Partitioning Logical Space

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Preface

In the present version of these lecture notes only a number of typos and a few glaring mistakes have been corrected. Thanks to Paul Dekker for his help in this respect.

No attempt has been made to update the original text or to incorporate new insights and approaches. For a more recent overview, see our 'Questions' in the *Handbook of Logic and Language* (edited by Johan van Benthem and Alice ter Meulen, Elsevier, 1997).

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Chapter 1

Interrogatives

In Section 1.1 of this chapter we first formulate general constraints on a semantics of interrogatives. In Section 1.2 we present the basic features of a theory that meets these constraints. Later chapters will elaborate on this.

1.1 Requirements on a semantics for interrogatives

1.1.1 Belnap's Requirements

In this section we formulate three general methodological constraints on a theory of questions and answers which are taken from Belnap 1981. Belnap uses them as a means to classify and evaluate different theories.

- **Independent meaning requirement** Interrogatives are entitled to a meaning of their own.
- Equivalence requirement Interrogatives and their embedded forms are to be treated on a par.
- **Answerhood requirement** The meaning of an interrogative resides in its answerhood conditions.

The independent meaning requirement goes against theories which analyze interrogatives in terms of indicative paraphrases, e.g., as explicit performative sentences, as in (1b), or as epistemic imperatives, as in (1c).

- (1) a. Does Mary walk in the garden?
 - b. I (hereby) ask you whether Mary walks in the garden
 - c. Bring it about that I know whether Mary walks in the garden

(An example of an explicit performative analysis of interrogatives is Lewis 1972. An imperative epistemic approach is presented, e.g., in Hintikka 1976.) The problem with such paraphrases is that they usually contain the embedded form of the interrogative that they intend to analyze. This is particularly problematic if we combine the independent meaning requirement with the equivalence requirement.

The latter takes it for granted that, since interrogatives ('direct questions') and embedded interrogatives ('indirect questions') come in pairs, they should be treated as being semantically intimately related, much in the same way as indicative sentences and their embedded forms are treated as being directly related to each other.

Interrogatives

The strongest way to meet the equivalence requirement is to treat interrogatives and their embedded forms as being identical in meaning. In this strong form we can use observations concerning semantic relations (such as entailment) between indicative sentences which contain embedded interrogatives, to determine the semantic properties of interrogatives. This may be helpful, since indicatives are a a more familiar domain of investigation.

One can take the independent meaning requirement to be so strong as to require that interrogatives are assigned a meaning different from that of indicatives. The meaning of an indicative is given by its truth conditions. Interrogatives are not true or false, hence their meanings cannot be identified with truth conditions. At the same time, there should be similarities between the meanings assigned to both kinds of sentences. They can be coordinated, and can be related to each other as question-answer pairs.

The answerhood requirement asks (at least) that a semantics of interrogatives enable us to account for the question-answer relation. As it is stated above, it is even stronger. If we treat the notions 'answerhood conditions' and 'truth conditions' on a par, the following picture emerges. In possible world semantics, the notion of a proposition captures that of truth conditions. A proposition is a function from possible worlds (more neutrally *indices*) to truth values. Similarly, we can associate answerhood conditions with semantic objects called questions, which would then be functions from indices to answers. If we take the latter to be of a propositional nature, a question is a function from indices to propositional objects, viz. the function which for every index tells us what the true answer(s) to that question is/are at that index.

It seems that something can be said against this. Viewed this way the semantics of interrogatives presupposes the existence of a notion of 'standard semantic answerhood'. It seems that this does not do justice to the observation that questions can be answered in many different ways, and that what is the best way to answer a question may depend on the information already available to the one who asks the question. So, we want to interpret the answerhood requirement in such a way that the notion of a standard semantic answer that a semantic theory of interrogatives characterizes, should form a suitable basis for a general semantic and pragmatic theory of answerhood, which takes into account that a question serves to indicate a gap in one's information, and that a suitable answer should contribute to closing this gap.

1.1.2 Entailment and Coordination

The answerhood requirement already provides us with an informal notion of the kind of semantic objects that is to be associated with interrogatives: the meaning of an interrogative is a question, a function which for every index tells us what the true answer at that index is. At the same time this suggests a notion of entailment between questions:

Definition 1 A question Q entails a question R iff for every index i it holds that, if a proposition P gives a true answer to Q at i, it gives a true answer to R at i.

A prime example of a questions which entails another is the following.

- (2) a. Who walk(s) in the garden?
 - b. Does Mary walk in the garden?

Other examples come from coordinated interrogatives. In general we have that a conjunction entails each of its conjuncts, and that a disjunction is entailed by each of its disjuncts. Consider the following examples:

- (3) a. Where are your father and your mother?
- b. Where is your father? And where is your mother?
- (4) a. Where is your father or your mother?
 - b. Where is your father? Or, where is your mother?

Example (3a) has a reading on which it can be paraphrased as (3b), which is a conjunction of two interrogatives. Similarly, (4a) has a reading which can be paraphrased as (4b), which is a disjunction of two interrogatives. And, indeed, according to our definition, the conjunction of two interrogatives entails each of its conjuncts, since to answer (3b) is to answer its two conjuncts. And, similarly, the disjunction (4b) is entailed by each of its disjuncts. To answer (4b), it suffices to answer one of its disjuncts.

We will deal with these issues in greater detail in the chapters to follow. But we want to remark now that these observations give rise to the following additional requirement:

Entailment requirement A semantics of interrogatives should account for entailment relations between (coordinated) interrogatives. And it should do so on the basis of the general notions of entailment and coordination that one's semantic framework provides.

The semantic framework we use here is that of possible worlds semantics. The logical language we use is that of two-sorted typetheory. Two-sorted typetheory is like ordinary type theory, but instead of having types e and t as basic types, it also has a third (sic) one, in our case, s. The domain corresponding to type s is the set of indices (possible worlds). Unlike the more usual language of intensional typetheory, the language of two-sorted type theory contains variables and constants of type s (among others). (We use i and j as variables of type s.) And, hence, it allows for explicit λ -abstraction and quantification over indices, thus making the modal operators and the cup- and cap-operator from intensional type theory superfluous. The interpretation function assigns extensions the non-logical constants. The indexdependent nature of most constants is taken care of by providing them with an extra argument place of type s. For example, the sentence Bill walks is translated as walk(i)(b), where walk is a constant of type $\langle s, \langle e, t \rangle \rangle$, i a variable of type s, and b is a constant of type e. Notice that in this way the proper name Bill is treated as a rigid designator, an index-independent expression, since it is not provided with an argument place of type s.

Before we can give the generalized notions of entailment and coordination of the framework, we first define an auxiliary notion:

Definition 2 (Relational types) The set of relational types is the smallest set such that:

- 1. t is a relational type
- 2. if b is a relational type, then $\langle a, b \rangle$ is a relational type

The relational types are the types that 'end in t'. Next we define notions of generalized conjunction, disjunction and entailment restricted to expressions of relational types.

Definition 3 (Generalized conjunction)

- 1. If α and β are of type t, then $\alpha \sqcap \beta = \alpha \land \beta$
- 2. If α and β are of relational type $\langle a, b \rangle$, then $\alpha \sqcap \beta = \lambda x_a[\alpha(x) \sqcap \beta(x)]$

Definition 4 (Generalized disjunction)

- 1. If α and β are of type t, then $\alpha \sqcup \beta = \alpha \lor \beta$
- 2. If α and β are of relational type $\langle a, b \rangle$, then $\alpha \sqcup \beta = \lambda x_a[\alpha(x) \sqcup \beta(x)]$

Definition 5 (Generalized entailment) If α and β are of the same relational type, then α entails β iff $\models \alpha \sqsubseteq \beta$, where \sqsubseteq is defined as follows:

- 1. If α and β are of type t, then $\alpha \sqsubseteq \beta = \alpha \rightarrow \beta$
- 2. If α and β are of relational type $\langle a, b \rangle$, then $\alpha \sqsubseteq \beta = \forall x_a[\alpha(x) \sqsubseteq \beta(x)]$

The entailment requirement forces upon us that the semantic objects that the theory assigns to interrogatives are such that by using the notions of coordination and entailment defined above, we can give a correct account of entailment relations such as the ones exemplified in (2)-(4).

1.1.3 The Categorial and the Propositional Approach

In this section we discuss the following two approaches to the semantics of interrogatives:

- The categorial approach The syntactic category and semantic type of an interrogative are determined by the category and type of its characteristic constituent answers.
- **The propositional approach** The semantic interpretation of an interrogative has to give its answerhood conditions, i.e., it should determine which propositions count as its semantic answers.

Examples of categorial approaches can be found in Hausser 1983; Hausser and Zaefferer 1978; Tichy 1978; Scha 1983, and of propositional approaches in Hamblin 1973; Karttunen 1977; Bennett 1979; Belnap 1982.

It will be clear from the requirements set out in Section 1.1.1, that we favor the propositional approach. However, we will indicate that we think that the basic insights of the categorial approach should be incorporated in an overall theory as well.

Categorial theories have in common that they prefer 'short' linguistic answers to, which we will refer to as *constituent answers*, to 'long' sentential answers. Constitutent answers are of different syntactic categories and semantic types in case they answer different kinds of interrogatives:

- (5) Whom did John kiss?
- (6) What happened in the kitchen last night?
- (7) Mary.
- (8) John kissed Mary.

The constituent answer (7) can be used to answer (5), but not (6). (8) on the other hand can be used to answer both (5) and (6). Constituent answers are closely tied to certain types of interrogatives, whereas the tie between sentential answers and different kinds of interrogatives seems much looser.

Categorial theories focus on the relation between interrogatives and constituent answers. The existence of a categorial match between interrogatives and their characteristic constituent answers is taken to determine their category. The categorial definition of interrogatives is chosen in such a way that in combination with the category of its constituent answers, the category of indicative sentences results.

As a result, different kinds of interrogatives are of distinct categories and semantic types. One of the consequences of this lack of a uniform interpretation of interrogatives is that entailment relations between interrogatives of different categories cannot be accounted for by means of the generalized notion of entailment defined above, since the latter requires such interrogatives to be of the same type. (A case in point is example (2) given above.) More generally, in focussing on the linguistic answerhood relation, constituted by categorial fit, the more interesting semantic notion of answerhood as a relation between propositions and questions is hardly touched upon.

Still, it can be argued that the relation between constituent answers and interrogatives not only should be accounted for in an overall theory, but even that any semantic theory should give it a prominent place in the analysis of interrogatives. Consider the following two interrogatives:

- (9) Who of John, Bill, and Mary will go to the party?
- (10) Who of John, Bill, and Mary will not go to the party?
- (11) John and Mary.

The two interrogatives (9) and (10) are equivalent in the sense that they have the same semantic answerhood conditions: each proposition which completely settles the one question will settle the other. However, in the context of (9), the constituent answer (11) expresses a different proposition than it does in the context of (10). In case of (9) it expresses the proposition that John and Mary are the ones that will attend the party, whereas in case of (10) it expresses that John and Mary are the ones that will not go to the party.

What the constituent answer (11) means depends on the context of the interrogative. But how can an analysis within the propositional approach account for this? Since the two interrogatives are equivalent, they have the same answerhood conditions, how can they provide a different context for the answer (11).

A categorial approach fares better in this respect. Both interrogatives require a term as constituent answer. A term and a property forms a sentence. So according to the categorial approach the two interrogatives express a property. And these properties, indeed, are different. In case of (9) it is the property of being John, Mary or Bill and going to the party. In case of (10) it is the property of being John, Mary or Bill, and *not* going to the party. And if we combine the answer (11) with these properties, a different proposition is expressed in each case.

On the other hand, since the categorial approach will identify the meaning of the two interrogatives with these two different properties, how is it to account for the fact that the two interrogatives are equivalent in the sense that they have the same answerhood conditions? This fact is accounted for in a propositional approach, which identifies the meaning of interrogatives with their answerhood conditions.

So, it seems that one has to be eclectic, one needs to combine both kinds of approaches to arrive at an overall theory.

1.2 A Semantics for Interrogatives

1.2.1 Index Dependency

First we consider sentential interrogatives, which express yes/no-questions, and their corresponding embedded forms, which under the strict version of the equivalence requirement denote the same semantic object. There is an intimate relation between embedded indicatives and embedded sentential interrogatives:

- (12) a. John knows whether Mary walks in the garden
 - b. Mary doesn't walk in the garden
 - c. John knows that Mary doesn't walk in the garden

Together, (12a) and (12b) entail (12c).

Embedded indicative sentences and embedded sentential interrogatives can be coordinated, as shown in (13):

(13) John knows that Peter left for Paris, and whether Mary went with him

The simplest assumption to account for the entailment in (12) is to assume that embedded interrogatives denote the same kind of semantic object as embedded indicatives, i.e., a *proposition*.

The denotation of an embedded indicative is *index independent*: at every index it denotes the same proposition. If we compare (12) with (14), we see that what proposition an embedded interrogative denotes depends on the actual facts.

- (14) a. John knows whether Mary walks in the garden
 - b. Mary walks in the garden
 - c. John knows that Mary walks in the garden

So, the denotation of embedded interrogatives is index dependent:

(a) whether ϕ denotes the proposition $\begin{cases} \text{that } \phi, & \text{if } \phi \text{ is true} \\ \text{that not } \phi, & \text{if } \phi \text{ is false} \end{cases}$

According to the equivalence requirement the same holds for non-embedded sentential interrogatives:

(b) ϕ ? denotes the proposition $\begin{cases} \text{that } \phi, & \text{if } \phi \text{ is true} \\ \text{that not-}\phi, & \text{if } \phi \text{ is false} \end{cases}$

This squares with the answerhood requirement. It means that the proposition denoted by an interrogative at a certain index i is its true answer at i. The sense of an interrogative is then a propositional concept, a function from indices to propositions.

	extension	intension	meaning
indicative	truthvalue	proposition	truth conditions
interrogative	proposition	propositional concept	answerhood conditions

Table 1.1: Interpretation of indicatives and interrogatives

In set-theoretical terms, the interpretation of a sentential interrogative can be defined as follows:

(c)
$$\llbracket \phi ? \rrbracket_i = \begin{cases} \{j \mid \llbracket \phi \rrbracket_j = 1\} & \text{if } \llbracket \phi \rrbracket_i = 1\\ \{j \mid \llbracket \phi \rrbracket_j = 0\} & \text{if } \llbracket \phi \rrbracket_i = 0 \end{cases}$$

Or, equivalently:

(d) $[\![\phi?]\!]_i = \{j \mid [\![\phi]\!]_i = [\![\phi]\!]_j\}$

This means that the interrogative ϕ ?, and the embedded interrogative whether ϕ translate in two-sorted type theory as follows:

(15) $\lambda j[\phi(i) = \phi(j)]$

A more concrete example *Does Mary walk?* and *whether Mary walks* both translate into:

(16) $\lambda j[\texttt{walk}(i)(\texttt{m}) = \texttt{walk}(j)(\texttt{m})]$

The intension of this expression is:

(17) $\lambda i \lambda j [walk(i)(m) = walk(j)(m)]$

It denotes a propositional concept, a function from indices to propositions, or equivalently, a relation between indices. The relation holds between two indices i and j if the denotation of *Mary walks* is the same at i and j:

(e) $\llbracket \phi ? \rrbracket = \{ \langle i, j \rangle \mid \llbracket \phi \rrbracket_i = \llbracket \phi \rrbracket_j \}$

Such relations are equivalence relations on the set of indices I, and correspond to bi-partitions of I. (See Chapter 2).

1.2.2 Extensional versus Intensional Interrogative Embedding Verbs

The validity of the entailments exemplified in (12) and (14) do not depend on the factivity of know. They also hold for a non-factive verb like *tell*:

- (18) a. John tells whether Mary walks in the garden
 - b. Mary walks in the garden
 - c. John tells that Mary walks in the garden
- (19) a. John tells whether Mary walks in the garden
 - b. Mary doesn't walk in the garden
 - c. John tells that Mary doesn't walk in the garden

One difference between factive and non-factive interrogative embedding verbs is illustrated by the difference between (20) and (21):

- (20) a. John knows that Mary walks in the garden
 - b. John knows whether Mary walks in the garden
- (21) a. John tells that Mary walks in the garden
 - b. Mary walks in the garden
 - c. John tells whether Mary walks in the garden

Whereas (20b) follows from (20a) alone, (21c) follows from (21a) only if the additional premises (22b) is added.

