Dynamic Semantics

Copenhagen handouts

Jeroen Groenendijk
Martin Stokhof
Frank Veltman

ILLC / Department of Philosophy
University of Amsterdam

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Handout 1

1 Introductory remarks

1.1 Static vs. dynamic view

- Static view: meaning equals truth conditions
- Dynamic view: meaning is context-change potential

1.2 Influences

- Speech act theory
- Procedural semantics (AI, NLP)
- Stalnaker on presupposition and assertion
- Non-monotonic reasoning, defaults
- Gårdenfors on belief revision

1.3 Dynamic interpretation

- Kamp's discourse representation theory
- Heim's file change semantics
- Game-theoretical semantics (Hintikka); discourse semantics (Seuren)

1.4 Dynamic semantics

Difference between dynamic *interpretation* and dynamic *semantics:*

Dynamic interpretation Locates the dynamics in the interpretation process as such.

Dynamic semantics Places the dynamics of interpretation in the semantics proper.

Assumption:

> There is a difference between 'interpretation' and 'semantics'.

Characteristics of dynamic semantics:

- Meaning is a function from contexts to contexts ('update').
- Updates are defined recursively.
- Entailment is defined in terms of updates.

1.5 The nature of contexts

Assumption (unwarranted):

Language is primarily a tool for conveying information

Hence:

A context is an information state.

Information may concern various items, such as:

- Aspects of the world
- Aspects of the discourse
- Information of other speech participants

1.6 Modeling information states

An information state is modeled by a *set of possibilities:* those possibilities which are open according to the information state.

Growth of information is elimination of possibilities.

[Caveats.]

1.7 Applications

Empirical issues:

- Incremental interpretation of discourse, c.q. texts
- Pronominal coreference: donkey anaphora, intersentential anaphora (plurals, generalized quantifiers, symmetric and asymmetric quantification)
- Modalities and modal subordination
- Tense and aspect (temporal structure of texts)
- Presuppositions, topic / focus
- Default reasoning (non-monotonic reasoning)
- Questions and answers (text coherence)
- Relational nouns and implicit arguments

Theoretical issues:

- The semantics – pragmatics boundary
- Formal properties of dynamic systems (expressiveness, completeness)
- Relations with proof systems
- The nature of variables

2 Setting up this course

2.1 The nature of this course

What the course is not:

- A general overview of all the empirical issues and theoretical approaches.

Starting points:

- A specific descriptive area: the interaction of pronouns and modals.
- A specific theoretical approach: our own.

Characteristics:

- A mixture of basic and more advanced issues.
- Partly work in progress.

2.2 The contents of this course

Tentative schedule:

Week 1: core bussiness

Class 1. Introduction; outline of descriptive area.
Class 2. Framework 1: Modeling information states.
Class 3. Framework 2: Updates; semantic notions.
Class 4. Predicate logic: coreference.
Class 5. Modal predicate logic: modality and coreference.

Week 2: diversification

Class 6. Comparison with standard modal predicate logic.
Class 7. Modal subordination.
Class 8. Dialogue structure.
Class 9. Questions and answers.
Class 10. Free for all.

The material of the first week will be available in a course book which is on its way now from Amsterdam to Copenhagen.

3 Modality, coreference, and discourse

3.1 Modality

Order matters:

(1) a. It might be raining outside [...] It isn't raining outside.
b. $\Diamond p \wedge \ldots \wedge \neg p$

(2) a. It isn't raining outside [...] *It might be raining outside.
b. $\neg p \wedge \ldots \wedge \Diamond p$

Non-equivalence needed for propositional modal logic:

(3) $\Diamond\phi \wedge \neg\phi \not\equiv \neg\phi \wedge \Diamond\phi$

3.2 Coreference

Intersentential anaphora:

(4) a. A man walks in the park. He wears a blue sweater.
b. $\exists x Px \wedge Qx$

(5) a. A man wearing a blue sweater is walking in the park.
b. $\exists x(Px \wedge Qx)$

Equivalence needed for predicate logic:

(6) $\exists x\phi \wedge \psi \equiv \exists x(\phi \wedge \psi)$

Donkeys!

(7) a. If a farmer owns a donkey, he beat it.
b. $\exists x(Px \wedge \exists y(Qy \wedge Rxy)) \rightarrow Sxy$

(8) a. Every farmer who owns a donker, beats it.
b. $\forall x(Px \wedge \exists y(Qy \wedge Rxy)) \rightarrow Sxy)$

Equivalence needed for predicate logic:

(9) $\exists x\phi \rightarrow \psi \equiv \forall x(\phi \rightarrow \psi)$

3.3 Modality and coreference

Binding *vs.* scope:

(10) a. There is someone hiding in the closet. He might have done it.
  b. $\exists x Qx \wedge \Diamond Px$

(11) a. There is someone hiding in the closet who might have done it.
  b. $\exists x(Qx \wedge \Diamond Px)$

Non-equivalence needed on modal predicate logic:

(12) $\exists x\phi \wedge \Diamond\psi \not\equiv \exists x(\phi \wedge \Diamond\psi)$

Modal donkeys:

(13) a. If there is someone hiding in the closet, he might have done it.
  b. $\exists x Qx \rightarrow \Diamond Px$

(14) a. Anyone who is hiding in the closet might have done it.
  b. $\forall x(Qx \rightarrow \Diamond Px)$

Non-equivalence needed in modal predicate logic:

(15) $\exists x\phi \rightarrow \Diamond\psi \not\equiv \forall x(\phi \rightarrow \Diamond\psi)$

The following discourses should be inconsistent, but in ordinary modal predicate logic (16b) and (17c) are not:

(16) a. Someone has done it. But it might be that noone has done it.
  b. $\exists x Px \wedge \Diamond\forall y\neg Py$

(17) a. Someone has done it. But it might be that he hasn't done it.
  b. $\exists x Px \wedge \Diamond\neg Px$
  c. $\exists x(Px \wedge \Diamond\neg Px)$

And note that although it is inconsistent, (17b) can not be concluded from (18), which is consistent:

(18) $\exists x Px \wedge \forall y\Diamond Py \wedge \forall y\Diamond\neg Py$

3.4 Modality and identity

This should be consistent:

(19) a. Someone has done it. It might be Alfred. But it also might not be Alfred.
b. $\exists x(Px \wedge \forall y(Py \rightarrow x = y)) \wedge \Diamond x = a \wedge \Diamond x \neq a$
c. ...It isn't Alfred. It's Bill.
d. $\ldots \wedge x \neq a \wedge x = b$

Also if you know who Alfred is, because you know who you are, and you want (20) to be consistent!

(20) Someone has done it. It might be you. But it also might not be you.

