not rain tomorrow then it is going to pour

(for example as a summary of a dismal weather forecast).

This would seem to convey that whether it pours or not, in any case it must be going to rain. So this sort of conditional would be asserted in category 8, where the antecedent must be false and the consequent is undecided. The reader will be able to work out these implicatures himself. (This case mirrors the last example.)

I have not been able to find any good (idiomatic) conditionals fitting into category 7.[14] Two possible examples are:

If I do not beat him then I'll thrash him

(a boxer boasting before his fight)

and (this time with an ironic tone)

If as you say John died last year then he looked pretty good to me when I ran into him yesterday

But it could well be that the first fits more naturally into 8 and the second into category 4. In any case I have not been able to prove any more than that the first sentence pragmatically implies the falsity of its antecedent and that the second one implies the truth of its consequent. Perhaps it is not surprising that there are no ready examples of conditionals which clearly belong in 7. Saying $A \rightarrow B$ and meaning by this both the more informative *must* $\neg A$ and the more informative *must* B involves violating the Maxim of Quantity not once but two times. It could be asking too much of a hearer to expect him to work out that he is being had twice.

4.2. *A Test for Pragmatic Incorrectness.*

Consider the following well known example:

If there is sugar in the coffee then it will taste good

∴ *If there is sugar in the coffee and diesel-oil as well then it will taste good*

There are a lot of theories of conditionals in which the logical validity of *Strengthening the Antecedent* is rejected on the grounds of this putative counter instance. I am more inclined to see it as a pragmatically unsound instance of a valid argument form. The reason for this is the following. I see an argument as something which is produced in order to establish the truth of its conclusion. You persuade someone that the conclusion is true by showing that it logically follows from some other things which he is willing to accept, the premises of your argument.

But if you want to convince anyone that a mixture of sugar and diesel-oil tastes good in coffee then you are not going to succeed with the above argument. It is a pragmatic implicature of the conclusion that the coffee which you are talking about may well contain diesel oil (along with the other things you usually take in your coffee). If this possibility is open then no one would be prepared to accept that the premise is true. Sugared coffee which may contain diesel oil as well does not in general taste good. For this reason presenting the above argument will do nothing to convince anyone.

In this paper there have been various arguments which I claimed to be pragmatically unsound, and for which a similar argument delivers the right results. (1) on page one is an example. There it is the first premise *If Jones wins the election then Smith will retire* which is not acceptable if you remember that the conclusion is such that you must reckon with the possibility of *Smith's decease*. (a) on page 16 is another example. If you add *the blue marble may not be in box 1* to the premise *it is in box 1* you get a contradiction. (c) on page 16, which is of the same form as (a), is without fault (though pointless). For its conclusion does not conversationally imply that the blue marble is in box 1. The speaker has explicitly indicated that he is breaching the Maxim of Quantity (its conclusion belongs to category 1).[15]

I would suggest generalizing these examples as a test for the pragmatic incorrectness of an argument: *Any argument of which the premises cannot hold if you take the conversational implicatures of the conclusion into account is pragmatically unsound.*
Any 'clearcut' counterexample to a 'putative' logical principle should at least pass this test if it is to be taken seriously.

Notes

*) I want to thank Fred Landman, Michael Morreau and Marjorie Pigge for their invaluable help with the preparation of this paper. This paper overlaps in some passages with my (1981). Therefore I once again thank all those who helped with that paper.

1) See Grice (1975).
2) This example is drawn from Adams (1975).
3) This strategy is followed in Stalnaker (1976).
4) See in particular Anderson & Belnap (1975).
5) See Adams (1975) and also Cooper (1978).
6) The information models defined here closely resemble the Kripke models for intuitionistic logic. See Kripke (1965).

7) I have made this stronger assumption just for technical convenience. As far as the logic is concerned, it does not make any difference which one you make. Actually, from a logical point of view, you might even make the still weaker assumption that for each $s \in S$ and atomic sentence A there is an $s' \geqslant s$ such that $V_{s'}(A)$ is defined.
8) By 'equivalent' I mean that the sentences in question have the same truth conditions, but not necessarily the same falsity conditions.
9) This does not mean that they cannot be criticized. For a defense of the clauses given for disjunction, see my (1981).
10) See Gazdar (1979), p. 46.
11) I mean the C.I. Lewis systems here. See Hughes & Cresswell (1972).
12) Compare the discussion in Gazdar (1979), pp. 83-87.
13) Gazdar (1979) arrives at the same conclusion albeit in a different way. See also Stalnaker (1976) for remarks in the same direction.
14) Not even in Lauerbach (1979) from which many of the above examples were taken.
15) See Chapter 8 of Cooper (1978) for a load of other examples.

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