Verbs like know and *tell* are extensional in the sense that they take the extension of an embedded interrogative as argument. Verbs like *wonder* and *guess* are *intensional*, they essentially take the *intension* of an embedded interrogative as argument.

- (22) a. John knows whether Mary walks
 - b. $\texttt{know}(i)(\texttt{j}, \lambda j[\texttt{walk}(i)(\texttt{m}) = \texttt{walk}(j)(\texttt{m})])$
- (23) a. John wonders whether Mary walks

b. wonder $(i)(j, \lambda i \lambda j [walk(i)(m) = walk(j)(m)])$

The validity of the entailments exemplified by (12), (14), (18)–(21) can easily be checked. In each case it crucially depends on the extensionality of the verbs involved.

The distinction between extensional and intensional interrogative embedding verbs is a matter of lexical semantics that can be accounted for by means of meaning postulates, or appropriate basic translations. Similarly, the fact that some verbs take only interrogatives as argument, such as *wonder*, that others, like *know*, take both indicative and interrogative arguments, and that there are also verbs which only take indicative arguments, like *believe*, is also to be accounted for by lexical semantics.

1.2.3 Exhaustiveness

Like embedded sentential interrogatives, constituent interrogatives are intimately related to embedded indicatives.

- (24) a. John knows who walk(s) in the garden
 - b. Mary walks in the garden
 - c. John knows that Mary walks in the garden
- (25) a. John knows who walk(s) in the garden
 - b. Mary doesn't walk in the garden
 - c. John knows that Mary doesn't walk in the garden

In these cases too, (c) follows from (a) and (b). (If, that is, we assume proper names to be rigid designators, and *know* to be closed under logical consequence.) And, again, this can most easily be accounted for if constituent interrogatives, like sentential ones, denote propositions. The examples also reflect the index dependency of the denotation of interrogatives: the proposition denoted by *who walks in the garden*?, should entail the proposition that Mary walks in the garden in case Mary actually walks in the garden, and the proposition that she doesn't in case she in fact doesn't. This means that (26a) entails (26b), and that (27a) and (27b) are equivalent (assuming that the 'wh-term' *who* concerns the universe of discourse as a whole, and is not restricted to a certain subset thereof, and assuming that John is informed about what constitutes the universe of discourse).

- (26) a. John knows who walk(s) in the garden
 - b. John knows whether Mary walks in the garden
- (27) a. John knows who walk(s) in the garden
 - b. John knows who doesn't/don't walk in the garden

Given the equivalence requirement, we also have that a proposition which gives a complete answer to (28a), will also give a complete answer to (28b). Which means that the interrogative (28a) entails the interrogative (28b), and that it does so on the basis of the general definition of entailment defined in Section 1.1.2.

- (28) a. Who walk in the garden?
 - b. Does Mary walk in the garden?

So, a simple interrogative like $Who \ walk(s)$ denotes the proposition which gives an exhaustive specification of the actual extension of the property of walking. In set-theoretical terms, its denotation can be represented as:

(f) $\llbracket \text{who walk}(s)? \rrbracket_i = \{j \mid \llbracket \text{walk} \rrbracket_i = \llbracket \text{walk} \rrbracket_j \}$

In two-sorted type theory we get the following representation:

(29) $\lambda j [\lambda x \texttt{walk}(i)(x) = \lambda x \texttt{walk}(j)(x)]$

This is equivalent to:

(30) $\lambda j [\forall x [\texttt{walk}(i)(x) \leftrightarrow \texttt{walk}(j)(x)]]$

This shows that who walks means something like for all x whether x walks. Suppose that Bill and Mary are the ones that walk at index i, then (29) and (30) denote the same proposition as is expressed by:

(31) $\forall x [\texttt{walk}(i)(x) \leftrightarrow [x = \texttt{b} \lor x = \texttt{m}]]$

This illustrates the exhaustiveness of constituent interrogatives.

1.2.4 The De Dicto Nature of Interrogatives

The following series of examples indicate that things get slightly different if we are dealing with a wh-term like *which man* instead of *who*.

- (32) a. John knows which man walks in the garden
 - b. Hilary is a man who walks in the garden
- c. John knows that Hilary is a man who walks in the garden
- (33) a. John knows which man walks in the gardenb. Hilary isn't a man who walks in the garden
- c. John knows that Hilary isn't a man who walks in the garden
- (34) a. John knows which man walks in the garden
 - b. John knows whether Hilary is a man who walks in the garden
- (35) a. John knows which men walk in the garden
 - b. John knows who the men are
 - c. John knows which men do not walk in the garden

Knowing which men walk in the garden involves some de dicto knowledge about who the men are. Of the men who actually walk in the garden, one should know not only that they walk in the garden, but also that they are men. More precisely, one should be able to give an exhaustive specification of the men who walk.

This means that the denotation of the simple one-constituent interrogative which man walks can be represented in set theoretical terms as follows:

(g) $\llbracket \text{which man walk}
bracket
bracket_i = \{j \mid \llbracket \text{man}
bracket_i \cap \llbracket \text{walk}
bracket_i = \llbracket \text{man}
bracket_j \cap \llbracket \text{walk}
bracket_j \}$

In two-sorted type theory this amounts to:

 $(36) \ \lambda j[\lambda x[\max(i)(x) \land \operatorname{walk}(i)(x)] = \lambda x[\max(j)(x) \land \operatorname{walk}(j)(x)]]$

Note that (36) is equivalent to (37), but not to (38):

- $(37) \ \lambda j \forall x [[\mathtt{man}(i)(x) \land \mathtt{walk}(i)(x)] = [\mathtt{man}(j)(x) \land \mathtt{walk}(j)(x)]]$
- $(38) \ \lambda j \forall x [\texttt{man}(i)(x) \to [\texttt{walk}(i)(x) = \texttt{walk}(j)(x)]]$

If we embed these propositions under know, we arrive at (39) and (40a) respectively. The latter is equivalent to (40b). Whereas (39) gives the de dicto reading of (41a), (40a) and (40b) represent its de re reading, which can be paraphrased as (41b). It is only the de dicto reading, i.e., (36)/(37), that corresponds to the interrogative (42).

- (39) $\operatorname{know}(i)(j, \lambda j \forall x[[\operatorname{man}(i)(x) \land \operatorname{walk}(i)(x)] = [\operatorname{man}(j)(x) \land \operatorname{walk}(j)(x)]])$
- $(40) \quad \text{a. } \operatorname{know}(i)(\mathbf{j})(\lambda j \forall x [\operatorname{man}(i)(x) \to [\operatorname{walk}(i)(x) = \operatorname{walk}(j)(x)]])$
- b. $\forall x[\mathtt{man}(i)(x) \to \mathtt{know}(i)(\mathtt{j}, \lambda j[\mathtt{walk}(i)(x) = \mathtt{walk}(j)(x)])]$
- (41) a. Bill knows which man walks
 - b. Of each man, Bill knows whether he walks
- (42) Which man walks?

One-constituent interrogatives denote propositions which give an exhaustive specification of the denotation of a property. Likewise, a two-constituent interrogative such as (43a) denotes a proposition that gives an exhaustive specification of a two-place relation.

- (43) a. Which girl kisses which boy?
 - b. $\lambda j[\lambda x \lambda y[girl(i)(x) \land boy(i)(y) \land kiss(i)(x,y)] = \lambda x \lambda y[girl(j)(x) \land boy(j)(y) \land kiss(j)(x,y)]]$

(43b) denotes the proposition which gives an exhaustive specification of the relation of loving, restricted in its first argument to the girls, and in its second argument to the boys.

1.2.5 Interrogative Formation

Having decided in the previous sections what the semantic interpretation of sentential, one-constituent and multiple constituent interrogatives is, we now turn to the question of how to construct interrogatives with these interpretations. Consider (again) the following four representative examples:

- (44) a. Does Mary walk?
- b. $\lambda j[\texttt{walk}(i)(\texttt{m}) = \texttt{walk}(j)(\texttt{m})]$
- (45) a. Who walks?
- b. $\lambda j[\lambda x \texttt{walk}(i)(x) = \lambda x \texttt{walk}(j)(x)]$
- (46) a. Which man walks?
- b. $\lambda j[\lambda x[\max(i)(x) \land \operatorname{walk}(i)(x)] = \lambda x[\max(j)(x) \land \operatorname{walk}(j)(x)]]$
- (47) a. Which girl kisses which boy?
 - b. $\lambda j [\lambda x \lambda y[girl(i)(x) \land boy(i)(y) \land kiss(i)(x, y)] = \lambda x \lambda y[girl(j)(x) \land boy(j)(y) \land kiss(j)(x, y)]]$

The four representations under (b) exhibit the following general form:

(h) $\lambda j[\lambda x_1 \dots \lambda x_n[\beta(i)(x_1, \dots, x_n)] = \lambda x_1 \dots \lambda x_n[\beta(j)(x_1, \dots, x_n)]], n \ge 0$ Each instance of the Schema (h) is a proposition which exhaustively specifies the denotation of an *n*-place relation. For n = 0, we are dealing with zero-place relations, i.e., propositions, the denotations of sentential interrogatives. For n > 0 we are dealing with the interpretations of constituent interrogatives.

As we have indicated in Section 1.1.3, and will argue for more extensively in Chapter 3, to be able to deal with the interpretation of linguistic answers, we need to cover both the basic insights of the propositional approach, and of the categorial approach. The latter requires that we assign n-place relations as interpretations to n-constituent interrogatives at some level of interpretation.

- This suggests that we form interrogatives in two steps:
- 1. construct n-constituent interrogatives as expressions denoting n-place relations

2. turn the relational interpretation of an interrogative into a propositional one As for step 2, Schema (h) already tells us how to do this.

As for step 1, we refer to *n*-constituent interrogatives interpreted at relational level as *n*-place abstracts. Compositionality requires that *n*-place abstracts be constructed stepwise. We do this in the following (rather orthodox and oldfashioned) way. We start out with zero-place abstracts, sentential structures with free variables. The rule of abstract formation then turns an *n*-place abstract into an n+1-place abstract. Semantically this amounts to λ -abstraction over a free variable in the *n*-place abstract. Thus, semantically, the rule of abstract formation turns an *n*-place relation into an n + 1-place relation. In case of wh-terms of the form which CN, λ -abstraction is restricted to the property denoted by the CN.

This leads to the following rule of abstract formation:

Regel 1 Let β be an *n*-place abstract, with translation β' . Then:

- 1. the n + 1-place abstract obtained from β by introducing the wh-term who translates as $\lambda x \beta'$
- 2. the n+1-place abstract obtained from β by introducing the wh-term which δ translates as $\lambda x [\delta'] \beta'$, where δ' is the translation of δ .

The notion of restricted λ -abstraction used in this rule of abstract formation is defined as follows:

Definition 6

- 1. If $x \in VAR_a, \alpha \in ME_{\langle a,t \rangle}$, and $\beta \in ME_b$, where b is a conjoinable type, then $\lambda x[\alpha]\beta \in ME_{\langle a,b \rangle}$.
- 2. $[\lambda x \lceil \alpha \rceil \beta]_g$ is that function $h \in D_{\langle a,b \rangle}$ such that for all $d \in D_a$: a. $h(d) = [\beta]_{g[x/d]}$ if $[\alpha]_g(d) = 1$; and b. $h(d) = \operatorname{zero}_b$ if $[\alpha]_g(d) = 0$ where $\operatorname{zero}_t = 0$; $\operatorname{zero}_{\langle a,b \rangle}$ is the constant function from D_a to zero_b

The rule which turns abstracts into interrogatives conforms to Schema (h):

Regel 2 If β' is the translation of an abstract β , then the translation of the corresponding interrogative is $\lambda j [\beta' = (\lambda i \ \beta')(j)]$

One example to illustrate these rules. The representation (47b) of the interrogative (47a) is obtained as follows. We start from a sentential structure x kisses y, which translates as kiss(i)(x, y). This expression is a zero-place abstract. If we apply the rule of abstract formation to this zero-place abstract and the wh-term which boy, we arrive at the one-place abstract x kisses which boy, which translates as (48a), and which is equivalent to (48b).

(48) a. $\lambda y \lceil boy(i) \rceil kiss(i)(x, y)$ b. $\lambda y \lceil boy(i)(y) \land kiss(i)(x, y) \rceil$

If we apply the rule of abstract formation to this one-place abstract and the whterm which girl, we arrive at the two-place abstract which girl kisses which boy. Its translation is (49a), which is equivalent to (49b):

(49) a. $\lambda x [girl(i)] \lambda y [boy(i)(y) \land kiss(i)(x, y)]$ b. $\lambda x \lambda y [girl(i)(x) \land boy(i)(y) \land kiss(i)(x, y)]$

By applying the rule for interrogative formation to (49b) we arrive at our semantic representation (47b) for (47a).

Interrogatives

Chapter 2

Questions and Answers

In this chapter we explore some formal aspects of the analysis of questions as partitions of the set of indices. Section 2.1 considers the notion of a partition and discusses some elementary properties. Section 2.2 deals with notions of answerhood. Section 2.3 is concerned with some aspects of the process of question-answering.

2.1 Questions as Partitions

In this section we point out some simple properties of partitions and discuss their relevance for questions.

Definition 7 \mathcal{A} is a partition of A iff

- 1. For all X in \mathcal{A} it holds that $X \neq \emptyset$
- 2. For all X, Y in A it holds that if $X \neq Y$ then $X \cap Y = \emptyset$
- 3. $\cup \mathcal{A} = A$

Questions are viewed as partitions of the set of indices I. The elements of a question are non-empty subsets of I, i.e., propositions. These propositions are the possible (complete semantic) answers of the question. A yes/no-question has two possible answers, hence it makes a bi-partition on I. See Fig. 2.1 for an illustration.

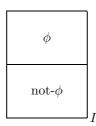


Figure 2.1: Partition made by whether- ϕ

A constituent question has as many distinct semantic answers as there are possible denotations of the relation on which it is based. Fig. 2.2 gives an illustration. The view that questions are partitions of I embodies the view that the semantic

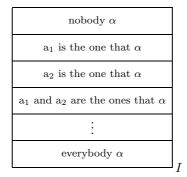


Figure 2.2: Partition made by Who α

interpretation of an interrogative determines what its (complete semantic) answers are. The latter view means adopting Schema (a) as the interpretation schema for interrogatives derived from *n*-place abstracts α :

(a) $\lambda i \lambda j [\alpha = (\lambda i \ \alpha)(j)]$

Any relation that instantiates Schema (a) is reflexive, symmetric, and transitive, i.e., it is an equivalence relation. To every equivalence relation R on a set A corresponds a partition of A, which consists of the equivalence classes of A under R. So, semantic objects of the type of Schema (a) do indeed correspond to partitions of I.

Notation. Let Q be an instance of Schema (a), then I/Q denotes the partition on I which is induced by Q:

Definition 8 $I/Q = \{[i]_Q \mid i \in I\}$

where $[i]_Q$, the set $\{j \in I \mid Q(i)(j)\}$, is the answer to Q at i. This expresses that the partition I/Q is the set of possible complete semantic answers to Q.

Two interesting partitions:

(1) a. $\{I\}$: the tautological question

b. $\{\{i\} \mid i \in I\}$: the most demanding question (What is the world like?) Examples of tautological questions:

- (2) a. whether $(\phi \text{ or not-}\phi)$
 - b. whether $(\phi \text{ and } \text{not-}\phi)$
 - c. who α or not α
 - d. which α is not α

We now consider some operations on partitions. (Througout we assume that all partitions are of the same set.) First intersection, which takes the non-empty intersections of all the elements of two partitions:

Definition 9 $\mathcal{A}_1 \sqcap \mathcal{A}_2 = \{X \cap Y \mid X \in \mathcal{A}_1 \& Y \in \mathcal{A}_2 \& X \cap Y \neq \emptyset\}$

Figure 2.3 illustrates how this intersection operation works.