But this is not consistent:

(21) $\exists x(Px \wedge \forall y(Py \rightarrow x = y) \wedge \Diamond x = \mathit{you} \wedge \Diamond x \neq \mathit{you})$

Notice: without uniqueness (and without the brackets), still having rigidly designating constants, things are consistent. An example:

(22) $\exists x(x^2 = 4) \wedge \Diamond x = 2 \wedge \Diamond x = -2$

And possible worlds are not in the picture here.

3.5 (Un)specificity

The ultimate form of unspecificity:

(23) a. Someone has done it. It might be anyone. And anyone might not be it.
b. $\exists x(Px \wedge \forall y(Py \rightarrow x = y)) \wedge \forall y \Diamond x = y \wedge \forall y \Diamond x \neq y$

(24) a. Alfred might be anyone. And anyone might not be Alfred.
b. $\forall x \Diamond x = a \wedge \forall x \Diamond x \neq a$

3.6 Modal subordination

Default: quantifiers inside the scope of negation or modals cannot bind variables outside:

(25) a. Noone has done it. *It is Alfred.
b. $\neg\exists x Px \wedge x = a$

(26) a. It might be the case that someone is hiding in the closet. *He has done it.
b. $\Diamond\exists x Px \wedge Qx$

But there are exeptions:

(27) a. It might be the case that someone is hiding in the closet. He might be the one who did it.
b. $\Diamond\exists x Px \wedge \Diamond Qx$

(28) a. A wolf might come in. It would eat you first.
b. $\Diamond\exists x Px \wedge \Box Qx$

(29) a. No man walks in the park at night. He might get robbed.
b. $\neg\exists x(Px \wedge Qx) \wedge \Diamond Rx$

These mean roughly the same as:

(30) a. It might be the case that someone is hiding in the closet. If there is someone hiding in the closet, then it might be the one who did it.
b. $\Diamond\exists x Px \wedge (\exists x Px \rightarrow \Diamond Qx)$

(31) a. A wolf might come in. If a wolf comes in, it will eat you first.
b. $\Diamond\exists x Px \wedge (\exists x Px \rightarrow Qx)$

(32) a. No man walks in the park at night. If a man walks in the park at night, then he might get robbed.
b. $\neg\exists x(Px \wedge Qx) \wedge (\exists x(Px \wedge Qx) \rightarrow \Diamond Rx)$

3.7 Discourse coherence

Compare:

(33) a. It might be raining outside [. . .] It isn't raining outside.
b. ?It might be raining outside. It isn't raining outside.
c. *It might be raining outside and it isn't raining outside.

The acceptability of a discourse may presuppose that different speakers, or different 'utterance occasions' are involved.

There are information states that can be consistently updated with (34), but there is no single consistent information state that can support it:

(34) $\Diamond p \wedge \neg p$

Another example:

(35) a. Alfred hasn't done it. There is someone hiding in the closet. He might have done it. It is Alfred.
b. $\neg Pa \wedge \exists x Qx \wedge \Diamond Px \wedge Qa$

Tthough this is consistent, it cannot be supported by a single information state. But if the closet is opened before the last sentence of the discourse is uttered (which gives another information state) it is fine, (35) is an acceptable discourse.

But notice the difference:

(36) a. Alfred hasn't done it. There is someone hiding in the closet who might have done it. It is Alfred.
b. $\neg Pa \wedge \exists x(Qx \wedge \Diamond Px) \wedge Qa$

This discourse is unacceptable. Any information state that has been updated with the first two sentences, can no longer support the last sentence. But the discourse is consistent, if we don't know who Alfred is.

Handout 2

4 Introduction

- Our *aim* is to provide an update semantics for the language of modal predicate logic.
- The *basic idea* is that the meaning of a sentence is its information change potential, rather than its truth conditions.
- The *issues* we hope to shed some new light on are those of coreference, modality and identity.
- The *central notions* are those of information, updating information, consistency and coherence of information.
- The *framework* we use is that of possible world semantics. Hence, our approach to information change is eliminative rather than constructive. And we don't know yet how to do better than that.
- The *status* of our way of doing semantics is pre-Montagovian. But here we do know how to do better.

5 Information

5.1 Two kinds of information

- To explicate the meaning of a sentence as its potential to change information states, we have to specify the nature of *information states*.
- An information state is a set of *possibilities*, those alternatives which are still open according to our information.
- What the possibilities are, depends on what information is about.
- First, there is *information about the world*, which provides a partial answer to the question what the world is like.
- One way of gathering such information is by linguistic communication.
- The interpretation process brings along its own questions concerning *discourse information*. For example, there are questions about anaphoric relations that we have to resolve.
- We have to keep track of items we have talked about. We use *pegs* for this. We hang information on them.
- Since discourse information is needed in the interpretation of discourse, which is an important source of information about the world, indirectly, discourse information also provides information about the world.

5.2 Three ingredients of information

Information about the world

- Our information about the world is represented by the set of worlds that are still possible alternatives according to our information.
- We think of worlds as total first order models.
- We will assume that we know the domain of discourse. We know the objects we could talk about. (Though we may not know their names).
- This means that our possible worlds share the same domain. We can think of them as interpretation functions of first order models.
- Extending information about the world amounts to eliminating worlds from the ones we considered still possible.

Discourse information

- The language we want to interpret is a logical language that has variables and quantifiers.
- Quantifiers introduce new pegs.
- Variables are the anaphoric expressions of our language.
- To resolve anaphoric relations, we not only have to keep track of which pegs we've got, but also of which variable is associated with which peg.
- Extending discourse information is adding variables and new pegs, i.e., it is adding new discourse questions.

Linking discourse information to information about the world

- An assignment of objects in the domain of discourse to the pegs—and hence indirectly to the variables asociated with them—links discourse information to information about the world.
- What is a suitable assignment of an object to a peg may depend on what information we have about the world.
- Usually, our information allows for several such possible assignments.
- Getting better informed on this score is to eliminate some such possible asignments, i.e., it means getting a better answer to some of our discourse questions.
- If a certain assignment that gets eliminated was the only possible one with respect to some world, then elimination of that assignment brings with it the elimination of that world.

5.3 Possibilities

The ingredients of information states are *possibilities*:

Definition 1
Let D, the *domain of discourse*, and W, the set of *possible worlds*, be two disjoint non-empty sets.
The set of *possibilities* based on D, W is the set I of quintuples $i = \langle v, n, r, g, w \rangle$, where:

1. v is a finite subset of variables
2. n is a natural number
3. r is an injection from v into n
4. g is a function from n into D
5. $w \in W$

We call $\langle v, n, r \rangle$, or r for short, the *referent system* of i.