We observe the following elementary fact:

Fact 1 For any partition \mathcal{A} there exist bi-partitions $\mathcal{B}_1, \ldots, \mathcal{B}_n, \ldots$ such that $\mathcal{A} = \mathcal{B}_1 \sqcap \mathcal{B}_2, \ldots, \sqcap \mathcal{B}_n, \ldots$

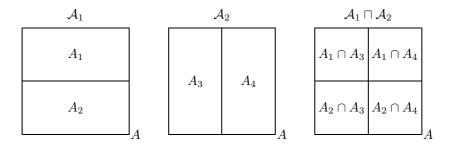


Figure 2.3: Intersection

Consequence for questions:

Fact 2 If I/Q is a non-tautological question, then it can be constructed by intersection from a number of single yes/no-questions.

Examples:

whether ϕ or ψ = whether $\phi \sqcap$ whether ψ

who α = whether $a_1 \alpha \sqcap$ whether $a_2 \alpha \sqcap, \ldots$, whether $a_n \alpha \sqcap, \ldots$

Next we consider union. The union of two partitions \mathcal{A}_1 and \mathcal{A}_2 is the set containing the smallest non-empty sets Z which equal a real subset of \mathcal{A}_1 and \mathcal{A}_2 :

Definition 10 $\mathcal{A}_1 \sqcup \mathcal{A}_2 = \{Z \mid Z \neq \emptyset \& \exists X \subset \mathcal{A}_1 \& \exists Y \subset \mathcal{A}_2 : Z = \cup X = \cup Y \& \neg \exists Z' : Z' \neq \emptyset \& Z' \subset Z \& \exists X \subset \mathcal{A}_1 \& \exists Y \subset \mathcal{A}_2 : Z' = \cup X = \cup Y \}$

See figure 2.4 for an illustration.

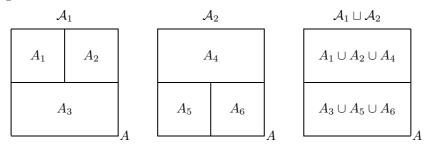


Figure 2.4: Union

The union operation seems to lack a straightforward linguistic analogue.

The most important notion in the present context is that of the relation of *inclusion* between partitions:

Definition 11 $\mathcal{A}_1 \sqsubseteq \mathcal{A}_2$ iff for all X in \mathcal{A}_1 there is a Y in \mathcal{A}_2 such that $X \subseteq Y$

Figure 2.5 gives an example.

The inclusion relation holds between two questions I/Q and I/R iff every semantic answer to Q entails a (unique) semantic answer to R. Hence it models the *entailment* relation between interrogatives. For, if every semantic answer to Qentails a semantic answer to R, then asking Q implies asking R.

We observe the following facts concerning inclusion:

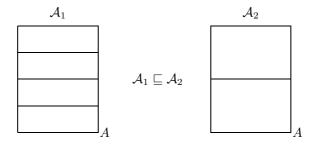


Figure 2.5: Inclusion

Fact 3 For all \mathcal{A}_1 , \mathcal{A}_2 : 1. $\mathcal{A}_1 \sqsubseteq \{A\}$ 2. $\{\{a\} \mid a \in A\} \sqsubseteq \mathcal{A}_1$ 3. $\mathcal{A}_1 \sqcap \mathcal{A}_2 \sqsubseteq \mathcal{A}_1$ 4. $\mathcal{A}_1 \sqsubseteq \mathcal{A}_2$ iff $\mathcal{A}_1 \sqcap \mathcal{A}_2 = \mathcal{A}_1$ 5. $\mathcal{A}_1 \sqsubseteq \mathcal{A}_1 \sqcup \mathcal{A}_2$ 6. $\mathcal{A}_1 \sqsubseteq \mathcal{A}_2$ iff $\mathcal{A}_1 \sqcup \mathcal{A}_2 = \mathcal{A}_2$

We note that \sqsubseteq is a partial order on the set of all partitions of a set A: \sqsubseteq is reflexive, anti-symmetric, and transitive.

The operations \sqcap and \sqcup satisfy commutativity, associativity, idempotency, and absorption:

Fact 4 For all A_1 , A_2 , A_3 :

- 1. $\mathcal{A}_1 \sqcap \mathcal{A}_2 = \mathcal{A}_2 \sqcap \mathcal{A}_1$, and $\mathcal{A}_1 \sqcup \mathcal{A}_2 = \mathcal{A}_2 \sqcup \mathcal{A}_1$
- 2. $(\mathcal{A}_1 \sqcap \mathcal{A}_2) \sqcap \mathcal{A}_3 = \mathcal{A}_1 \sqcap (\mathcal{A}_2 \sqcap \mathcal{A}_3)$, and $(\mathcal{A}_1 \sqcup \mathcal{A}_2) \sqcup \mathcal{A}_3 = \mathcal{A}_1 \sqcup (\mathcal{A}_2 \sqcup \mathcal{A}_3)$
- 3. $\mathcal{A}_1 \sqcap \mathcal{A}_1 = \mathcal{A}_1$, and $\mathcal{A}_1 \sqcup \mathcal{A}_1 = \mathcal{A}_1$
- 4. $\mathcal{A}_1 \sqcap (\mathcal{A}_1 \sqcup \mathcal{A}_2) = \mathcal{A}_1$, and $(\mathcal{A}_1 \sqcap \mathcal{A}_2) \sqcup \mathcal{A}_1 = \mathcal{A}_1$

In view of Facts 3 and 4, the set of all partitions of a set A forms a complete lattice under \sqsubseteq . In this lattice $\{A\}$ is the maximal element (cf. Fact 3.1), and $\{\{a\} \mid a \in A\}$ is the minimal element (cf. Fact 3.2). The bi-partitions are the dual atoms of the lattice, and \sqcap and \sqcup are the meet and the join.

Projecting this on the set of all questions, i.e., the set of all partitions of the set of indices I, we observe that $\{I\}$, the least demanding question, is the maximal element, and that its counterpart $\{\{i\} \mid i \in I\}$, the most demanding question, is the minimal element. The single yes/no-questions, from which all non-tautological questions can be derived by intersection (Fact 2), are the dual atoms. Assuming we have enough expressions at our disposal the realm of all questions can thus be pictured as in Fig. 2.6.

So we see that the set of all questions has a certain structure, which reflects the logical relationships between natural language interrogatives.

2.2 **Propositions as Answers**

In this section we turn to an analysis of various relations of answerhood that the present approach permits. Section 2.2.1 deals with semantic notions, and Section 2.2.2 with their pragmatic counterparts.

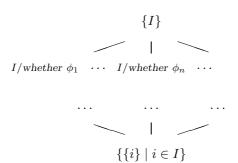


Figure 2.6: Lattice of questions

2.2.1 Semantic Notions of Answerhood

In what follows we use P as a meta-variable over propositions, i.e., subsets of I, and Q as a meta-variable over questions, i.e., partitions of I.

The two most basic notions:

Definition 12

- 1. P is a semantic answer to Q iff $P \in I/Q$
- 2. P is a partial semantic answer to Q iff $P\neq \emptyset$ and there is an $X\subset I/Q$ such that $P=\cup X$

A complete answer is the possible denotation of an interrogative. A partial answer is the disjunction of some, but not all, such denotations. We notice the following facts:

Fact 5

- 1. All complete answers to non-tautological questions are also partial answers.
- 2. Partial answers to yes/no-questions are complete answers.

Two more liberal notions:

Definition 13

- 1. P gives a semantic answer to Q iff $P\neq \emptyset$ and there is a $P'\in I/Q$ such that $P\subseteq P'$
- 2. P gives a partial semantic answer to Q iff $P\neq \emptyset$ and there is an $X\subset I/Q$ such that $P\subseteq \cup X$

A proposition gives a (partial) answer iff it is non-contradictory and implies a (partial) semantic answer.

Fact 6 If a proposition is a (partial) semantic answer, it gives a partial semantic answer.

Fig. 2.7 gives illustrations of the four notions we have defined.

In general, there is not just one partial answer given by P to Q. There is, however, always a strongest partial answer given by P, which is the disjunction of the semantic answers it is compatible with. This strongest answer we will call the partial answer which P gives:

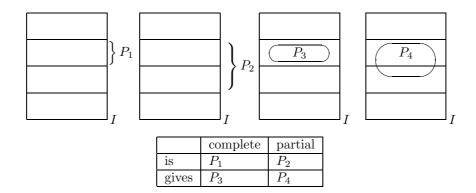


Figure 2.7: Examples of semantic answers

Definition 14 Let P give a partial semantic answer to Q. The partial semantic answer to Q that P gives is $\cup \{P' \in I/Q \mid P' \cap P \neq \emptyset\}$

If P is a (partial) semantic answer, then the answer that P gives is P itself.

We now consider the notion of a true semantic answer at a given index. Parallel to definitions 7–8, four cases can be distinguished, captured in the one following definition:

Definition 15 P is/gives a true (partial) semantic answer to Q at an index i iff P is/gives a (partial) semantic answer to Q and the partial semantic answer to Q that P gives is true at i

Notice that if P is a true (partial) answer, then P itself must be true. But if P merely gives such an answer, this need not be so. The actual index may lie inside the answer P gives, but outside P itself. Notice that for the analogous case of being/giving a false answer, the falsity of P follows in both cases. Cf. the situations depicted in Fig. 2.8.

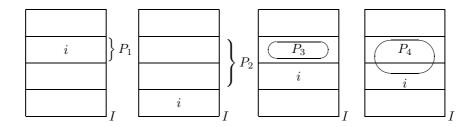


Figure 2.8: True and false answers given

 P_1 is a true, complete answer, and is true itself; P_2 is a false, partial answer, and is false itself; P_3 gives a false, complete answer, and is false itself; and P_4 gives a true, partial answer, and is false itself.

2.2.2 Pragmatic notions of answerhood

In order to obtain pragmatic notions of answerhood, we relativize questions and answers to information sets, i.e., to non-empty subsets of I. Cf. Fig 2.9.

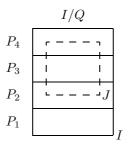


Figure 2.9: Relativization

We denote the set of semantic answers to Q which are compatible with J as $I/Q^J\colon$

Definition 16 $I/Q^J = \{X \in I/Q \mid X \cap J \neq \emptyset\}$

We observe the following fact:

Fact 7 $I/Q^J \subseteq I/Q$, for all J.

A second notion is that of the partition that a question Q restricted to J makes on J. We write this as J/Q:

Definition 17 $J/Q = \{X \cap J \mid X \in I/Q \& X \cap J \neq \emptyset\}$

The notions I/Q^J and J/Q are related as follows:

Fact 8 $X \in I/Q^J$ iff there is a Y in J/Q such that $Y \subseteq X$

The definition of the inclusion relation between partitions can now be generalized:

Definition 18 $J/Q \sqsubseteq K/R$ iff for all X in J/Q there is a Y in K/R such that $X \subseteq Y$

Now we observe the following fact:

Fact 9 $J/Q \sqsubseteq K/R$ iff $J \subseteq K$ and $J/Q \sqsubseteq J/R$

Notice that this implies that we have:

Fact 10 $J/Q \sqsubseteq I/Q$

This means that the partition that a question Q makes on I is preserved when Q is restricted to J, in the sense that it may be compatible with less semantic answers, but every answer to Q given the information J (i.e., every element of J/Q) will imply a semantic answer. Next we define:

Definition 19 Q is a question in an information set J iff there are $X, Y \in J/Q$ such that $X \neq Y$

Conversely, a question Q is answered in J if J/Q has only one element, being J itself.

Now we are ready to define the pragmatic counterparts of the semantic notions of answerhood defined in the previous section.

First we define the notions of a (complete) pragmatic answer, and of a partial pragmatic answer:

Definition 20 Let Q be a question in J.

- 1. P is a pragmatic answer to Q in J iff $P \cap J \in J/Q$
- 2. P is a partial pragmatic answer to Q in J iff $P \cap J \neq \emptyset$ and there is an $X \subset J/Q$ such that $P \cap J = \bigcup X$

P is a (complete) pragmatic answer to *Q* in *J* if adding *P* to the information set *J* results in an information set in which the question *Q* is answered, i.e., if $(P \cap J)/Q = \{P \cap J\}$. And *P* is a partial pragmatic answer if adding it to the information set *J* excludes at least one answer which hitherto was admitted.

The two corresponding notions of giving a complete or a partial answer are captured in the following definition:

Definition 21 Let Q be a question in J.

- 1. P gives a pragmatic answer to Q in J iff $P \cap J \neq \emptyset$ and there is a $P' \in J/Q$ such that $P \cap J \subseteq P'$
- 2. P gives a partial pragmatic answer to Q in J iff $P \cap J \neq \emptyset$ and there is an $X \subset J/Q$ such that $P \cap J \subseteq \cup X$

Each of the four situations depicted in Fig. 2.10 illustrates one of the pragmatic notions of answerhood.

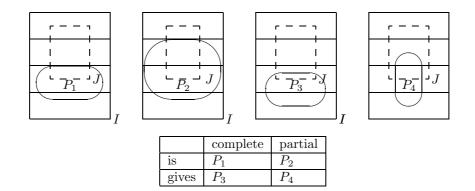


Figure 2.10: Examples of pragmatic answers

We observe that the semantic notions are limiting cases of the pragmatic ones. For J = I, the two sets of definitions coincide (disregarding the tautologous

22

question).

Similar dependencies as hold between notions of semantic answerhood hold here: to be a pragmatic answer implies to give a pragmatic answer, and to be or to give a complete pragmatic answer implies to be or to give a partial pragmatic answer.

An important fact to be observed is the following:

Fact 11 Let J' be a subset of J, Q a question in J', P compatible with J'. Then the followings hold: If P stands in a certain type of pragmatic answerhood relation to Q in J, then P stands in that same type of relation to Q in J'

Moreover, since semantic answerhood is a limit of pragmatic answerhood, it follows that (under the same provisos):

Fact 12 If P bears a certain semantic answerhood relation to Q, it bears the corresponding pragmatic answerhood relation to Q in any information set.

So, answerhood relations are preserved under information update. Another fact to be noticed is the following:

Fact 13 Let J' be a subset of J. If P stands in a certain answerhood relation to Q in J, it may stand in a 'stronger' relation to Q in J'.

In this sense, too, answerhood is monotone with respect to increase of information.

We now turn to the notion of a true pragmatic answer. Its definition is a bit complicated. Notice that, given suitable information, false propositions are not only able to give true pragmatic answers, they can also be such answers. And furthermore, it holds that even if not all of our information happens to be true, i.e., even if the information set J, being the conjunction of all our information, is false, this does not prevent us from getting true answers. Cf. the situations depicted in Fig. 2.11. Hence, if we want to decide whether P gives a true pragmatic answer in J to Q, we have to inspect whether the (partial) semantic answer determined by Pwith respect to J is true. So, we define:

Definition 22 Let P give a partial pragmatic answer to Q in J. The partial semantic answer to Q determined by P in $J = \bigcup \{P' \in I/Q \mid P' \cap P \cap J \neq \emptyset\}$.

This leads us to the following definition of true pragmatic answerhood:

Definition 23 P is (gives) a true (partial) pragmatic answer to Q in J at i iff P is (gives) a (partial) pragmatic answer to Q in J and the partial semantic answer to Q determined by P in J is true at i.

Fig. 2.11 provides some illustrations. The answer given by P_1 is P_5 , that given by P_2 is P_6 , that given by P_3 is P_7 , and P_4 gives P_8 . P_1 is false, but it is a true, complete answer. P_2 is true, and gives a true, partial answer. P_3 is false, and gives a true complete answer. And P_4 is true, and gives a false, partial answer.

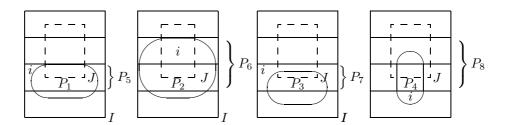


Figure 2.11: True and false answers given

2.3 Comparing answers

This last section deals with one aspect of the correctness of question-answering. The latter involves the well-known Gricean maxims:

Quality A proposition satisfies Quality if the answer it gives is true.

Relevance A proposition satisfies Relevance if it bears some answerhood relation to the question, given the information of the questioner, and if it does not contain, or contains only a minimum of, irrelevant information. What is (ir)relevant depends on the question.

Quantity A proposition satisfies Quantity if it is the best answer that can be given, *modulo* Quality.

We concentrate on the last topic, that of choosing a quantitatively best answer. The problem is how one is to choose between several rival propositions which are all qualitatively in order. Is there always a best answer? It will turn out that, given certain additional conditions, the answer is 'yes'. And we outline a kind of evaluation procedure that leads to this result.