The variables in v are the variables in active use. The number n stands for the number of pegs that have been introduced. The pegs themselves are the numbers smaller than n. The function r associates variables with pegs. It is an injection because a variable may get disconnected from a peg. The function g assigns to each peg an object. Variables are assigned an object, indirectly, *via* the peg they are associated with. The composition of g and r assigns values to variables: $g(r(x)) \in D$. Since v and n are just the domain and range of r, they are kind of superfluous. When handy, we refer to possibilities as triples $\langle r, g, w \rangle$.

5.4 Information states

Information states are (real) subsets of the set of possibilities:

Definition 2
Let I be the set of possibilities based on D and W.
The set of information states based on I is the set S such that $s \in S$, iff

1. $s \subseteq I$
2. $\forall i, i' \in s$: i and i' share their referent system

Variables and pegs are introduced globally with respect to an information state. That is why an information state has a unique referent system.

5.5 Extending possibilities

A possibility can be extended by adding a variable, associating it with the next peg, assigning that peg a value, and leaving the rest as it was.

Definition 3
Let $i, i' \in I, i = \langle v, n, r, g, w \rangle$ and $i' = \langle v', n', r', g', w' \rangle$.
$i \leq i'$, i' is an *extension* of i iff

1. $v \subseteq v'$
2. $n \leq n'$
3. (a) If $x \in v$ then $r(x) = r'(x)$ or $n \leq r'(x)$
   (b) If $x \notin v$ and $x \in v'$ then $n \leq r'(x)$
4. $\forall m < n\colon g(m) = g'(m)$
5. $w = w'$

We allow to re-use a quantifier. If that happens, we still introduce a new peg and associate the variable of the quantifier with that new peg. The old peg it was connected with has no variable associated with it anymore.

Fact 1

1. $\leq$ is a partial order on I
2. The minimal possibilities are: $\{\langle \emptyset, \emptyset, w \rangle \mid w \in W\}$
3. There are no maximal possibilities

There are no maximal possibilities, because one can always add a new peg.

5.6 Extending information states

An information state can be extended in two ways: by extending possibilities in it, and by eliminating possibilities from it.

Definition 4
Let S be the set of information states based on I, $s, s' \in S$.
$s \leq s'$, s' is an *extension of* s, iff $\forall i' \in s'\colon \exists i \in s\colon i \leq i'$

An information state s' is an extension of an information state s if every possibility in s' is an extension of some possibility in s.

Fact 2

1. $\leq$ is a partial order on S
2. There is a unique state of minimal information: $\{\langle\emptyset, \emptyset, w\rangle \mid w \in W\}$
3. There is a unique state of maximal information: $\emptyset$

Non-empty subsets of the minimal information state are called *initial states*. The maximal information state is called the *absurd state*. States of *total information* are states consisting of just one possibility. They are just below the absurd state, and above anything else in the extension hierarchy.

5.7 Subsistence

Subsistence is weaker notion of 'identity' between states, disregarding the possible addition of variables and new pegs:

Definition 5
Let $s, s' \in S$. $i \in s$.

1. *i subsists in s'* iff $\exists i' \in s' \colon i \leq i'$
2. *s subsists in s'*, $s \preceq s'$ iff $s \leq s'$ and $\forall i \in s \colon i$ subsists in s'

If s subsists in s', then not only every possibility in s' is an extension of some possibility in s, but also every possibility in s has an extension in s'. Subsistence is a partial order.

5.8 Similarity

We not just keep track of the variables and the pegs that have been introduced, but also of the order in which they were. And there can be pegs around that got disconnected from a variable. Similarity abstracts from that:

Definition 6

1. Let $i, i' \in I, i = \langle v, n, r, g, w\rangle, i' = \langle v', n', r', g', w'\rangle$.
   i is similar to i', $i \approx i'$ iff $v = v' \,\&\, w = w' \,\&\, \forall x \in v \colon g(r(x)) = g'(r'(x))$
2. Let $s, s' \in S$.
   s is similar to s', $s \approx s'$ iff
   (a) $\forall i \in s \colon \exists i' \in s' \colon i \approx i'$
   (b) $\forall i' \in s' \colon \exists i \in s \colon i' \approx i$

Similarity is an equivalence relation.

5.9 Updates

The meaning of a sentence is its information change potential.

Definition 7
Let S be the set of information states based on a set of possibilities I.
A *state transformer on* S is a partial function from S to S.

We use postfix notation: $s[\tau]$ is the result of transforming s by τ. $s[\tau][\tau']$ is the result of first transforming s by τ, and next transforming $s[\tau]$ by τ'. Whether s can be transformed to some state s' by τ may depend on the fulfillment of certain constraints. If a state s does not meet them, then $s[\tau]$ *does not exist*.

Some properties of state transformers:

Definition 8
Let τ be a state transformer on S.

1. τ is *safe* iff $\forall s \in S$: $s[\tau]$ exists
2. τ is an *update* iff $\forall s \in S$ such that $s[\tau]$ exists: $s \leq s[\tau]$
3. τ is *eliminative* iff $\forall s \in S$ such that $s[\tau]$ exists: $s[\tau] \subseteq s$
4. τ is a *test* iff $\forall s \in S$ such that $s[\phi]$ exists: $s \simeq s[\tau]$ or $s[\tau] = \emptyset$
5. τ is *distributive* iff $\forall s \in S$: $s[\tau] = \cup_{i \in s}\{i\}[\tau]$

We will interpret sentences as *updates*. We do not deal with downdating or revision of information.

Fact 3
Let P be any of the properties of: being safe, being an update, being eliminative, being a test, being distributive.
If τ and τ' have the property P, then their composition has the property P

A sequence of similar updates preserves their properties.

Eliminative tests just test (non-eliminative ones may also add new pegs):

Fact 4
If τ is eliminative and τ is a test, then $\forall s \in S$ such that $s[\tau]$ exists: $s[\tau] = s$ or $s[\tau] = \emptyset$

5.10 Classical updates

Definition 9
τ is a (safe) *classical update* iff τ is a (safe) distributive eliminative update

Fact 5
If τ is a safe eliminative update, then τ is a eliminative test on each state of total information: $\forall i \in I\colon \{i\}[\tau] = \{i\}$ or $\emptyset$

Distributivity of classical updates furthermore guarantees that for a (safe) classical update we can define the proposition that it expresses:

Definition 10
If τ is a safe classical update, then the *proposition expressed by* τ, $P_\tau = \{i \in I \mid \{i\}[\tau] \neq \emptyset\}$

The non-dynamic nature of classical updates can then be stated as follows:

Fact 6 For all safe classical updates $\tau\colon s[\tau] = s \cap P_\tau$

5.11 Consistency

To be willing to update with a sentence, updating with it should not lead to the absurd state:

Definition 11
Let s be an information state, ϕ a sentence.

1. s *allows* ϕ iff $s[\phi]$ exists and $s[\phi] \neq \emptyset$
2. s *forbids* ϕ iff $s[\phi] = \emptyset$

Definition 12
Let ϕ be a sentence, S the set of information states.

1. ϕ is *consistent* iff $\exists s \in S\colon s$ allows ϕ
2. ϕ is *inconsistent* iff $\forall s \in S$: if $s[\phi]$ exists, then s forbids ϕ

No information state allows for an update with an inconsistent sentence. Consistency is a necessary condition for a sentence to be acceptable.

5.12 Coherence

To assert a sentence correctly, our information state should support it:

Definition 13
Let s be an information state, ϕ a sentence.
s *supports* ϕ iff $s[\phi]$ exists and $s \leftharpoondown s[\phi]$

A necessary condition for a sentence to be acceptable is that it is coherent.

Definition 14
Let ϕ be a sentence, S the set of information states.
ϕ *is coherent* iff $\exists s \in S\colon s \neq \emptyset$ and s supports ϕ

We note the following:

Fact 7
For all ϕ such that $s[\phi][\phi] = s[\phi]$ it holds that ϕ is consistent iff ϕ is coherent

5.13 Entailment and equivalence

If updating with the sequence of sentences $\phi_1, \ldots, \phi_n$ always results in a state which supports ψ, we may say that $\phi_1, \ldots, \phi_n$ entail ψ. The order of the premisses may matter and entailment is not monotone.

Definition 15
Let $\phi_1, \ldots, \phi_n, \psi$ be sentences, S the set of information states.
$\phi_1, \ldots, \phi_n \models \psi$ iff $\forall s \in S$: if $s[\phi_1] \ldots [\phi_n][\psi]$ exists, then $s[\phi_1] \ldots [\phi_n]$ supports ψ

Two sentences are equivalent if they have similar update effects:

Definition 16
Let ϕ and ψ be sentences, S the set of information states.
$\phi \equiv \psi$ iff $\forall s \in S\colon s[\phi] \approx s[\psi]$

Equivalence ignores the order in which variables and pegs are introduced, and pays no attention to pegs with which no variable is associated.

Handout 3

6 Co-reference

We now start stating an update semantics for the language of modal predicate logic. In this section we restrict ourselves to the predicate logical fragment.

6.1 Terms and predicates

A possibility gives all we need to interpret our basic expressions:

Definition 17
Let α be a basic expression, $i = \langle v, n, r, g, w \rangle \in I$, I based upon W and D.

1. If α is an individual constant, then $i(\alpha) = w(\alpha) \in D$
2. If α is a variable such that $\alpha \in v$, then $i(\alpha) = g(r(\alpha)) \in D$, else $i(\alpha)$ does not exist
3. If α is an n-place predicate, then $i(\alpha) = w(\alpha) \subseteq D^n$

The non-existence of variables in a state will be the only source of partiality of our updates.

6.2 Atomic updates

The interpretation of atomic formulas:

Definition 18 $s[Rt_1 \ldots t_n] = \{i \in s \mid \langle i(t_1), \ldots, i(t_n) \rangle \in i(R)\}$

Definition 19 $s[t_1 = t_2] = \{i \in s \mid i(t_1) = i(t_2)\}$

In case one of the terms in an atomic formula is a variable that is not in the set v in s, the update with that formula does not exist. Atomic updates are partial updates, unless they contain no variables.

Observation:

- Atomic updates are consistent and coherent, partial, classical updates.

6.3 (Re-)Assignment

We define an update that (re-)assigns a value to a variable.

Definition 20
Let $i = \langle v, n, r, g, w\rangle \in I, s \in S, x \in Var, d \in D$.

1. $i[x/d] = \langle v \cup \{x\}, n+1, r[x/n], g[n/d], w\rangle$
2. $s[x/d] = \{i[x/d] \mid i \in s\}$

Fact 8

1. $i < i[x/d]$
2. $s \leq s[x/d]$
3. If $s \neq \emptyset$, then $s < s[x/d]$
4. $s \leftharpoonup s[x/d]$

Observations:

- (Re-)assignment is a non-classical, distributive and non-eliminative update
- (Re-)assignment is allowed and supported by every non-absurd information state

6.4 Existential quantification

Definition 21 $s[\exists x\phi] = \cup_{d\in D}(s[x/d][\phi])$

If we update a state s with $\exists x\phi$, we pick an object d from the domain, and we (re-)assign d to x in s. The state $s[x/d]$ that results from this is updated with ϕ. After we have done this for every object d, we collect the results.

Fact 9

1. If ϕ is an update, then for all $s \in S$: if $s[\exists x\phi]$ exists and $s \neq \emptyset$, then $s < s[\exists x\phi]$
2. If ϕ is a distributive update, then $\exists x\phi$ is a distributive update
3. $\exists x\phi$ is not a classical update

Fact 10

1. $\exists xPx \models \exists yPy$
2. $\exists xPx \not\equiv \exists yPy$

3. $\exists x Px \models Px$
4. $\exists x \exists y Rxy \equiv \exists y \exists x Rxy$

Observations:

- ∘ Existential quantification is a non-eliminative, real update.
- ∘ It is distributive if its complement is.
- ∘ It is allowed (forbidden) in a state if that state extended with a new peg, under some (every) assignment of an object to that peg, allows (forbids) the complement of the quantifier.
- ∘ It is supported by a state if that state extended with a new peg, under every assignment of an object to that peg, supports the complement.

6.5 Sequencing

We interpret conjunction as a sequence of updates.

Definition 22 $s[\phi \wedge \psi] = s[\phi][\psi]$

Sequencing passes on dynamic effects. Since order matters in dynamic semantics it cannot be expected to be a commutative operation.

Fact 11

1. $((\phi \wedge \psi) \wedge \chi) \equiv (\phi \wedge (\psi \wedge \chi))$
2. $\exists x\phi \wedge \psi \equiv \exists x(\phi \wedge \psi)$, if ϕ and ψ are distributive updates
3. $\phi \wedge \psi \equiv \psi \wedge \phi$, if ϕ and ψ are classical updates
4. $x = a \wedge \exists x(x = b) \not\equiv \exists x(x = b) \wedge x = a$
5. $x = a \wedge \exists x(x = b) \not\models x = a \wedge \exists x(x = b)$
6. $\exists x Px \equiv \exists x Px \exists x Px$

Observations:

- ∘ Sequencing inherits its properties from its conjuncts.
- ∘ It is eliminative (distributive), if both of its conjuncts are.
- ∘ A conjunction is allowed (forbidden) (supported) by an information state, if the first conjucnt is allowed (forbidden) (supported) by that information state, and if the second conjunct is allowed (forbidden) (supported) by the information state that results from updating the orginal state with the first conjunct.