We start with the procedure. The theory of answerhood gives the following oppositions:

- 1. Complete answers vs. partial answers
- 2. Propositions that are answers vs. propositions that give an answer

3. Pragmatic answers vs. semantic answers

The question how to choose between rival propositions is no other than the question how these oppositions between answers are to be evaluated.

There is no general answer to this question, so we adopt the following background perspective: question-answering concerns filling in gaps in the information of individuals.

This answers the question with respect to 1. and 3. How about 2? This is a matter of relevance: if P gives an answer to Q without being an answer to Q, then P contains information that is strictly speaking irrelevant for Q. So, given our background perspective, we have:

1. We prefer complete answers over partial ones.

2. Propositions that are answers are better than propositions that give answers.

3. Pragmatic considerations are more important than semantic ones.

The question now becomes: how are we to evaluate the three oppositions among themselves?.

Notice that they work in opposite directions:

- 1. Preferring complete over partial answers favours stronger propositions
- 2. Preferring propositions that are answers over propositions that give answer favours weaker propositions

So, the evaluation procedure should apply these preferences in a definite order. The background perspective indicates which order this is to be: pragmatic considerations take precedence over semantic ones.

The evaluation procedure:

- I. *Pragmatic Quantity*: Choose the answer that stands the best chance of filling in the gap in the information of the questioner which is indicated by the question, i.e., leave semantic considerations aside, for the time being.
- There are two sides to this:
 - 1. Choose the answer that excludes the greatest number of possible answers which are still allowed by the information of the questioner, i.e., choose the answer which is the least partial one with respect to the information of the questioner.
 - 2. If two or more answers come out the same according to 1, choose the one that contains the least superfluous information, given what the question asks for.

If two propositions come out equal under I, apply II:

II. Semantic Quantity: Choose the answer which is best from a purely semantic point of view, i.e., which is the best one on the basis of conventional meaning only.

Again, two aspects can be discerned:

- 1. Choose the answer which is most informative with respect to the question asked.
- 2. Should two or more answers still rate equally: choose the one which contains the least superfluous information given what the question asks for.

In what follows we will:

- a. formalize the procedure
- b. show that it yields a best answer

We consider the following case:

Assumption: P_1 and P_2 are two different, mutually compatible propo-

sitions, which both give a partial answer to the question Q.

Now recall that the procedure involves two opposing forces:

• towards weaker propositions ('relevance', i.e., I.2 and II.2);

• towards stronger propositions ('informativeness', i.e., I.1 and II.1)

This means that, beside P_1 and P_2 themselves, their conjunction $P_1 \cap P_2$ and their disjunction $P_1 \cup P_2$ have to be taken into account. What we shall show, then, is:

To be shown: There is a best answer to Q among P_1 , P_2 , their con-

junction $P_1 \cap P_2$, and their disjunction $P_1 \cup P_2$.

We start with the semantic evaluation procedure (II). First, we define some notation:

Definition 24 $[P, I/Q] = \bigcup \{ P' \in I/Q | P \cap P' \neq \emptyset \}$

In definition 24 $\{P' \in I/Q | P \cap P' \neq \emptyset\}$ is the set of answers to Q which are compatible with P, so by $\lceil P, I/Q \rceil$ we denote the disjunction of these answers.

We use $\mathcal{A}(P, I/Q)$ as a notation for 'P gives a partial answer to Q', and we notice:

Fact 14 $\mathcal{A}(P, I/Q)$ iff $\emptyset \neq [P, I/Q] \neq I$

Next we define relative semantical informativeness:

Definition 25

- 1. P is semantically more informative than P' with respect to Q iff $[P, I/Q] \subset [P', I/Q]$.
- 2. *P* and *P'* are semantically equally informative with respect to *Q* iff [P, I/Q] = [P', I/Q].

Notice that if a proposition entails another, this does not imply that it is more informative. What does hold is that if a proposition properly entails another one, then the former is at least as informative as the latter with respect to any question:

Fact 15 If $P \subset P'$, then $[P, I/Q] \subseteq [P', I/Q]$

Notice also:

Fact 16 If P gives a complete semantic answer to Q, then no other proposition P' gives a semantically more informative answer to Q than P

However, this does not determine a unique best answer.

We now turn to relevance, or, as we call it in this semantic context, that of *semantical standardness*: containing less superfluous information, is, from a semantic point of view, coming closer to being a standard identification of some object, or state of affairs. A comparative notion of semantic standardness is defined as follows:

Definition 26 P is semantically more standard than P' with respect to Q iff P' and P' are semantically equally informative with respect to Q and $P' \subset P$

Of two propositions which are equally informative with respect to some question the more standard one is the one which is the weakest. Standardness and informativeness work in opposite directions: the former favours weaker propositions, the latter stronger ones. Notice:

Fact 17 A proposition that is a semantic partial answer to a question is semantically more standard than any other semantically equally informative proposition.

The next step is to combine the notions of semantic informativeness and semantic standardness in a comparative notion.

Definition 27 P is semantically quantitatively better than P' with respect to Q iff

- 1. P is more informative than P' with respect to Q; or
- 2. P and P' are equally informative with respect to Q and P is more standard than P' with respect to Q

We shall use $P \gg_Q P'$ to denote this relation, and we observe:

Fact 18 $P \gg_Q P'$ iff 1. $\lceil P, I/Q \rceil \subset \lceil P', I/Q \rceil$; or 2. $\lceil P, I/Q \rceil = \lceil P', I/Q \rceil$ and $P' \subset P$

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Definition 27 embodies the semantic evaluation procedure with its two subprocedures, of semantic informativeness and semantic standardness, properly ordered. Notice:

- 1. Propositions that are (semantic standardness) complete (semantic informativeness) semantic answers are semantically quantitatively best.
- 2. There may be more than one such proposition.

In view of 2 it may seem that we are not getting anywhere. However, we now show that the following holds:

Fact 19 Let P_1 and P_2 be distinct and compatible propositions, which both give a partial answer to the question Q. Then there is a semantically quantitatively best answer to Q in the set $\{P_1, P_2, P_1 \cap P_2, P_1 \cup P_2\}$.

We need to distinguish various cases. By the supposition of compatibility of P_1 and P_2 we know that either:

- a. $P_1 \subset P_2$ (or similarly, $P_2 \subset P_1$); or
- b. $P_1 \cap P_2 \subset P_1, \subset P_2$.

The a-case is straightforward (since Q is a fixed parameter, we leave out reference to Q and I/Q in what follows):

Since $P_1 \subset P_2$, we have $\lceil P_1 \rceil \subseteq \lceil P_2 \rceil$ (Fact 15); hence either $P_1 \gg_Q P_2$, in case $\lceil P_1 \rceil \subset \lceil P_2 \rceil$, by informativeness; or $P_2 \gg_Q P_1$, in case $\lceil P_1 \rceil = \lceil P_2 \rceil$, by standardness. In the b-case the conjunction and disjunction come into play. The following facts will prove useful:

Fact 20 If $\mathcal{A}(P, I/Q)$ and $P \cap P' \neq \emptyset$, then $\mathcal{A}(P \cap P', I/Q)$

Fact 21 If $\mathcal{A}(P, I/Q)$, $\mathcal{A}(P', I/Q)$, and $\lceil P \rceil = \lceil P' \rceil$, then $\mathcal{A}(P \cup P', I/Q)$

In the b-case we know that, since $P_1 \cap P_2 \subset P_1$, $\subset P_2$, $\lceil P_1 \cap P_2 \rceil \subseteq \lceil P_1 \rceil$, $\subseteq \lceil P_2 \rceil$. This leaves 4 subcases to be considered:

- 1. $\lceil P_1 \cap P_2 \rceil = \lceil P_1 \rceil, \subset \lceil P_2 \rceil$. Then $\lceil P_1 \rceil \subset \lceil P_2 \rceil$ and hence, by informativeness, $P_1 \gg_Q P_2$
- 2. $\lceil P_1 \cap P_2 \rceil = \lceil P_2 \rceil, \subset \lceil P_1 \rceil$. Then $\lceil P_2 \rceil \subset \lceil P_1 \rceil$ and hence, by informativeness, $P_2 \gg_Q P_1$
- 3. $\lceil P_1 \cap P_2 \rceil \subset \lceil P_1 \rceil, \subset \lceil P_2 \rceil$. Then, by Fact 20, $\mathcal{A}(P_1 \cap P_2, I/Q)$, and hence, by informativeness, $P_1 \cap P_2 \gg_Q P_1, \gg_Q P_2$
- 4. $[P_1 \cap P_2] = [P_1], = [P_2]$. Then also $[P_1 \cup P_2] = [P_1], = [P_2]$, so $\mathcal{A}(P_1 \cup P_2, I/Q)$, by Fact 21, and hence, by standardness, $P_1 \cup P_2 \gg_Q P_1, \gg_Q P_2$

This proves Fact 19 stated above, that given two compatible propositions there is a semantically quantitatively best one among these two, their conjunction and their disjunction.

Now we turn to the pragmatic procedure, which, as is to be expected, turns out to be structurally analogous to the semantic one.

First, some notation:

Definition 28 $[P, J/Q] = \bigcup \{ P' \in J/Q | P \cap P' \cap J \neq \emptyset \}$

The set $\{P' \in J/Q | P \cap P' \cap J \neq \emptyset\}$ consists of those answers to Q restricted to J which are compatible with P, so by $\lceil P, J/Q \rceil$ we denote the disjunction of these answers.

We use $\mathcal{AP}(P, J/Q)$ as a notation for 'P gives a partial pragmatic answer to Q relative to J', and we notice:

Fact 22 If $\mathcal{AP}(P, J/Q)$, then $\emptyset \neq [P, J/Q] \neq J$

We get a relative notion of *pragmatic informativeness* as follows:

Definition 29

- 1. P is pragmatically more informative than P' with respect to Q and J iff $[P, J/Q] \subset [P', J/Q]$.
- 2. P and P' are pragmatically equally informative with respect to Q and J iff [P, J/Q] = [P', J/Q].

Similar remarks as we made above apply here. Entailment, here given the information in J, is not sufficient for being pragmatically more informative. What does hold is that if the restriction of P to J properly entails the restriction of P' to J, then P is at least as pragmatically informative with respect to J as P' is, for any Q:

Fact 23 If $(P \cap J) \subset (P' \cap J)$, then $[P, J/Q] \subseteq [P', J/Q]$

Like in the semantic case, there are 'upper bounds' of informativeness, viz., the propositions that give complete pragmatic answers:

Fact 24 If P gives a complete pragmatic answer to Q, then no other proposition P' gives a pragmatically more informative answer to Q than P

Notice that in general there is no unique such limit.

A definition of *pragmatic standardness* can be obtained as a restriction of the corresponding semantic notion:

Definition 30 P is pragmatically more standard than P' with respect to Q and J iff P' and P' are pragmatically equally informative with respect to Q and J and $(P' \cap J) \subset (P \cap J)$

Notice that if a proposition is weaker than another one in J, it is to be preferred. What happens 'outside J', so to speak, i.e., whether the proposition is also semantically weaker than its rival, is not considered at all: it is only as far as the information contained in J goes, that things (propositions) are measured and compared.

Given this definition of pragmatic standardness, we can combine it with that of pragmatic informativeness into a comparative notion, like we did in the semantic case.

Definition 31 P is pragmatically quantitatively better than P' with respect to Q and J iff

- 1. P is pragmatically more informative than P' with respect to Q and J; or
- P and P' are pragmatically equally informative with respect to Q and J and P is more standard than P' with respect to Q and J

We use $P \gg_{Q,J} P'$ to denote this relation, and we note:

Fact 25 $P \gg_{Q,J} P'$ iff 1. $\lceil P, J/Q \rceil \subset \lceil P', J/Q \rceil$; or 2. $\lceil P, J/Q \rceil = \lceil P', J/Q \rceil$ and $(P' \cap J) \subset (P \cap J)$

Further we note the following:

- 1. Propositions that are (pragmatic standardness) complete (pragmatic informativeness) pragmatic answers are pragmatically quantitatively best.
- 2. Two propositions may come out equally well.

Propositions that are complete pragmatic answers are propositions that, given the information that is available, completely settle the matter raised by the question with respect to this information, without giving any other ('irrelevant') information.

However, the following fact can be proved:

Fact 26 Let P_1 and P_2 be distinct and compatible propositions, which both give a partial pragmatic answer to the question Q given J. Then there is a pragmatically quantitatively best answer to Q given J in the set $\{P_1, P_2, P_1 \cap P_2, P_1 \cup P_2\}$.

The proof runs completely parallel to that of the corresponding semantic fact. By supposition we know that:

a. $(P_1 \cap J) \subset (P_2 \cap J)$ (or vice versa); or

b. $(P_1 \cap P_2 \cap J) \subset (P_1 \cap J), \subset (P_2 \cap J).$

Again, the a-case follows immediately (again, we leave out reference to Q and I/Q in what follows):

 $P_1 \gg_{Q,J} P_2$, in case $\lceil P_1 \rceil \subset \lceil P_2 \rceil$; and $P_2 \gg_{Q,J} P_1$, in case $\lceil P_1 \rceil = \lceil P_2 \rceil$ (since $(P_1 \cap J) \subset (P_2 \cap J)$). In the b-case there are again four subcases to be considered, and two general facts to take advantage of:

Fact 27 If $\mathcal{AP}(P, J/Q)$ and $P \cap P' \cap J \neq \emptyset$, then $\mathcal{AP}(P \cap P', J/Q)$

Fact 28 If $\mathcal{AP}(P, J/Q)$, $\mathcal{AP}(P', J/Q)$ and $\lceil P, J/Q \rceil = \lceil P', J/Q \rceil$, then $\mathcal{AP}(P \cup P', J/Q)$

The b-case is then proved as follows:

1. $\lceil P_1 \cap P_2 \rceil = \lceil P_1 \rceil, \subset \lceil P_2 \rceil \colon P_1 \gg_{Q,J} P_2$

- 2. $\lceil P_1 \cap P_2 \rceil = \lceil P_2 \rceil, \subset \lceil P_1 \rceil \colon P_2 \gg_{Q,J} P_1$
- 3. $[P_1 \cap P_2] \subset [P_1], \subset [P_2]: P_1 \cap P_2 \gg_{Q,J} P_1, \gg_{Q,J} P_2$, by Fact 27
- 4. $[P_1 \cap P_2] = [P_1], = [P_2]: P_1 \cup P_2 \gg_{Q,J} P_1, \gg_{Q,J} P_2$, by Fact 28

Now that we have defined the semantic and the pragmatic evaluation procedures, it is important to note (once again) that the two may give different outcomes. However, given the background perspective we assume, we must first apply the pragmatic criterion, and only if that does not give us a unique outcome, the semantic one.

Hence the overall procedure combines the semantic procedure (Definition 27) and the pragmatic procedure (Definition 31) as follows:

Definition 32 P is better than P' with respect to Q and J iff

- 1. P is pragmatically quantitatively better than P' with respect to Q and J; or
- 2. P and P' are pragmatically quantitatively equal with respect to Q and J, and P is semantically quantitatively better than P' with respect to Q

Our final result follows immediately from Definition 32 and Facts 19 and 26:

Fact 29 Let P_1 and P_2 be distinct and compatible propositions, which both give a partial pragmatic answer to the question Q given J. Then there is a best answer to Q given J in the set $\{P_1, P_2, P_1 \cap P_2, P_1 \cup P_2\}$

Chapter 3

Linguistic Answers

This chapter is devoted to an analysis of interrogatives and linguistic answers. Two kinds of answers are distinguished: constituent answers and sentential answers. It is argued that both kinds of answers can be interpreted properly only in the context of an interrogative, and that this interpretation is exhaustive (Section 3.1). A general operation of exhaustivization is defined and applied to various constructions (Section 3.2): answers to single constituent interrogatives (Section 3.2.1); answers to multiple constituent interrogatives (Section 3.2.2); answers to sentential interrogatives (Section 3.2.3). In Section 3.3 the relationships between various properties of constituent answers and the various notions of answerhood, studied in Chapter 2, are explored.

3.1 Answers and Exhaustiveness

It is argued in this section that neither the category of constituent answers, nor that of sentential answers, is 'basic' with respect to the other, since the interpretation of both is context-dependent and exhaustive.