6.6 Negation

Definition 23 $s[\neg\phi] = \{i \in s \mid \neg\exists i'\colon i \leq i' \text{ and } i' \in s[\phi]\}$

A negation $\neg\phi$ eliminates those possibilities in s that would subsist after updating s with ϕ.

Apart from negating, negation blocks the binding of quantifiers in its scope:

(37) a. It is not the case that a man is walking in the park. *He is wearing a blue sweater.
b. $\neg\exists x Px \wedge Qx$

But cf.:

(38) No man walks in the park at night. He would get robbed.

Fact 12

1. $\neg\phi$ is an eliminative update
2. If ϕ is a distributive update, then $\neg\phi$ is too

Fact 13

1. $\neg\neg\phi \models \phi$
2. $\phi \models \neg\neg\phi$, if ϕ is an eliminative update
3. $Px \wedge \exists x\neg Px \not\models \neg\neg(Px \wedge \exists x\neg Px)$
4. $\neg\neg\phi \equiv \phi$, if ϕ is an eliminative update,
5. $\exists x Px \not\equiv \neg\neg\exists x Px$

6.7 Implication, disjunction and universal quantification

We add as non-basic operations:

Definition 24

1. $(\phi \rightarrow \psi) =_{\mathrm{df}} \neg(\phi \wedge \neg\psi)$
2. $(\phi \vee \psi) =_{\mathrm{df}} \neg(\neg\phi \wedge \neg\psi)$
3. $\forall x\phi =_{\mathrm{df}} \neg\exists x\neg\phi$

Calculating their interpretation we arrive at:

Fact 14

1. $s[\phi \to \psi] =$
   $\{i \in s \mid \forall i'$: if $i \leq i'$ and $i' \in s[\phi]$, then $\exists i''\colon i' \leq i''$ and $i'' \in s[\phi][\psi]\}$
2. $s[\phi \vee \psi] = \{i \in s \mid \exists i'\colon i \leq i'$ and $i' \in s[\phi]$ or $i' \in s[\neg\phi][\psi]\}$
3. $s[\forall x\phi] = \{i \in s \mid \forall d \in D\colon \exists i'$ such that $i \leq i'$ and $i' \in s[x/d][\phi]\}$

If we update a state with $\phi \to \psi$, then a possibility will remain just in case: if it subsists after an update with ϕ, then all its extensions after updating with ϕ should subsist after a further update with ψ.

If we update a state with $\phi \vee \psi$, those possibilities remain which subsist after an update with ϕ, or which do not subsist after an update with ϕ but which do subsists after an update with ψ.

If we update a state with $\forall x\phi$, then those possibilities remain, which after every (re-)assignment of x to some object, subsist after an update with ϕ.

Fact 15
If ϕ and ψ are distributive updates, then $\exists x\phi \to \psi \equiv \forall x[\phi \to \psi]$

Fact 16
$\phi \models \psi$ iff $\models \phi \to \psi$

7 Summing up

Observations:

- ○ All formulas of predicate logic constitute distributive updates.
- ○ Not all formulas of predicate logic constitute eliminative updates.
- ○ Dynamic predicate logic is not classical.

So, providing the language of predicate logic with an update semantics makes sense. Some of the effects of that we have seen above.

Handout 4

8 Modality

In this section we add epistemic modalities to the language.

Definition 25

∘ $s[\Diamond\phi] = s$ if s allows ϕ
$= \emptyset$ if s forbids ϕ

The *might*-operator tests for consistency with an information state.

Fact 17

1. $\Diamond\phi$ is an eliminative test
2. $\Diamond\phi$ is a non-distributive update

Consistency testing essentially involves looking at an information state globally, and not pointwise with respect to the possibilities it contains.

Note: inc ombination with an existential quantifier eliminiation effects do occur: $\exists x\Diamond Px$ may eliminate possibilities from an information state.

Order matters: An example:

(39) a. It might be raining outside [...] It isn't raining outside.
b. It isn't raining outside [...] *It might be raining outside.

Its explanation:

(40) a. $\Diamond p \wedge \neg p$ is consistent
b. $\neg p \wedge \Diamond p$ is inconsistent

Why the dots are important:

(41) *It might be raining outside and it is not raining outside.

The unacceptability is accounted for by the following observation:

∘ $\Diamond\phi \wedge \neg\phi$ is incoherent, though consistent

The unacceptability of (3) follows because it is viewed as a (single) *sentence*, and hence is incoherent. That unlike (3), (1a) *is* acceptable, is because it is viewed as a *discourse*, which, unlike a sentence, allows for different utterance occasions.

Consequence:

- $\Diamond p \wedge \neg p \not\equiv \neg p \wedge \Diamond p$

Other observations:

- $\Diamond p \wedge \neg p \not\models \Diamond p \wedge \neg p$
- $\Diamond p \models \Diamond p$, but $\Diamond p \wedge \neg p \not\models \Diamond p$

Usually, one defines a $\Box$-operator in terms of the $\Diamond$-operator:

Definition 26

- $\Box\phi =_{\mathrm{df}} \neg\Diamond\neg\phi$
- $s[\Box\phi] = s$ if s supports ϕ
  $= \emptyset$ if $s[\phi]$ exists and s does not support ϕ

Whereas $\Diamond\phi$ tests for acceptance, $\Box\phi$ tests for support. One way to read $\Box\phi$ is as ... *So*, ϕ.

$\Diamond\phi$ being a test means that we do not (yet) account for:

- Update of second order information
- Update of information about what is possible

9 Coreference and modality

Extended binding inside the scope of modal operators:

(42) $\exists x Px \wedge \Diamond Qx$

Due to that, the following is inconsistent:

(43) $\exists x Px \wedge \neg\Diamond Px$

Also inconsistent is:

(44) $\exists x Px \wedge \Diamond\forall y\neg Py$

But this one is *not:*

(45) $\exists x Px \wedge \forall y\Diamond\neg Py$

Yet inconsistent is:

(46) $\exists x Px \wedge \forall y\Diamond\neg Py$

So we see that $\exists x\phi \wedge \psi$ and $\exists x(\phi \wedge \psi)$ are not generally equivalent. And the same holds for $\exists x\phi \rightarrow \psi$ and $\forall x(\phi \rightarrow \psi)$.

Two more illustrations.

(47) a. $\exists x(Qx \wedge \Diamond Px)$
b. There is someone hiding in the closet who might be the one who did it.

(48) a. $\exists xQx \wedge \Diamond Px$
b. There is someone hiding in the closet. He might be the one who did. it.