3.1.1 Constituent Answers

Some typical examples of interrogatives and constituent answers:

- (1) Who walk in the garden?— John and Mary.
- (2) Whom did John kiss?
- A girl and two boys.(3) Which boy kissed which girl?
 - The tall boy, Mary; and the small boy, the two redheads.

Fig. 3.1 illustrates the context-dependency of constituent answer interpretation: the constituent interpretation is combined with that of the interrogative to derive the interpretation of the answer.

3.1.2 Sentential Answers

Naively we might think that the interpretation of sentential answers is contextindependent, as in Fig. 3.2. The interpretation strategy of Fig. 3.2 works only for Linguistic Answers

Figure 3.1: Interpretation of constituent answers (simplified)

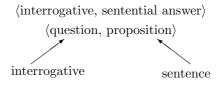


Figure 3.2: Naive interpretation of sentential answers

explicitly exhaustive answers:

- (4) Who walk in the garden?
 - Only John and Mary walk in the garden.
- (5) Whom did John kiss?
 - John kissed a girl and two boys and no one else.
- (6) Which boy kissed which girl?

— The tall boy kissed just Mary, and the small boy kissed only the two redheads, and no other boy kissed another girl.

The default interpretation of sentential answers is exhaustive:

- (7) Who walk in the garden?
 - John and Mary walk in the garden.
- (8) Whom did John kiss?
 - John kissed a girl and two boys.
- (9) Which boy kissed which girl?
 - The tall boy kissed Mary, and the small boy the two redheads.

Non-exhaustiveness of answers is a marked phenomenon:

- (10) a. John and Mary, for example, walk in the garden.
 - b. (I don't know, but) at least John and Mary walk in the garden.
 - c. John and Mary are among the ones that walk in the garden.

Conclusion: the interpretation of sentential answers also depends on the context of the interrogative. Some further examples to illustrate this:

- (11) a. Who kissed Mary?
 - b. Whom did John kiss?
 - c. Who kissed whom?
 - d. What did John do?
- (12) John kissed Mary.
- (13) a. John is the one who kissed Mary.
 - b. Mary is the one that John kissed.
 - c. The only one who kissed was John and the only one he kissed was Mary.
 - d. The thing that John did was kiss Mary.

A particularly interesting example:

- (14) a. Who walks in the garden?
 - b. Which boy walks in the garden?
- (15) Hilary walks in the garden.
- (16) a. The one who walks in the garden is Hilary.
 - b. The boy who walks in the garden is Hilary.

Conclusion: both constituent and sentential answers need to be interpreted in the context of an interrogative. Question: what is the relevant interpretation of the interrogative?

- (17) a. Who walk?
 - b. Who do not walk?
- (18) John and Mary.
- (19) a. What time is it now in Amsterdam?
- b. What time is it now in Moscow?
- (20) 5 P.M.

These examples show that we need the relation (abstract) underlying the question (interrogative) in determining the proposition expressed by the answer. So the proper schema for the interpretation of answers is as in Fig. 3.3.

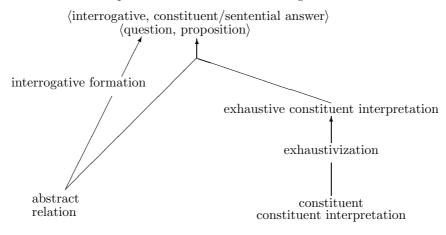


Figure 3.3: Interpretation of answers

Notice that according to Fig. 3.3 sentential answers, too, are derived from underlying constituents.

3.2 Semantics of Linguistic Answers

The implementation of the interpretation schema of Fig. 3.3 requires the definition of two rules and one operation:

- 1. The rule of interrogative formation.
- 2. The rule of answer formation.
- 3. The operation of exhaustivization.

The first rule has been dealt with in Chapter 1, the second and the third are the concern of what follows. Both rules will have to be formulated quite generally, since they have to cover single and multiple constituent interrogatives, sentential interrogatives, and their answers. For expository reasons we first discuss single constituent

interrogatives (Section 3.2.1). Then we generalize to multiple constituent interrogatives (Section 3.2.2.), and show how sentential interrogatives can be accomodated by viewing them as zero-constituent interrogatives (Section 3.2.3).

3.2.1 Answers to Single Constituent Interrogatives

In this section we concentrate on single constituent interrogatives and their answers. As abstracts, the former express properties of objects of some kind, and the latter generalized quantifiers over those objects. Following the schema of Fig. 3.3, this means that the answer formation rule is as follows:

Regel 3 If β' is the relational interpretation of a single constituent interrogative, α' the interpretation of a term, EXH the semantic operation of exhaustivization, then the interpretation of the constituent answer α . is $(EXH(\alpha'))(\beta')$

Now for the definition of the operation of exhaustiveness. (To facilitate reading we will use extensional representations in what follows, whenever intensionality isn't really involved.) The following examples make clear what we are after:

- (21) Who walk(s)?
- (22) a. John.
 - b. John walks.
 - c. Only John walks.
 - d. $\forall x [walk(x) \leftrightarrow x = j]$
- (23) a. John and Mary.
 - b. John and Mary walk.
 - c. Only John and Mary walk.
 - d. $\forall x [\texttt{walk}(x) \leftrightarrow [x = j \lor x = m]]$
- (24) a. Every boy.
 - b. Every boy walks.
 - c. Only every boy walks.
 - d. $\forall x [\texttt{walk}(x) \leftrightarrow \texttt{boy}(x)]$
- (25) a. John or Mary.
 - b. John or Mary walks.
 - c. Only John or Mary walks.
 - d. $\forall x [[walk(x) \leftrightarrow x = j] \lor [walk(x) \leftrightarrow x = m]]$
- (26) a. A girl.
 - b. A girl walks.
 - c. Only a girl walks.
 - d. $\exists x[girl(x) \land \forall y[walk(y) \leftrightarrow x = y]]$
- (27) $John \rightsquigarrow \lambda PP(j)$ $EXH(\lambda PP(j))$ $\lambda P \forall x [P(x) \leftrightarrow x = j]$
- (28) John and Mary $\rightsquigarrow \lambda P[P(\mathbf{j}) \land P(\mathbf{m})]$ $\operatorname{EXH}(\lambda P[P(\mathbf{j}) \land P(\mathbf{m})])$
 - $\lambda P \forall x [P(x) \leftrightarrow [x = \mathbf{j} \lor x = \mathbf{m}]]$
- (29) every boy $\rightsquigarrow \lambda P \forall x [boy(x) \rightarrow P(x)]$ $\operatorname{EXH}(\lambda P \forall x [boy(x) \rightarrow P(x)])$ $\lambda P \forall x [boy(x) \leftrightarrow P(x)]$

(30) John or Mary $\rightsquigarrow \lambda P[P(j) \lor P(m)]$ $EXH(\lambda P[P(j) \lor P(m)])$ $\lambda P \forall x[[P(x) \leftrightarrow x = j] \lor [P(x) \leftrightarrow x = m]]$ (31) a girl $\rightsquigarrow \lambda P \exists x[girl(x) \land P(x)]$ $EXH(\lambda P \exists x[girl(x) \land P(x)])$

$$\lambda P \exists x [\texttt{girl}(x) \land \forall y [P(y) \leftrightarrow x = y]]$$

Exhaustivization is the operation which filters the minimal elements from a set of sets:

Definition 33 EXH = $\lambda \mathcal{P}\lambda P[\mathcal{P}(P) \land \neg \exists P'[\mathcal{P}(P') \land P \neq P' \land \forall x[P'(x) \to P(x)]]]$

An example to illustrate how Definition 33 works: EXH applied to the translation of John and Mary.

(32) $\lambda P[P(\mathbf{j}) \wedge P(\mathbf{m}) \wedge \neg \exists P'[P'(\mathbf{j}) \wedge P'(\mathbf{m}) \wedge P \neq P' \wedge \forall x[P'(x) \rightarrow P(x)]]]$

Application of (32) to walk, the relational interpretation of Who walks?, results in (33), which is equivalent to (23d).

(33) $[walk(j) \land walk(m) \land \neg \exists P'[P'(j) \land P'(m) \land walk \neq P' \land \forall x[P'(x) \rightarrow walk(x)]]]$ In order for the operation of exhaustivization to work properly, we need to assume that some terms are 'group denoting'. An example:

- (34) Who walk(s)?
- (35) a. John or Mary.
 - b. John or Mary or both (John and Mary).
- (36) a. Only John or only Mary.
 - b. Only John or only Mary or only both (John and Mary).

At the individual level (35a) and (35b) are equivalent, hence their exhaustivization would be the same too, viz., (36a). However, the exhaustivization of (35b) is (36b), so we conclude that (35b) mentions two individuals and the group consisting of them. We assume some theory of plurality which distinguishes between individuals and groups,¹, and we denote the group consisting of the individuals John and Mary by [John, Mary].

- (37) a. John or Mary or both (John and Mary)
 - b. $\{X \mid \{John\} \subseteq X \lor \{Mary\} \subseteq X \lor \{[John, Mary]\} \subseteq X\}$
- (38)~a. Only John or only Mary or only both (John and Mary).
 - b. $\{\{John\}, \{Mary\}, \{[John, Mary]\}\}$

A similar analysis shows how the group denoting term At least one girl. gives a different answer than does the individual denoting term A girl.

A slightly more complex example to finish the discussion of single constituent interrogatives and their answers. First the example:

(39) Which guests did John kiss?

— (John kissed) Bill or Peter, and two girls.

The constituent underlying both the constituent and the sentential answer, and its translation:

(40) a. Bill or Peter, and two girls

b. $\lambda P[[P(\mathbf{b}) \lor P(\mathbf{p})] \land \exists x \exists y [x \neq y \land girl(a)(x) \land girl(a)(y) \land P(x) \land P(y)]]$

And the abstract underlying the interrogative and the property it expresses:

^{1.} See e.g., Scha 1981; Link 1983; Landman 1989.

(41) a. which guests John kissed

b. $\lambda x[\texttt{guest}(a)(x) \land \texttt{kiss}(a)(\texttt{j}, x)]$

Application of exhaustivization to the constituent:

- - b. $\forall I [\exists x \exists y | x \neq y \land g \exists \exists (u)(x) \land g \exists \exists (u)(y) \land [\forall z | 1(z) \leftrightarrow [z = x \lor z = y \lor z = b]] \lor \forall z [P(z) \leftrightarrow [z = x \lor z = y \lor z = p]]]]]$

Finally, application of Rule 3 to derive the interpretation of the answer:

 $\begin{array}{l} (43) \quad \exists x \exists y [x \neq y \land \texttt{girl}(a)(x) \land \texttt{girl}(a)(y) \land \\ \quad [\forall z [[\texttt{guest}(a)(z) \land \texttt{kiss}(a)(\texttt{j})(z)] \leftrightarrow [z = x \lor z = y \lor z = \texttt{b}]] \lor \\ \forall z [[\texttt{guest}(a)(z) \land \texttt{kiss}(a)(\texttt{j})(z)] \leftrightarrow [z = x \lor z = y \lor z = \texttt{p}]]] \end{array}$

3.2.2 Answers to Multiple Constituent Interrogatives

We now turn to a generalization of the analysis to cover also answers to multiple constituent interrogatives. First a simple example:

- (44) a. Which man loves which woman?
 - b. John, Mary.
 - John loves Mary.

More complex examples involve conjunctions and disjunctions of constituent sequences:

- (45) a. Which man loves which woman?
 - b. John, Mary; and Bill Suzy.
 - John loves Mary, and Bill (loves) Suzy.
 - c. John Mary; or Bill Suzy.
 - John loves Mary, or Bill (loves) Suzy.

In accordance with the interpretation strategy of Fig. 3.3 we derive the interpretation of the answer by applying the exhaustivization of the interpretation of the sequences of constituents to the interpretation of the multiple constituent interrogative. The latter is a relation. For example, the interpretation of the interrogative (45a) is:

(46) $\lambda x \lambda y[\operatorname{man}(a)(x) \wedge \operatorname{woman}(a)(y) \wedge \operatorname{love}(a)(x,y)]$

In order to implement the schema of Fig. 3.3 completely generally, we need two more things:

- 1. An interpretation of constituent sequences
- 2. A generalization of the operation of exhaustivization

Sequences of n terms are called n-place terms and are constructed from n ordinary terms. The interpretation is given in Rule 4:

Regel 4 If $\alpha_1 \ldots \alpha_n$ are are *n* terms which translate as $\alpha'_1 \ldots \alpha'_n$ respectively, then the sequence of these terms translates as:

 $\lambda R^n[\alpha'_1(\lambda x_1[\ldots \alpha'_n(\lambda x_n[R^n(x_1\ldots x_n)])\ldots])]$

where \mathbb{R}^n ranges over *n*-place relations

Some examples to illustrate this. First a simple one, the two-place term John, Mary:

(47) a. $\lambda R[\lambda PP(\mathbf{j})(\lambda x_1[\lambda PP(\mathbf{m})(\lambda x_2[R(x_1, x_2]])])]$ b. $\lambda RR(\mathbf{j}, \mathbf{m})$

Conjunctions, John, Mary; and Bill, Suzy, and disjunctions, John, Mary; or Bill, Suzy are dealt with by means of generalized conjunction and disjunction rules (see Chapter 1):

(48) a. $\lambda R[[R(j, m) \land R(b, m)]]$ b. $\lambda R[[R(j, m) \lor R(b, m)]]$

A complex example:

(49) a. John and Bill, Mary or Suzy; and Peter or Fred a redhead b. $\lambda R[[[R(j,m) \land R(b,m)] \lor [R(j,s) \land R(b,s)]] \land \exists x [redhead(a)(x) \land [R(p,x) \lor R(f,x)]]]$

Now the generalization of the operation of exhaustivization. We define a family of operations EXH^n , $n \ge 0$:

Definition 34 EXHⁿ = $\lambda \mathcal{R}^n \lambda \mathcal{R}^n [\mathcal{R}^n(\mathcal{R}^n) \land \neg \exists S^n [\mathcal{R}^n(S^n) \land \mathcal{R}^n \neq S^n \land \forall x_1 \dots x_n [S^n(x_1 \dots x_n) \to \mathcal{R}^n(x_1 \dots x_n)]]]$

By way of example we show the application of EXH^2 to the multiple terms in (45b), (46b,c) and (50a):

- (50) a. John, Mary $\lambda R \forall x \forall y [R(x, y) \leftrightarrow [x = j \land y = m]]$ b. John, Mary; and Bill, Suzy $\lambda R \forall x \forall y [R(x, y) \leftrightarrow [[x = j \land y = m] \lor [x = b \land y = s]]]$ c. John, Mary; or Bill, Suzy $\lambda R \forall x \forall y [R(x, y) \leftrightarrow [x = j \land y = m]] \lor [R(x, y) \leftrightarrow [x = b \land y = s]]]$ d. John and Bill, Mary or Suzy; and Peter or Fred a redhead
 - $\begin{array}{l} \lambda R \exists z_1 \exists z_2 \exists z_3 [[z_1 = \mathtt{m} \lor z_1 = \mathtt{s}] \land [z_2 = p \lor z_2 = \mathtt{f}] \land [\mathtt{redhead}(a)(z_3) \land \\ \forall x \forall y [R(x,y) \leftrightarrow [[[x = \mathtt{j} \lor x = \mathtt{b}] \land y = z_1] \lor [x = z_2 \land y = z_3]]]]] \end{array}$

With respect to group denoting terms a similar strategy has to be followed as was indicated above.

The generalization of Rule 3 is simple:

Regel 5 If α' is the interpretation of an *n*-place term, and β' is the relational interpretation of an *n*-constituent interrogative, the interpretation of the linguistic answer based on α in the context of the interrogative β is $(\text{EXH}^n(\alpha'))(\beta')$

A simple example to illustrate Rule 5: the derivation of (45b). It consist of the application of the exhaustified interpretation of the conjunctive two-place term to the relational interpretation of the interrogative:

- (51) a. $(\text{EXH}^2(\lambda R[R(j, m) \land R(b, s)]))(\lambda x \lambda y[man(a)(x) \land woman(a)(y) \land love(a)(x, y)])$, which reduces to:
 - b. $\lambda R \forall x \forall y [R(x, y) \leftrightarrow [[x = j \land y = m] \lor [x = b \land y = s]]] (\lambda x \lambda y [man(a)(x) \land woman(a)(y) \land love(a)(x, y)])$, which reduces to:
 - c. $\forall x \forall y [[man(a)(x) \land woman(a)(y) \land love(a)(x, y)] \leftrightarrow [[x = j \land y = m] \lor [x = b \land y = s]]]$

3.2.3 Answers to Sentential Interrogatives

Answers to sentential interrogatives are covered by Rule 5 as well: sentential interrogatives can be viewed as zero-constituent interrogatives, and constituent answers to such interrogatives as zero-place terms. An example: (52) a. Does John walk?

Sentences are of type t, and hence are zero-constituent interrogatives. They express propositions, i.e., zero-place relations. Yes and No are extensional sentential adverbs, i.e., expressions of type $\langle t, t \rangle$. They can be viewed as sets of (properties of) zeroplace relations, i.e., as zero-place terms. Their interpretations are (p is a variable of type t):

(53) yes
$$\rightsquigarrow \lambda p p$$

no
$$\rightsquigarrow \lambda p \neg p$$

Derivation of the interpretation of (52b) by means of Rule 5:

- (54) a. $(\text{EXH}^0(\lambda p \ p))(\text{walk}(a)(j))$, which is equivalent to:
 - b. $(\lambda p[p \land \neg \exists p[p' \land p' \neq p \land [p' \rightarrow p]])(\texttt{walk}(a)(\texttt{j}))$, which reduces to:
 - c. $\operatorname{walk}(a)(j) \land \neg \exists p'[p' \land p' \neq \operatorname{walk}(a)(j) \land [p' \to \operatorname{walk}(a)(j)]]$, which is equivalent to:
 - d. walk(a)(j)

The derivation of (52c) is analogous.

The reason that exhaustivization has no effects is that Yes and No always denote exhaustive sets:

(55) a. $\lambda p \ p = \{1\} = \{\{\emptyset\}\}\$ b. $\lambda p \ \neg p = \{0\} = \{\emptyset\}\$

An example of a non-exhaustive answer to a sentential interrogative:

- (56) a. Does John walk?
 - b. If Mary walks.
 - John walks if Mary walks.

In cases such as these, too, the default interpretation is exhaustive: in the context of (56a), (56b) means:

(57) John walks if and only if Mary walks

The derivation according to Rule 5:

- (58) a. $(\text{EXH}^0(\lambda p[\texttt{walk}(a)(\texttt{m}) \to p]))(\texttt{walk}(a)(\texttt{j}))$, which reduces to: b. $[\texttt{walk}(a)(\texttt{m}) \to \texttt{walk}(a)(\texttt{j})] \land \neg \exists p[[\texttt{walk}(a)(\texttt{m}) \to p] \land [p \neq m])$
 - $\begin{aligned} & \texttt{walk}(a)(\texttt{m}) \rightarrow \texttt{walk}(a)(\texttt{j}) \\ & \texttt{walk}(a)(\texttt{j}) \rceil \land [p \rightarrow \texttt{walk}(a)(\texttt{j})]] \end{aligned}$

So, the exhaustivization of *if Mary walks* means the same as *if and only if Mary walks*. The reason is that *if Mary walks* does not always denote an exhaustive set: *if Mary walks* denotes $\{\emptyset, \{\emptyset\}\}$.

Notice that this effect depends on the interrogative:

- (59) a. Is it true that John walks if Mary walks?
 - b. (Yes,) John walks if Mary walks.

In a similar vein it can be shown that disjunctive answers in the context of certain interrogatives express *exclusive disjunctions*. Cf.:

- (60) a. Are there cookies in the box?
 - b. (Yes,) or chocolates.

— (Yes,) there are cookies in the box or chocolates.

- (61) a. Are there cookies or chocolates in the box?
 - b. (Yes,) there are cookies or chocolates in the box.

3.2.4 Qualified Answers and Qualified Interrogatives

In this section we briefly indicate that certain aspects of the meanings of answers and of interrogatives are best considered to be not part of their semantic content in the truth-conditional sense, but *qualifications* thereof.

Some examples:

- (62) a. Who walks?b. John, I believe.
 - John walks, I believe.
- (63) a. Does John walk?
 - b. (Yes,) I believe so.
 - (No,) I believe not.
 - If Mary walks, I believe.
 - John walks if Mary walks, I believe.

And some more:

- (64) a. Who walks?
 - b. John, obviously.
 - John, maybe.
 - John, of course.
- (65) a. Does John walk?
 - b. Possibly, yes.
 - Maybe so.
 - Certainly not.

Interrogatives, too, can be qualified so as to express an epistemic or doxastic attitude. Interesting examples are *negative* sentential interrogatives:

(66) a. Doesn't John walk? b. — No.

Application of Rule 5 to the zero-place relation \neg walk(a)(j) and (the exhaustiviza-

tion of) no predicts that (66b) means that John walks. So, the negation is not part of the semantic content, but qualifies the question being asked: the questioner expects a negative reply.

Some support comes from the following observations.

Positive answers to negative interrogatives are marked, as are negative answers to positively marked interrogatives:

- (67) a. Doesn't John walk?
 - b. But yes, he does!
- (68) a. John does walk, doesn't he?
 - b. Yes.
 - c. But no, he doesn't!

Interrogatives can be marked in other ways, too:

- (69) Does John come, perhaps?
- (70) Do you have a pen, by any chance?

Cf. also the following contrast:

- (71) a. Are you not happy?
 - b. No. (= I am not happy)
 - But yes, I am. (= I am happy)
- (72) a. Are you unhappy?

b. — No. (= I am happy) — Yes. (= I am unhappy)

3.3 Linguistic Answers and Answerhood

This section is devoted to an application of the abstract theory of answerhood relations that was developed in Chapter 2, to interrogatives and their characteristic answers. The main point to be illustrated is that although answerhood is a relation which may hold between any question and any (contingent) proposition given some suitable information set, characteristic linguistic answers have certain properties which guarantee that certain answerhood relations will obtain between the question expressed by an interrogative and the proposition they express in the context of that interrogative. This provides a (partial) explanation of why these answers form a natural class.

As we shall see, two factors may be involved in determining connections between properties of linguistic answers and relations of answerhood: (i) the contextdependent interpretation of an answer; and (ii) properties of the constituent involved.

We first study semantic notions of answerhood (Section 3.3.1), and then turn to the pragmatic counterparts thereof (Section 3.3.2). For expository reason we first deal with answers to single constituent interrogatives, leaving the generalization to the multiple and zero-constituent case for the final Section 3.3.3.

3.3.1 Linguistic Answers and Semantic Answerhood

A proposition which is a semantic answer to a single-constituent question identifies one of the possible denotations of the related property. This means that it is:

- 1. exhaustive
- 2. rigid
- 3. definite

These notions are defined as follows:

Definition 35 α is exhaustive iff it holds for all *i* that if $X \in [\![\alpha]\!]_i$, then there is no $Y \in [\![\alpha]\!]_i$ such that $X \subset Y$.

Definition 36 1. α is rigid with respect to *i* and *j* iff $[\![\alpha]\!]_i = [\![\alpha]\!]_j$. 2. α is rigid iff α is rigid with respect to all *i* and *j*.

Definition 37 1. α is definite with respect to *i* iff there exists an $X \in [\![\alpha]\!]_i$ such that for all $Y \in [\![\alpha]\!]_i, X \subseteq Y$.

2. α is definite iff α is definite with respect to all *i*.

Fact 30 For all terms α , EXH(α) is exhaustive.

Fact 31 The following are rigid terms:

- 1. proper names; unrestricted quantifiers everyone, someone, no-one; quantifiers restricted by means of rigid properties
- 2. conjunctions, disjunctions, negations, exhaustivizations of rigid terms

Fact 32 The following are definite terms:

- 1. proper names; universally quantified terms; definite descriptions
- 2. conjunctions and exhaustivizations of definite terms

Rule 5 guarantees that an answer is exhaustive, but this is not sufficient:

- (73) a. Who walk(s)?
 - b. The ones who walk.

Since (73b) is a tautology it bears no answerhood relation to (73a).

Assuming a rigid designator treatment of proper names, the following answer is exhaustive and rigid:

(74) a. Who walk(s)?

b. — John or Bill.

This is only a partial answer.

The following is both exhaustive, rigid, and definite:

(75) a. Who walk(s)?

b. — John and Bill.

The following fact holds:

Fact 33 Let β be a one-place interrogative, and α a one-place term. Let β'_r be the interpretation of β as a relation, and β'_q its interpretation as a question, α' the interpretation of α . If α is exhaustive, rigid and definite, and $[\![\alpha'(\beta'_r)]\!]$ is not a contradiction, then $[\![\lambda a(\alpha'(\beta'_r))]\!]$ is a (complete) semantic answer to β'_q .

Obviously, in virtue of Rule 5 the case where α is a non-exhaustive term that is used as an answer to β , is a subcase of Fact 33.

The following counterexample shows that the reverse of Fact 33 does not hold:

(76) a. Which prime number did John write on the blackboard?

b. — An even number.

According to Fact 33, (exhaustive,) rigid, and definite terms always are answers. To give an answer, a proposition must contain additional, contingent information, and hence cannot be derived from a term which is rigid and definite.

There are properties connected with giving, instead of being, an answer:

Definition 38 A term α is semi-rigid iff for all *i* and *j* either α is rigid with respect to *i* and *j* or $[\![\alpha]\!]_i = \emptyset$.

Definition 39 A term α is semi-definite iff for all *i* either α is definite with respect to *i* or $[\![\alpha]\!]_i = \emptyset$.

Now we can state the following fact:

Fact 34 If α is exhaustive, semi-rigid, and semi-definite, and $[\![\alpha'(\beta'_r)]\!]$ is not a contradiction, then $[\![\lambda a(\alpha'(\beta'_r))]\!]$ gives a (complete) semantic answer to β'_q .

Consider the following example:

- (77) a. Who kissed Mary?
 - b. John, who really loves her.
 - c. John is the one who kissed Mary, and John really loves Mary

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d. — John is the one who kissed Mary

The difference between complete and partial answers is the property of definiteness:

Fact 35 If α is rigid and exhaustive, and $[\![\alpha'(\beta'_r)]\!]$ is a contingency, then $[\![\lambda a(\alpha'(\beta'_r))]\!]$ is a partial semantic answer to β'_q .

Fact 36 If α is semi-rigid and exhaustive, and $[\![\alpha'(\beta'_r)]\!]$ is a contingency, then $[\![\lambda a(\alpha'(\beta'_r))]\!]$ gives a partial semantic answer to β'_q .

The various facts are summed up in Table 3.1.

	be an answer	give an answer
complete	rigid	semi-rigid
	definite	semi-definite
partial	rigid	semi-rigid

Table 3.1: Properties of terms and notions of answerhood

3.3.2 Linguistic Answers and Pragmatic Answerhood

Pragmatic notions of answerhood are restrictions, to an information set J, of the various semantic notions. Pragmatic analogues of the correlations between properties of terms and semantic notions of answerhood are obtained by restricting the former to information sets:

Definition 40 A term α is exhaustive in J iff it holds for all j in J that if $X \in [\![\alpha]\!]_j$, then there is no $Y \in [\![\alpha]\!]_j$ such that $X \subset Y$.

Definition 41 A term α is rigid in J iff α is rigid with respect to all j and k in J.

Definition 42 A term α is definite in J iff α is definite with respect to all j in J.

In a similar way we obtain notions of semi-rigidness in J and of semi-definiteness in J.

We then observe the following facts:

Fact 37 If α is exhaustive, rigid and definite in J, and $[\![\alpha'(\beta'_r)]\!]$ is compatible with J, then $[\![\lambda a(\alpha'(\beta'_r))]\!]$ is a (complete) pragmatic answer to β'_q in J.

Fact 38 If α is exhaustive, semi-rigid, and semi-definite in J, and $[\![\alpha'(\beta'_r)]\!]$ is compatible with J, then $[\![\lambda a(\alpha'(\beta'_r))]\!]$ gives a (complete) pragmatic answer to β'_q .

Fact 39 If α is rigid and exhaustive in J, and $[\![\alpha'(\beta'_r)]\!]$ is compatible with J but not included in J, then $[\![\lambda a(\alpha'(\beta'_r))]\!]$ is a partial pragmatic answer to β'_q .

Fact 40 If α is semi-rigid and exhaustive, and $[\![\alpha'(\beta'_r)]\!]$ is compatible with J but not included in J, then $[\![\lambda a(\alpha'(\beta'_r))]\!]$ gives a partial pragmatic answer to β'_q .

Some examples to illustrate this. First, pragmatic rigidness:

(78) a. Whom did you talk to?

b. — Your father.

Another example of pragmatic rigidness:

- (79) a. Who won the Tour de France in 1980?
 - b. The one who ended second in 1979.

Finally, pragmatic definiteness:

- (80) a. Who served you when you bought these boots?
 - b. An elderly lady wearing glasses.

As was the case with Fact 33, the reverse of Fact 37 (and the others, of course) does not hold. Consider:

- (81) a. ¿From which authors did the editors already receive their contribution to the proceedings?
 - b. ¿From whom did the organizers already receive a letter of acceptance to attend the conference?
 - c. (I don't know, but) at least from Professor A.

3.3.3 Multiple- and Zero-Constituent Answers and Answerhood

Here we consider the n-constituent case, and note some nice results concerning answers to sentential interrogatives.

The generalization of the notions of exhaustiveness, (semi-)rigidness, and (semi-)definiteness, and their relativized counterparts, is straightforward: replace X, which ranges over sets of individuals, by \mathbb{R}^n , which ranges over sets of *n*-tuples of individuals. The relevant facts also generalize in a completely straightforward way.

Finally, two observations with respect to answers to sentential interrogatives. First of all, yes and no as defined in (53) are exhaustive, rigid, and definite,

which explains why these phrases are standard answers to sentential interrogatives. Secondly, phrases of the form $if \phi$ are always definite but not always rigid.

Those that are function in the same as yes and no:

- (82) a. Will you come to the party?
 - b. If 2+2 = 4.
 - c. If 2+2 = 5.
 - d. If Mary comes to the party.

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Chapter 4

Coordinating Interrogatives

This chapter is devoted to an analysis of coordination of interrogatives and related phenomena. Section 4.1 discusses two of these: the phenomenon of *pair-list*-readings and that of *choice*-readings of interrogatives, and shows that they are intimately tied to coordination. Both phenomena require an extension of the core theory developed in the previous chapters. Section 4.2 shows how an analysis of pair-list readings can be provided in a simple extension of the core theory. Section 4.3 deals with disjunction and choice-readings, and shows that a proper analysis of them requires a higher level of analysis of interrogatives. Section 4.4, finally, is concerned with another phenomenon, that of *mention-some* interpretations.

4.1 Some Phenomena

4.1.1 Pair-list Readings

An example:

- (1) a. Which student was recommended by each professor?
 - b. Direct reading: — John.

— John was recommended by each professor.

c. Pair-list reading:
 — Professor Jones, Bill; professor Williams, Mary; and professor Peters, John.

— Professor Jones recommended Bill, professor Williams recommended Mary, and professor Peters recommended John.

Compare the pair-list reading of (1) with the two-constituent interrogative:

(2) Which professor recommended which student?

The ambiguity also arises with embedded interrogatives, which are in fact three ways ambiguous:

- (3) John knows which student was recommended by each professor
- (4) a. Direct reading:
 - John knows which student is such that each professor recommended him

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- b. Pair-list reading:
 - John knows which professor recommended which student
- c. 'Quantificational' reading:

Of each professor, John knows which student was recommended by him The difference between (4b) and (4c) is that (4b) requires *de dicto* knowledge about who the professors are.

The connection with conjunction is quite obvious in the following example:

- (5) a. Whom do John and Mary love?
 - b. Direct reading:
 - Suzy.
 - Suzy is (the one who is) loved by John and Mary.
 - c. Pair-list reading:
 - John, Suzy; and Mary, Suzy and Bill.
 - John loves Suzy, and Mary loves Suzy and Bill.

Cf. the following conjunction of interrogatives:

(6) Whom does John love? And, whom does Mary love?

Conclusion: pair-list readings have the following characteristics:

- 1. they are answered like two-constituent interrogatives
- 2. they involve de dicto interpretation of the NP
- 3. they are connected with conjunction

4.1.2 Choice Readings

Choice readings are important because they seem to contradict an essential feature of the core theory: viz., that every question has a unique answer. An example:

- (7) a. Whom does John or Mary love?
 - b. Direct reading:
 - Suzy and Bill.