Imagine the following situation. You and your spouse have three sons. One of them broke a vase. Your spouse is very anxious to find out who did it. You are not interested at all. Both you and your spouse know that your eldest didn't do it, he was playing outside when it must have happened. You are looking for him to help you do the washing up. He might be hiding somewhere. In the meantime your spouse has gone upstairs.

Suppose your spouse hears a noise coming from the closet. If it is the shuffling of feet, your spouse will know that someone is hiding in there, but will not be able to decide which of your three sons it is. In that case your spouse could utter (10), but *not* (9).

But if the noise she hears is a high-pitched voice she knows it can't be you eldest son, he already has a frog in his throat. In that case she *can* say (9).

So, if your spouse yells (9) from upstairs, you stay were you are, but if it is (10) that your spouse screams, you run upstairs to check whether perhap your aid is hiding there.

Next consider:

(49) a. $\forall x(Qx \rightarrow \Diamond Px)$
b. Anyone who is hiding in the closet might be the one who did it.

(50) a. $\exists xQx \rightarrow \Diamond Px$
b. If there is someone hiding in the closet, he might be the one who did it.

Take the same situation again. Only in case your spouse just heard some high-pitched voice, (11) is a correct utterance.In the other case (11) is not supported, and only (12) is left.

So, there is a difference between (9) and (10), and between (11) and (12). Our semantics predicts this difference:

Fact 18

1. $\exists x(Px \wedge \Diamond Qx) \not\equiv \exists x Px \wedge \Diamond Qx$
2. $\exists x Qx \rightarrow \Diamond Px \not\equiv \forall x(Qx \rightarrow \Diamond Px)$

Two important features of $\exists x$ and $\Diamond$

- ○ $\Diamond\phi$ is interpreted as a *global* consistency test, both with respect to information about the world, and with respect to discourse information.
- ○ $\exists x\phi$ is involves *distributive* (w.r.t. the re-assigment of x) update with the matrix ϕ.

Consequence: Updating with $\exists x\Diamond Px$ outputs as possible values of x only those d such that in some w compatible with our information d has the property P in w. If $\Diamond Px$ is within the scope of $\exists x$, the consistency test is performed one by one for each $d \in D$, where we eliminate those d as possible values for x for which the test fails. (The formula $\exists x\Diamond Px$ is not a test.)

10 Identity, coreference and modality

Consider the following example:

(51) a. $\exists! x Px \wedge \Diamond x = a \wedge \Diamond x \neq a$
b. Someone has done it. It might be Alfred. It might not be Alfred.

(52) a. $x \neq a \wedge x = b$
b. It is not Alfred. It is Bill.

The sequence of sentences (7) is consistent and coherent. If we continue it with (8), everything together is still consistent. But viewed as one sentence, it would be incoherent.

The consistency of (7) does not depend on not knowing who a is. You know who you are, still you want (9) to be consistent. It is.

(53) a. $\exists! x Px \wedge \Diamond x = \mathit{you} \wedge \Diamond x \neq \mathit{you}$
b. Someone has done it. It might be you. But it might also not be you.

Even in case only one world would be left, *might* can still have a non-trivial meaning.

(54) $\exists x(x^2 = 4) \wedge \Diamond x = 2 \wedge \Diamond x = -2$

This is a valid formula.

11 Identity and identification

The following two sentences are consistent:

(55) $\exists! x P x \wedge \forall y \Diamond(x = y) \wedge \forall y \Diamond(x \neq y)$
(56) $\forall x \Diamond(x = a) \wedge \forall x \Diamond(x \neq a)$

These are ultimate forms of non-identification.

Definition 27
Let α be a term, s an information state.

1. α is *identified in* s iff $\forall i, i' \in s \colon i(\alpha) = i'(\alpha)$
2. α is an *identifier* iff $\forall s \colon \alpha$ is identified in s.

If α is identified in s, then s contains the information who α is, in at least some sense of *knowing who*. If α is not identified in s, then there is at least some doubt about who α is.

Fact 19

1. α is identified in s iff s supports $\exists x \Diamond x = \alpha \wedge \forall y(\Diamond y = \alpha \rightarrow y = x)$
2. α is an identifier iff $\models \exists x \Diamond x = \alpha \wedge \forall y(\Diamond y = \alpha \rightarrow y = x)$

The formula which characterizes identification is a (non-eliminative) *test* for whether α is identified in an information state or not.

12 Why we need identifiers

We need identifiers. Otherwise, if we start out being ignorant we can never really find out who is who, in the sense of knowing the names of the objects we are talking about.

Definition 28
Let $\langle r, g, w\rangle \in I, \langle r, g', w'\rangle \in I$
$\langle r, g, w\rangle \simeq \langle r, g', w'\rangle$ iff there exists a bijection f from D onto D such that:

1. For every peg m in the domain of g: $g'(m) = f(g(m))$
2. For every individual constant a: $w'(a) = f(w(a))$
3. For every n-place predicate P:
   $\langle d_1, \ldots, d_n\rangle \in w(P)$ iff $\langle f(d_1), \ldots, f(d_n)\rangle \in w'(P)$

Fact 20
$\forall s \in S$ such that $s = \mathbf{0}[\phi_1] \ldots [\phi_n]$: if $i \in s$ then $i' \in s$, for every $i' \simeq i$

13 We need something like *this*

To fill the need for identifiers, we add demonstratives to our language.

Definition 29
We add *this* to the logical inventory of the language.

- Let $d \in D$, then $this_d$ is a term.
- Let $i \in I$, then $i(this_d) = d$

Fact 21

1. $this_d$ is an identifier
   Let *this* and *that* be two different identifiers.
2. $\models \Diamond(this = that) \rightarrow (this = that)$
3. $\models (this = that) \rightarrow \Box(this = that)$

Suppose the domain consists of two individuals. We update a state of ignorance with the following sequence:

$$(a \neq b) \wedge (this \neq that) \wedge \neg\exists x((x \neq this) \wedge (x \neq that))$$

Our information state will support:

$$\Diamond(this = a) \wedge \Diamond(that = a) \wedge \Diamond(this = b) \wedge \Diamond(that = b)$$

Our information state will forbid:

$$\forall x\Diamond(x = this)$$

Whereas it supports:

$$\forall x\Diamond(x = a)$$

And at the same time, of course, our information state forbids:

$\Diamond(b = a)$

Like universal instantiation is not always allowed, existential generalization is not either:

$\forall y \Diamond y \neq a \not\models \exists x \forall y \Diamond y \neq x$

Fact 22

1. If α is an identifier, then $\forall x \phi \models [\alpha/x]\phi$
2. If α is an identifier, then $[\alpha/x]\phi \models \exists x \phi$

Handout 5

14 Leibniz' law

Consider:

- ∘ If s supports $a = b$, then $s[\phi(a)] = s[\phi(b)]$

This holds in a standard modal semantics only if a and b are rigid.