— Suzy and Bill (are the ones that) are loved by John or Mary.

- c. Choice reading:
 - i. John, Suzy.
 - John loves Suzy.
 - ii. Mary, Suzy and Bill.
 - Mary loves Suzy and Bill.

Cf. the following disjunction of interrogatives:

(8) Whom does John love? Or, whom does Mary love?

Another example:

- (9) a. What did two of John's friends give him for Christmas?
 - b. Direct reading:
 - A watch.

c. Choice reading:

— Bill, a watch and a ball; Peter, a book and a pen.

— Bill gave him a watch and a ball, and Peter gave him a book and a pen

Again, the ambiguity also shows up with embedded interrogatives:

- (10) a. John knows whom two girls love
 - b. Direct reading: John knows who is such that two girls love him

- c. *Choice reading*: John knows of two girls, that they are girls and whom they love
- d. 'Quantificational reading': Of two girls, John knows whom they love

Again, the difference between the choice reading and the quantificational reading is that the latter does not, and the former does involve de dicto knowledge about girls. So choice readings share the first two characteristrics of pair-list readings. and they are connected with disjunction of interrogatives. Notice that whether a pair-list reading a choice reading or neither occurs is determined by the semantic characteristics of the term involved. Cf.:

- (11) a. Which student did each professor recommend? (pair-list)
 - b. Which student did two professors recommend? (choice)
 - c. Which student did no professor recommend? (neither)

Summing up:

- 1. On its choice reading an interrogative is associated with more than one question, and, for that reason, has more than one complete and true semantic answer.
- 2. Both pair-list readings and choice readings are a matter of scope, and both induce an n+1-constituent interpretation of what superficially is an n-constituent interrogative.
- 3. Both pair-list readings and choice readings are preserved under complement embedding verbs.
- 4. Whether a pair-list reading or a choice reading results when we assign a term wide scope with respect to a *wh*-phrase, depends on the semantic properties of the term.

4.1.3 Mention-some interpretations

Other cases of interrogatives which seem to have more than one semantic answer: *mention-some* interpretations. Example:

- (12) a. Where do they sell Italian newspapers in Amsterdam?
 - b. Who has got a light?
 - c. Where can I find a pen?

Notice that depending on the context exhaustive ('mention-all') answers are possible too. Most important issue: is this a separate reading, i.e, a matter of semantics, or is this a pragmatic phenomenon? (See Section 4.4.) In any case, mention-some interpretations differ from choice readings. Two observations support this claim. First of all, they are answered differently:

(13) — At the Central Railway Station.

- At the Central Railway Station they sell Italian newspapers.

Secondly, mention-some interpretations also occur with universal and negative terms:

- (14) Where do they have all books written by Nooteboom in stock?
- (15) On which route to Rotterdam is there likely to be no police control?

4.2 Pair-list readings

Adding pair-list readings requires only a minor extension of the core theory.

The derivation of a two-constituent interrogative. Take the one-place abstract:

(16) a. whom he_0 loves

b. $\lambda y[\texttt{love}(a)(x_0, y)]$

Introduce the wh-term which man to get the two-place abstract:

- (17) a. whom which man loves
 - b. $\lambda x_0[\max(a)]\lambda y[\operatorname{love}(a)(x_0, y)]$, which is equivalent to:

c. $\lambda x \lambda y[\operatorname{man}(a)(x) \wedge \operatorname{love}(a)(x,y)]$

Turn this into the interrogative:

(18) a. Whom does which man love? b. $\lambda i [\lambda x \lambda y [\operatorname{man}(a)(x) \wedge \operatorname{love}(a)(x, y)] = \lambda x \lambda y [\operatorname{man}(i)(x) \wedge \operatorname{love}(i)(x, y)]]$

Pair-list readings. Consider:

- (19) a. Whom do John and Mary love?

 - b. $\lambda x \lambda y[[x = \mathbf{j} \lor x = \mathbf{m}] \land \mathbf{love}(a)(x, y)]$ c. $\lambda i[\lambda x \lambda y[[x = \mathbf{j} \lor x = \mathbf{m}] \land \mathbf{love}(a)(x, y)] = \lambda x \lambda y[[x = \mathbf{j} \lor x = \mathbf{m}] \land$ love(i)(x,y)||

Conclusion: like two-constituent interrogatives, pair-list readings are cases of restricted λ -abstraction, the term delivering the property which functions as restriction.

General procedure for extracting the required property from the term:

Definition 43 Let α be a pair-list term. Then $\text{LIVE}(\alpha) = \lambda a \lambda x \forall P[\alpha(P) \rightarrow P(a)(x)]$

Then the following generalization of the rule for forming abstracts suffices:

Regel 6 If α is an NP, translating as α' , and β is an *n*-place abstract, translating as β' , then the n+1-place abstract formed from them translates as $\lambda x_n [LIVE(\alpha')(\alpha)]\beta'$

An example. Combining (16b) with the NP every man using Rule 6 gives:

(20) $\lambda x_0 [LIVE(\lambda P \forall x [man(a)(x) \rightarrow P(a)(x)])(a)] \lambda y [love(a)(x_0, y)]$

Application of Definition 43 gives:

- (21) a. $(\lambda a \lambda x \forall P [\forall x [man(a)(x) \rightarrow P(a)(x)] \rightarrow P(a)(x)])(a)$, which is equivalent
 - b. $(\lambda a \lambda x [man(a)(x)])(a)$

Whence (20) is equivalent to:

(22) $\lambda x_0 \lceil \operatorname{man}(a) \rceil \lambda y \lceil \operatorname{love}(a)(x_0, y) \rceil$

The entire core theory, including pair-list readings, consists of three rules.

- 1. The rule which turns an *n*-place abstract and a term into an n+1-place abstract (Rule 6).
- 2. The rule which turns an *n*-place abstract into an interrogative (Rule 2 from Chapter 2).
- 3. The rule which turns an *n*-place abstract and an *n*-place term into a characteristic linguistic answer (Rule 5 from Chapter 3).

4.3Choice readings

A characteristic of the core theory is that questions have a unique complete semantic answer at an index. An interrogative on a choice reading has more than one such

answer. So? So we conclude that such interrogatives express more than one question (and not that questions may have more than one answer). Again, an extension of the core theory is called for.

4.3.1 Type-shifting

In this section we add two (intensional) type-shifting principles. (See Partee and Rooth 1983; Hendriks 1988; Groenendijk and Stokhof 1988 for more rules and more discussion). This extension will enable us to deal with interrogatives which are associated with more than one question. Consider the following simple examples:

(23) a. Whom does John love? And, whom does Mary love?b. Whom does John love? Or, whom does Mary love?

Generalized conjunction and disjunction predict the following interpretations:

(24) a. $\lambda i[\lambda x[love(a)(j, x)] = \lambda x[love(i)(j, x)] \land \lambda x[love(a)(m, x)] = \lambda x[love(i)(m, x)]]$ b. $\lambda i[\lambda x[love(a)(j, x)] = \lambda x[love(i)(j, x)] \lor \lambda x[love(a)(m, x)] = \lambda x[love(i)(m, x)]]$

(24a) expresses a question, i.e., an equivalence relation on the set of indices I, but (24b) does not: it is reflexive and symmetric, but not transitive.

The solution that suggests itself is to lift interrogatives to the level of expressions denoting sets of properties of questions. We do so by application of the operation LIFT:

Definition 44 If α is of type a, then $\text{LIFT}(\alpha)$ is of type $\langle \langle s, \langle \langle s, a, \rangle, t, \rangle \rangle, t \rangle$. If α translates as α' , then $\text{LIFT}(\alpha)$ translates as $\lambda X_{\langle s, \langle \langle s, a \rangle, t \rangle}[X(a)(\lambda a \alpha')]$.

Applied to interrogatives ϕ from the core theory:

(a) LIFT(ϕ) translates as $\lambda Q[Q(a)(\lambda a \phi')]$

where Q is a variable of type $\langle s, \langle \langle s, \langle s, t \rangle \rangle, t \rangle \rangle$. In accordance with Schema (a) we obtain for (23a,b):

(25) a. $\begin{aligned} \lambda Q[Q(a)(\lambda a\lambda i[\lambda x[\mathsf{love}(a)(\mathbf{j}, x)]] &= \lambda x[\mathsf{love}(i)(\mathbf{j}, x)]]) \wedge \\ Q(a)(\lambda a\lambda i[\lambda x[\mathsf{love}(a)(\mathbf{j}, x)]] &= \lambda x[\mathsf{love}(i)(\mathbf{j}, x)]])]] \\ \text{b.} \quad \lambda Q[Q(a)(\lambda a\lambda i[\lambda x[\mathsf{love}(a)(\mathbf{j}, x)]] &= \lambda x[\mathsf{love}(i)(\mathbf{j}, x)]]) \vee \\ Q(a)(\lambda a\lambda i[\lambda x[\mathsf{love}(a)(\mathbf{j}, x)]] &= \lambda x[\mathsf{love}(i)(\mathbf{j}, x)]])] \end{aligned}$

We check this result with the appropriate notion of entailment, provided by the generalized entailment schema:

Definition 45 An interrogative Φ entails an interrogative Ψ iff $\forall a \forall Q [\Phi(Q) \rightarrow \Psi(Q)]$

where Φ , Ψ are of type $\langle \langle s, \langle s, t \rangle \rangle, t \rangle$. According to Definition 45, (16a) entails each of its conjuncts, and its of its disjuncts entails (16b). For embedded interrogatives we need the operation of argument-lifting:

Definition 46 Let c be a conjoinable type. If α is of type $\langle a, c \rangle$, then ARG-LIFT (α) is of type $\langle \langle \langle s, \langle \langle s, a \rangle, t \rangle \rangle, t \rangle, c \rangle$. If α translates as α' , then ARG-LIFT (α) translates as $\lambda X_{\langle \langle s, a \rangle, t \rangle \rangle, t \rangle}$ [QUANT $(X, y_a, \alpha'(y))$], where

$$QUANT(X, y, \delta) = \begin{cases} X(\lambda y \delta) & \text{if } \delta \text{ is of type } t \\ \lambda x_d[QUANT(X, y, \delta(x_d))] & \text{if } \delta \text{ is of type } \langle d, f \rangle \end{cases}$$

Distribution of coordinated interrogatives over extensional verbs is guaranteed, since the following is universally valid (Q is a variable of type $\langle s, \langle \langle s, \langle s, t \rangle \rangle, t \rangle \rangle$:

(26) $\forall i \forall x \forall \mathcal{Q}[(\text{ARG-LIFT}(\texttt{know}))(i)(x, \mathcal{Q}) = \mathcal{Q}(i)(\lambda a \lambda q[\texttt{know}(a)(x, q(a))])]$ Intensional verbs are lexically typed on the higher level. This guarantees that no distribution over them takes place. But notice that all interrogatives which the core theory deals with denote sets of properties of unique questions. In those cases reduction without meaning postulates goes through. So for such \mathcal{Q} the following holds for example:

(27) $\forall i \forall x \forall \mathcal{Q}[\texttt{wonder}(i)(x, \mathcal{Q}) = \mathcal{Q}(i)(\lambda a \lambda q[(\texttt{ARG-LOW}(\texttt{wonder}))(a)(x, q(a))])]$

where ARG-LOW indicates the type-shifting operation of 'argument-lowering', which we do not bother to define here.

We now illustrate the effects of adding type-shifting to the core theory by investigating what happens on the level of lifted interrogatives with:

- 1. answerhood relations
- 2. entailment
- 3. linguistic answers

All facts concerning answerhood relations carry over to lifted interrogatives. We treat one case, that of a proposition giving a complete and true semantic answer (the other cases are completely analogous). It is convenient to define an objectlanguage expression to denote this relation:

Definition 47 ans(a)(p,q) iff $\forall i[p(i) \rightarrow q(a)(i)]$

Next, we define its analogue on the lifted level:

Definition 48 ANS(a)(p, Q) iff $Q(a)(\lambda a \lambda q[ans(a)(p, q)])$

We observe the following fact:

Fact 41 For all *i* and *p*: $ans(i)(p, \phi)$ iff $ANS(i)(p, LIFT(\phi))$

As for answerhood of disjunctive interrogative, Definition 48 predicts such results as:

(28) ANS(a)(p, $\lambda Q[Q(a)(\lambda a \lambda i[\lambda x[love(a)(j, x)] = \lambda x[love(i)(j, x)]]) \lor Q(a)(\lambda a \lambda i[\lambda x[love(a)(j, x)] = \lambda x[love(i)(j, x)]])]]$ if and only if ans(a)(p, $\lambda a \lambda i[\lambda x[love(a)(j, x)] = \lambda x[love(i)(j, x)]]) \lor$ ans(a)(p, $\lambda a \lambda i[\lambda x[love(a)(j, x)] = \lambda x[love(i)(j, x)]])$

With respect to entailment, differences exists between the core level analysis of interrogatives and their lifted analogues. Notice that the strong connection between entailment and answerhood is loosened:

Fact 42 ϕ entails ψ iff $\forall i \forall p[ans(i)(p, \lambda a \phi) \rightarrow ans(i)(p, \lambda a \psi)]$

Fact 43 If Φ entails Ψ then $\forall i \forall p [ANS(i)(p, \Phi) \rightarrow ANS(i)(p, \Psi)]$

The reverse of Fact 43 does not hold. Given Fact 42 this means that not all facts concerning entailment carry over. An example of something we loose is the prediction that (29) entails (30):

- (29) Who walks?
- (30) Does John walk?

Conclusion: we cannot confine ourselves to the higher level analysis, both levels of analysis are needed.

Finally, linguistic answers. According to Definition 44, the following result obtains for *n*-place abstracts β (R^n ranges over properties of *n*-place relations): (b) LIFT(β) translates as $\lambda R^n [R^n(a)(\lambda a \beta')]$

The rule for turning lifted abstracts into lifted interrogatives becomes $(r^n \text{ ranges} over n-\text{place relations})$:

Regel 7 Let β' be the translation of a lifted *n*-place abstract. Then the translation of the corresponding lifted interrogative is $\lambda Q[\beta'(\lambda a \lambda r^n[Q(a)(\lambda a \lambda i[r^n(a) = r^n(i)])])]$

The rule for deriving answers becomes:

Regel 8 Let β' be the translation of a lifted *n*-place abstract β and α' the translation of an *n*-place term α . Then the answer expressed by α in the context of β is $\beta'(\lambda a[\text{EXH}^n(\lambda a \alpha')])$

A simple example:

- (31) a. whom John loves
 - b. $\lambda R^1[R^1(a)(\lambda a \lambda x[\texttt{love}(a)(j, x)])]$
 - c. $\forall x [\texttt{love}(a)(j, x) \leftrightarrow x = s]$
 - d. Whom does John love?
 - e. Suzy.

(31a) is the abstract underlying (31d), (31b) its lifted translation, (31c) the result of combining (31b) with the term Suzy using Rule 8, which is what (31e) means in the context of (31d).

4.3.2 Choice Readings

An implementation of choice readings uses both extensions of the core theory developed sofar: a lifted generalized version of Rule 6.

Relevant observations:

- 1. On a choice reading an interrogative poses more than question.
- 2. The questions are the result of combining the same relation with several properties.
- 3. These properties are determined by the term. This leads to:

Regel 9 $\lambda R^{n+1}[\beta'(\lambda \alpha \lambda r^n[\exists P[CHOICE(\alpha')(P) \land R^{n+1}(a)(\lambda \alpha \lambda x_k[P(a)]r^n(a))])]$

As before, r^n ranges over *n*-place relations, and R^n over properties of *n*-place relations. Needed: a definition of CHOICE. Consider:

- (32) a. Whom does John or Mary love?
 - b. John, Suzy.
 - c. Mary, Suzy and Bill.
 - d. John, Suzy; and Mary, Suzy and Bill.