In our system it holds also for non-identified names.

But we do *not* have:

- ∘ If s supports $\Diamond a = b$, then $s[\phi(a)] = s[\phi(b)]$

Counterexample:

- ∘ There are non-absurd states s such that s supports $\Diamond a = b$, and such that s also supports $\Diamond a \neq b$. But no non-absurd state s supports $\Diamond b \neq b$.

Of course, if a and b are both identified in s, then it can never occur that s supports both $\Diamond a = b$ and $\Diamond a \neq b$.

15 Barcan formulae

Consider the following arguments:

1. $\exists x \Diamond \phi(x) \,/\, \Diamond \exists x \phi(x)$
2. $\Diamond \exists x \phi(x) \,/\, \exists x \Diamond \phi(x)$
3. $\forall x \Diamond \phi(x) \,/\, \Diamond \forall x \phi(x)$
4. $\Diamond \forall x \phi(x) \,/\, \forall x \Diamond \phi(x)$

In a standard modal semantics (e.g., in Montague's IL) which has one fixed domain for all worlds, and in which the assignments are independent of the worlds, arguments 1, 2, and 4 come out valid, while argument 3 is invalid.

The same holds for our system.

Yet, besides with (3), there is also something wrong with (2), and (4). Only the validity of (1) is beyond reasonable doubt.

15.1 No fixed domain

We drop the assumption that we know what the domain of discourse is, that we know which objects exist in the real world.

- Instead of having a single domain D shared by all worlds, we view D as the set of possible individuals. Each possible world $w \in W$ has its own domain $D_w \subseteq D$.
- This means that we can no longer identify a world with just an interpretation function. A world w is now a full fledged first order model, identified by a domain D_w, and an interpretation function F_w, which determines the extensions of the constants and predicates of our language with respect to D_w.
- Extending world knowledge still amounts to eliminating possible worlds. But now we can learn, e.g., that there is more than one object, which would mean eliminating all those worlds w where D_w is a singleton set.
- We assume that one of the possible worlds is the real world (if only we knew which one!), and that its domain consists of the objects that really exist.
- These objects, whichever they are, are the only objects that we can possibly get acquainted with, and that we might be able to point at while uttering demonstratives.

To implement this in our framework, we have to adapt a few of the more basic definitions. The other definitions that are based on them can remain 'literally' the same.

First, we add a definition that explicitly defines what possible worlds are:

Definition 30 (Possible worlds)
Let D be a non-empty set of *possible objects*, and B the set of non-logical constants of a first order language.
The set of *possible worlds* based on D and B, is the set W of pairs $w = \langle D_w, F_w \rangle$, where:

1. $\emptyset \neq D_w \subseteq D$
2. F_w is an ordinary (total) interpretation function assigning extensions to the elements of B with respect to D_w:
   a. If $\alpha \in B$ and α is an individual constant, then $F(\alpha) \in D_w$
   b. If $\alpha \in B$ and α is an n-place predicate, then $F(\alpha) \subseteq (D_w)^n$

Among the elements of W, there is a distinguished element $\mathbf{w}$, the real world. E is the set of real objects: $D_{\mathbf{w}} =_{\text{df}} E$.

Note that F_w is a total function. In particular, we assume that we know that each name denotes an object, but we may not know which one. We try to find out what the real world (an ordinary first order model) looks like: what the real objects are, and what properties they have. The language is given to us. And names denote objects, we know that.

We don't allow for the possibility that an individual constant denotes an object that does not exist. If we were to play this game, we would still let F_w be a total function, but $F_w(c)$ could be just an element of D, and not necessarily of D_w. If $F_w(c) \notin D_w$, this means that the object that the name c refers to does not exist in w. Why not let $F_w(c)$ be undefined in this case? A Kripkean answer: a name is a name because by some act of initial baptism the expression is introduced to refer to a particular object. From an ontological/metaphysical point of view it rigidly refers to some object. From an epistemological point of view, we may not know to which object it rigidly refers (since we don't know what the metaphysically possible worlds are). The bearer of a name may cease to exist, then the name still refers to the same object, it only does not exist anymore. Whether or not the bearer of a name actually exists or not, is just one of the things we may want to find out. If the bearer of a name does not exist, this is no reason for the name not to be defined. There extension is always defined, that is how they come to be. A name is not a name unless at some point it has been used to baptize a particular object. Meaningless sounds are not in the domain of F. (This alternative view, where name do always refer to some possible object, but not necessarily to an existing one, only makes sense if next to epistemic possibilities we also introduce ontological possibilities. Then the aim of the game is not only to find out what the real world is like, but also what the real metaphysically possible world are. We will return to this later.)

Next, we make some minor changes in the definition of what the possibilities are:

Definition 31 (Possibilities)
Let D, B and W be as defined above.
The set of *possibilities* based on D, B and W, is the set I of quintuples $i = \langle v, n, r, g, w\rangle$, where:

1. v is a finite subset of variables

2. n is a natural number
3. r is an injection from v into n
4. g is a function from n into D_w
5. $w \in W$

We call $\langle v, n, r \rangle$, or r for short, the *referent system* of i.

The only change is in clause 4: pegs (and the variables associated with them) are assigned an object in the domain *of the world* of the possibility.

The definitions of the notion of an information state; of the relation of extension between possibilities and between information states; the notions of subsistence and similarity; the notion of a state transformer, and the properties we distinguished; the notions of consistency, support, and coherence; and the definitions of entailment and equivalence, all remain unaltered.

We make one change in the definition of the language:

Definition 32 (Demonstratives)
Let $d \in E$, then $this_d$ is a term

As we noted above, we can only point at some object d, and say *this*, if that object exists in the *real world*, i.e., if $d \in E$, the domain of the real world.

The definition of the interpretation of basic expressions remains virtually the same, but we have added demonstratives:

Definition 33 (Extensions of basic expressions)
Let α be a basic expression, an individual constant, variable, demonstrative or n-place predicate; and let $i = \langle v, n, r, g, w \rangle \in I$, I based upon D, B and W.

1. If α is an individual constant, then $i(\alpha) = F_w(\alpha)$
2. If α is a variable, then $i(\alpha) = g(r(\alpha))$, if $\alpha \in v$, else $i(\alpha)$ is not defined
3. If α is a demonstrative $this_d$, then $i(\alpha) = d$, if $d \in D_w$, else $i(\alpha)$ is not defined
4. If α is an n-place predicate, then $i(\alpha) = F_w(\alpha)$

For the extension of a demonstrative to be defined in a possibility, it is required that the object pointed at exists in the domain of the world in that possibility. That makes sense given the assumption that demonstratives can only be used to refer to existing objects.