(32a) can be derived from:

(33) $\lambda R^2[R^2(a)(\lambda a \lambda x \lambda y[x = j \land love(a)(x, y)]) \lor R^2(a)(\lambda a \lambda x \lambda y[x = m \land love(a)(x, y))]$

In order to derive this we need the following two relations:

(34) a. $\lambda x_0 \lceil \lambda x [x = j] \rceil \lambda y [\texttt{love}(a)(x_0, y)]$ b. $\lambda x_0 \lceil \lambda x [x = m] \rceil \lambda y [\texttt{love}(a)(x_0, y)]$

And these can be obtained from:

(35) a. $\lambda y[\text{love}(a)(x_0, y)])]$

- b. $\lambda x[x = \mathbf{j}]$
- c. $\lambda x[x = m]$

Another example:

- (36) a. Whom do two girls love?
 - b. Mary, Bill and Suzy; and Hilary, Peter.

One of the relations underlying (36a) should be:

(37)
$$\lambda x_0 \lceil \lambda x[\texttt{girl}(a)(x) \land [x = \texttt{m} \lor x = \texttt{h}]] \rceil \lambda y[\texttt{love}(a)(x_0, y)]$$

So generally we should have:

(38) $\lambda x_0 \lceil \lambda x[girl(a)(x) \land [x = g_1] \lor x = g_2] \rceil \lambda y[love(a)(x_0, y)]$

for every two girls g_1, g_2 . Notice that we get de dicto readings this way.

- In order to determine the choice properties of α we need:
- 1. the set of properties of being an element of a minimal element of α 2. the property on which α lives

A choice property is the conjunction of a property in 1 with the property in 2. We generalize the definition of LIVE:

Definition 49 Let α be a pair-list or a choice term. Then: LIVE $(\alpha) = \lambda a \lambda x \exists P[\text{EXH}(\alpha)(P) \land P(a)(x)]$

And we define the operation CHOICE:

Definition 50 CHOICE(α) = $\lambda P \exists X [EXH(\alpha)(\lambda a X) \land P = \lambda a \lambda x [X(x) \land LIVE(\alpha)(x)]]$

This implements Rule 9.

One worked-out example.

- (39) a. whom he_0 loves
 - b. $\lambda R^1[R^1(a)(\lambda a \lambda y[\texttt{love}(a)(x_0, y)])]$
- (40) a. two girls
 - b. $\lambda P \exists x \exists y [x \neq y \land girl(a)(x) \land girl(a)(y) \land P(a)(x) \land P(a)(y)]$

Combining these with Rule 9 we get:

(41) a. $\lambda R^2[(39b)(\lambda a \lambda r^1[\exists P[CHOICE(40)(P) \land$

 $R^{2}(a)(\lambda a\lambda x_{0}[P(a)]r^{1}(a))])]$, which can be reduced to:

b. $\lambda R^2[\exists P[CHOICE(40)(P) \land R^2(a)(\lambda a \lambda x_0 \lceil P(a) \rceil \lambda y[love(a)(x_0, y)])]$

Suppose there are three girls: Mary, Hilary , and Jane. Then:

(42) a. $EXH(40)(a) = \{\{m, h\}, \{m, j\}, \{h, j\}\}\$ b. $LIVE(40) = \lambda a \lambda x [girl(a)(x)]$

c. CHOICE(40) =

$$\{ \lambda a \lambda x [\texttt{girl}(a)(x) \land [x = \texttt{m} \lor x = \texttt{h}]], \\ \lambda a \lambda x [\texttt{girl}(a)(x) \land [x = \texttt{m} \lor x = \texttt{j}]], \\ \lambda a \lambda x [\texttt{girl}(a)(x) \land [x = \texttt{h} \lor x = \texttt{j}]] \}$$

So, in this situation, (41) is equivalent to:

(43)
$$\lambda R^{2}[R^{2}(a)(\lambda a \lambda x \lambda y[\texttt{girl}(a)(x) \land [x = \texttt{m} \lor x = \texttt{h}] \land \texttt{love}(a)(x, y)]) \lor R^{2}(a)(\lambda a \lambda x \lambda y[\texttt{girl}(a)(x) \land [x = \texttt{m} \lor x = \texttt{j}] \land \texttt{love}(a)(x, y)]) \lor R^{2}(a)(\lambda a \lambda x \lambda y[\texttt{girl}(a)(x) \land [x = \texttt{h} \lor x = \texttt{j}] \land \texttt{love}(a)(x, y)])]$$

Application of Rule 7 then results in:

(44) a.
$$\lambda Q[(41)(\lambda a \lambda r^2[Q(a)(\lambda a \lambda i[r^2(a) = r^2(i)])])]$$
, which reduces to:
b. $\lambda Q \exists P[CHOICE(40)(P) \land Q(a)(\lambda a \lambda i[\lambda x_0 \lceil P(a) \rceil \lambda y[love(a)(x_0, y)] = \lambda x_0 \lceil P(i) \rceil \lambda y[love(a)(x_0, y)]])]$

In the given situation, this denotes:

$$\begin{array}{ll} (45) & \lambda Q[Q(a)(\lambda a\lambda i[\lambda x\lambda y[\texttt{girl}(a)(x) \land [x=\texttt{m} \lor x=\texttt{h}] \land \texttt{love}(a)(x,y)] = \\ & \lambda x\lambda y[\texttt{girl}(i)(x) \land [x=\texttt{m} \lor x=\texttt{h}] \land \texttt{love}(i)(x,y)]]) \lor \\ & Q(a)(\lambda a\lambda i[\lambda x\lambda y[\texttt{girl}(a)(x) \land [x=\texttt{m} \lor x=\texttt{j}] \land \texttt{love}(a)(x,y)] = \\ & \lambda x\lambda y[\texttt{girl}(i)(x) \land [x=\texttt{m} \lor x=\texttt{j}] \land \texttt{love}(i)(x,y)]]) \lor \\ & Q(a)(\lambda a\lambda i[\lambda x\lambda y[\texttt{girl}(a)(x) \land [x=\texttt{h} \lor x=\texttt{j}] \land \texttt{love}(a)(x,y)] = \\ & \lambda x\lambda y[\texttt{girl}(i)(x) \land [x=\texttt{h} \lor x=\texttt{j}] \land \texttt{love}(a)(x,y)] = \\ & \lambda x\lambda y[\texttt{girl}(i)(x) \land [x=\texttt{h} \lor x=\texttt{j}] \land \texttt{love}(i)(x,y)]]) \end{aligned}$$

We observe that an interrogative on a choice reading:

- 1. is associated with more than one question
- 2. functions as a two-constituent interrogative
- 3. is interpreted de dicto

which were the requirements we formulated earlier.

- To finish off, we analyse the following answer to (36a):
- (46) Hilary, Peter; and Jane, Suzy.

Notice that in the context of (36a) this answer expresses that Hilary is a girl. The answer is based on the following two-place term:

(47) $\lambda R^2[R^2(a)(\mathbf{h},\mathbf{p}) \wedge R^2(a)(\mathbf{j},\mathbf{s})]$

Application of Rule 8 gives:

(48) (41)($\lambda a[EXH(\lambda a[(47)])])$

The exhaustivization of the two-place term is:

(49) $\lambda R^2 \forall x \forall y [R^2(a)(x,y) \leftrightarrow [[x = \mathbf{h} \land y = \mathbf{p}] \lor [z = \mathbf{j} \land y = \mathbf{s}]]]$

So we get:

 $\begin{array}{lll} (50) & \text{a.} & \exists P[\text{EXH}(40)(P) \land \forall x \forall y[[P(a)(x) \land \texttt{love}(a)(x,y)] \leftrightarrow \\ & & [[x = \texttt{h} \land y = \texttt{p}] \lor [z = \texttt{j} \land y = \texttt{s}]]]], \text{ which is equivalent to:} \\ & \text{b.} & \forall x \forall y[[\texttt{girl}(a)(x) \land [x = \texttt{h} \lor x = \texttt{j}] \land \texttt{love}(a)(x,y)] \leftrightarrow \\ & & [[x = \texttt{h} \land y = \texttt{p}] \lor [z = \texttt{j} \land y = \texttt{s}]]] \end{array}$

Conclusion. The core theory, including pair-list readings, consists of three rules:

- 1. The rule which turns an n-place abstract and a term into an n+1-place abstract (Rule 6).
- 2. The rule which turns an *n*-place abstract into an interrogative (Rule 2 from Chapter 2).
- 3. The rule which turns an *n*-place abstract and an *n*-place term into a characteristic linguistic answer (Rule 5 from Chapter 3).

In order to deal with choice readings it suffices:

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- 1. To add two type-shifting principles, viz.:
 - (a) Lifting (Definition 44).
 - (b) Argument-lifting (Definition 46).
- 2. To define a lifted and generalized version of the rule which turns an n-place abstract and a term into an n+1-place abstract (Rule 9).
- 3. To define a lifted version of the rule which turns an *n*-place abstract into an interrogative (Rule 7).
- 4. To define a lifted version of the rule which turns an *n*-place abstract and an *n*-place term into a characteristic linguistic answer (Rule 8).

4.4 Mention-Some Interpretations

In this section we treat mention-some interpretations of interrogatives. In Section 4.4.1 we outline a possible pragmatic analysis, and show why it will not work. Section 4.4.2. gives a semantic analysis, which appears to be quite successfull. In Section 4.4.3, however, some problems for the semantic approach are indicated.

Mention-some interpretations differ from choice readings. Both are associated with more than one complete semantic answer. However, these answers are of a different nature. Cf.:

- (51) a. Where is a pen?
 - b. On my desk.

This answer is typically that to a one-constituent interrogative. Cf. also:

- (52) a. John knows where a pen is
 - b. John knows a place where a pen is
 - c. John knows of a place where a pen is, that there is a pen there
 - d. John knows of a pen where that pen is

Only (52b) and (52c) are paraphrases of (52a), (52d) is not.

4.4.1 Problems for a Pragmatic Approach

Suggestion: the mention-some interpretation can be derived by pragmatic reasoning, which infers from the context that an interrogative does not call for a complete, i.e., an exhaustive answer, but only for a partial one. This keeps the semantics of interrogatives uniform, but makes answers ambiguous: do, or do not exhaustify.

This does not work. For it requires that the following holds:

- ${\cal P}$ is a complete, true mention-some answer to ${\cal Q}$ at an index i iff ${\cal P}$ is
- a partial, true mention-all answer to Q at i

But this fails. Consider:

- (53) a. Where is a pen?
 - b. Not in the drawer.
 - c. Nowhere

(53b) is a partial mention-all answer to (53a), and (53c) is a complete mention-all answer. Both fail to be a complete mention-some answer.

Another argument against a pragmatic approach. The mention-all/mentionsome distinction is preserved under embedding:

- (54) John knows where a pen is
- (55) a. For all places where a pen is, John knows that there is a pen at that place

- b. For some places where a pen is, John knows that there is a pen at that place
- (56) John wonders where a pen is
- (57) a. John wants for all places where a pen is, to know whether there is a pen at that place
 - b. John wants for some place where a pen is, to know whether there is a pen at that place

The a- and b-paraphrases have distinct truth-conditions.

4.4.2 A Semantic Approach

Characteristics of the mention-some interpretation of an interrogative derived from a one-place abstract:

- 1. It behaves like a one-constituent interrogative.
- 2. It has more than one complete true semantic answer.
- 3. Each answer is a *positive* specification of an individual which has the property expressed by the abstract.

Illustration:

- (58) a. Who has a pen?
 - b. John.

The abstract and the term involved are:

- (59) a. who has a pen
 - b. $\lambda x[\exists y[pen(a)(y) \land has(a)(x,y)]]$, or lifted:
 - c. $\lambda R^1[R^1(a)(\lambda a \lambda x[\exists y[pen(a)(y) \land has(a)(x, j)]])]$
- (60) a. John b. $\lambda P[P(j)]$

The meaning of (58b) in the context of (58a) is:

(61) $\exists y [\texttt{pen}(a)(y) \land \texttt{has}(a)(y, j)]]$

Another rule for deriving interrogatives from abstracts:

Regel 10 $\lambda Q[\exists x[\beta'(x) \land Q(a)(\lambda a \lambda i[\beta'(x) = (\lambda a \beta'(x))(i)])]]$

Application of Rule 10 to the abstract (59) gives:

(62) $\lambda Q[\exists x[\exists y[pen(a)(y) \land has(a)(x,y)] \land$

 $Q(a)(\lambda a \lambda i [\exists y [pen(a)(y) \land has(a)(x,y)] = \exists y [pen(i)(y) \land has(i)(x,y)]])]]$

Notice that at an index at which nobody has a pen, this denotes the empty set. This accounts for the fact that in such a situation, the interrogative does not have a true mention-some answer.

Examples involving embedding. First an intensional verb:

- (63) a. John wonders who has a pen
 - b. wonder $(a)(j, \lambda a \lambda Q[\exists x [\exists y [pen(a)(y) \land has(a)(x, y)] \land has(a)(x, y)]) \land has(a)(x, y) \land$

 $Q(a)(\lambda a \lambda i [\exists y [\texttt{pen}(a)(y) \land \texttt{has}(a)(x,y)] = \exists y [\texttt{pen}(i)(y) \land \texttt{has}(i)(x,y)]])]]$

Decomposing wonder into want to know we get:

(64) a. want $(a)(j, \lambda a[\exists x[\exists y[pen(a)(y) \land has(a)(x, y)] \land$

 $\mathtt{know}(\mathtt{j},\lambda i[\exists y[\mathtt{pen}(a)(y) \wedge \mathtt{has}(a)(x,y)] = \exists y[\mathtt{pen}(i)(y) \wedge \mathtt{has}(i)(x,y)]])])$

b. John wants to know of someone who has a pen whether he has a pen which reduces to:

(65) a. want $(a)(j, \lambda a[\exists x[\exists y[pen(a)(y) \land has(a)(x, y)] \land know(j, \lambda a[\exists y[pen(a)(y) \land has(a)(x, y)])])$

b. John wants to know of someone who has a pen that he has a pen

Next an extensional verb:

- (66) a. John knows who has a pen
 - b. $\exists x [\exists y [\texttt{pen}(a)(y) \land \texttt{has}(a)(x,y)] \land$
 - $\texttt{know}(\texttt{j},\lambda i[\exists y[\texttt{pen}(a)(y) \land \texttt{has}(a)(x,y)] = \exists y[\texttt{pen}(i)(y) \land \texttt{has}(i)(x,y)]])]$
 - c. Of someone who has a pen, John knows whether he has a pen

In fact this reduces to:

- (67) a. $\exists x [\exists y [\texttt{pen}(a)(y) \land \texttt{has}(a)(x,y)] \land$
 - $\texttt{know}(\texttt{j},\lambda a[\exists y[\texttt{pen}(a)(y) \land \texttt{has}(a)(x,y)]])]$
 - b. Of someone who has a pen, John knows that he has a pen

Notice that if nobody has a pen and John knows this, (66) is false on its mentionsome interpretation but true on its mention-all reading. Observe furthermore:

- \circ The choice reading of (66) implies its mention-some reading.
- The mention-all reading of (66) implies the mention-some reading if there is someone who has a pen

Likewise we have for:

(68) Who has a pen?

- \circ If P gives a complete and true answer to the choice reading, it gives a complete and true answer to the mention-some reading.
- If P gives a complete and true answer to the mention-all reading, it gives a complete and true answer to the mention-some reading, except when P is the proposition that nobody has a pen
- $\circ~$ If P gives a complete and true answer to the mention-some reading, it gives a partial and true answer to the mention-all reading

4.4.3 Problems for a Semantic Approach

The semantic approach to mention-some interpretations faces some difficulties.

Main problem: the semantic approach predicts an ambiguity which in fact seems to occur only with verbs which take a human subject and which express a limited set of relations. Cf.:

- (69) a. What the average grade is depends on what grade each student got
 - b. Where you can get gas depends on what day it is
 - c. Does it matter where a pen is?
 - d. Who will come to the party is partly determined by who is invited

Here only mention-all readings are likely.

Particularly striking is the following example:

- (70) a. Where can I get gas around here?
 - b. That depends on what time it is.

The natural interpretation of the interrogative is the mention-some interpretation. But the anaphor in the answer refers back to the mention-all interpretation.

Finally, we note that some languages have the tendency to lexicalize mentionsome interpretations:

(71) a. ?Jan weet wie een vuurtje heeft

(John knows who has a light)

- b. better: Jan weet iemand die een vuurtje heeft (John knows someone who has a light)
- (72) a. ?Wat is een voorbeeld van een priemgetal? (What is an example of a prime number?)
 - b. better: Geef een voorbeeld van een priemgetal (Give an example of a prime number)

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