The definition of the interpretation of atomic formulae remains the same. It looks different because this time we are more explicit about when the update with an atomic formula is undefined:

Definition 34 (Atomic updates)
$s[Rt_1 \ldots t_n] = \{i \in s \mid \langle i(t_1), \ldots, i(t_n)\rangle \in i(R)\}$, if $\forall k\colon 1 \leq k \leq n$ and $\forall i \in s\colon i(t_k)$ is defined; else $s[Rt_1 \ldots t_n]$ is not defined

For a variable x, the precondition amounts to the requirement that x is present in the reference system of s, which is the same in each of its possibilities. For a demonstrative $this_d$, the precondition requires that d should be present in each of the domains of the possible worlds occurring in s: $d \in \cap_{\langle r,g,w\rangle \in s} D_w$. This means nothing else than that according to our information we are sure that d exists. Actually, since for $this_d$ to be a term it is required that $d \in E$, this piece of information is real knowledge.

Definition 35 (Identity)
$s[t_1 = t_2] = \{i \in s \mid i(t_1) = i(t_2)\}$, if $\forall i \in s\colon i(t_1)$ and $i(t_2)$ are defined; else $s[t_1 = t_2]$ is not defined.

We also have to make some adjustments in the definition of (re-)assignment:

Definition 36 ((Re-)assignment)
Let $i = \langle v, n, r, g, w\rangle \in I, s \in S, x \in Var, d \in D$.

1. $i[x/d] = \langle v \cup \{x\}, n+1, r[x/n], g[n/d], w\rangle$, if $d \in D_w$, else $i[x/d]$ is undefined.
2. $s[x/d] = \{i[x/d] \mid \exists i \in s$ such that $i[x/d]$ is defined$\}$

Here, too, the only change is that a peg (and a variable associated with it) can only be assigned an object as value in a possibility if that object is in the domain of the world in that possibility. This means that some possibilities $i \in s$ may not subsist in $s[x/d]$. This will happen if d is not in the domain of the world in i. (So, with varying domains, it is no longer true that $s \preceq s[x/d]$.)

All the other update clauses remain the same, except that we have to be careful with the use of our demonstratives. For, to be able to interpret a term $this_d$, our information state has to be such that we acknowledge the existence of d. If something is being pointed at in the discourse situation, then apparently that object does exist, and we are aware of that. (The latter is covered by the requirement that d should be in the domain of each world that is still possible according to our information.

However, so far our framework does not incorporate a mechanism to get acquainted with an object. As we saw in the previous lecture, states are closed under isomorphisms, as long as they are only updated with sentences not containing demonstratives. That means, if a certain possibility is in s, all possibilities isomorphic to it will be in s. Hence, in this way we will never get at an object which is an element of the domains of all possibilities in s. So, we will never get in a position where we can use a demonstrative properly.

As a solution we offer the possibility of getting acquainted with a set of objects:

Definition 37 (Acquaintance)
Let $s \in S$, and $A \subseteq E$.
$s[A] = \{\langle r, g, w\rangle \in s \mid A \subseteq D_w\}$

Now, if an assertion such as $P(this_d)$ has been preceeded, e.g., by an update $s[\{d\}]$ it will be interpretable in any extension of $s[\{d\}]$.

Fact 23

1. $\exists x \Diamond Px \models \Diamond \exists x Px$
2. $\Diamond \exists x Px \not\models \exists x \Diamond Px$, yet $\Diamond \exists x Px \wedge \neg \exists x \Diamond Px$ is incoherent
3. $\forall x \Diamond Px \not\models \Diamond \forall x Px$, and $\forall x \Diamond Px \wedge \neg \Diamond \forall x Px$ is coherent
4. $\Diamond \forall x Px \not\models \forall x \Diamond Px$, yet $\Diamond \forall x Px \wedge \neg \forall x \Diamond Px$ is incoherent

Handout 6

Got lost

Handout 7

16 Definite descriptions continued

16.1 E-type variables

A variable which is bound by a certain existential quantifier, but is not in the scope of it, can not always be replaced by a suitable anaphoric definite description.

(57) a. If Pedro owns a donkey then he beats it.
b. If Pedro owns a donkey, then he beats the donkey that he owns.

(58) a. Pedro owns two donkeys, Ali and Baba. If Pedro owns a donkey then he beats it.
b. Pedro owns two donkeys. Ali and Baba. ? If Pedro owns a donkey then he beats the donkey that he owns.

The discourse should not explicitly provide for more than one different peg that satisfies the description. In that case the precondition for the anaphoric use is not met.

If a variable can be replaced by a suitable description, i.e., if the precondition for the anaphoric use of the definite decription is met, then the result is equivalent to the orginal.

Formally:

- If $s[\exists x \phi(x) \wedge \imath x(\phi(x), \psi(x)]$ is defined, then $s[\exists x \phi(x) \wedge \imath x(\phi(x), \psi(x)] = s[\exists x \phi(x) \wedge \psi(x)]$

Fact 24

1. $\models \imath x(Px, Px)$
2. $\exists! x Px \wedge \forall y \Diamond \neg Py \not\models \Diamond \neg \imath x(Px, \neg Px)$

Fact 25 (Problem)

1. $\models \exists x(\imath y(Py, x = y)$
   Reminder, this means that if the formula is defined in a state, it is supported by it.
2. $\neg \exists x(\imath y(Py, x = y)$ is either inconsistent with s or undefined in s.

Fact 26
For absolute use of a description:
$this \neq that \wedge \Diamond \imath x(Px, x = this) \wedge \Diamond \imath x(Px, x = that)$ is consistent, whereas $this \neq that \wedge \Diamond \imath x(\Diamond Px, x = this) \wedge \Diamond \imath x(\Diamond Px, x = that)$ is inconsistent. For anaphoric use this is different. Both are consistent.

16.2 Accommodation

One might uphold that even though 'The queen of Holland is rich' has the precondition that Holland has a queen and no more then one, this still does not mean that an information state has to *support* this piece of information. That one can *accommodate* the presupposition by simply *updating* with this information. One could make the precondition into the requirement that an information state *can* be updated with that information, i.e., that updating with it does not lead to the absurd state.

Notice, there still is a difference with between $\imath(Qx, Rx)$ and $\exists!xPx \wedge Rx$. If the information state s is such that it does not allow $\exists!xPx$, then updating $\exists!xPx \wedge Rx$ leads to the absurd state, whereas the update of s with $\imath(Qx, Rx)$ is undefined.

The following definition captures this idea:

Definition 38 (Semantics, absolute use, accommodating)

1. Provided that the precondition in 2 is fulfilled,
   $s[\imath x(\phi, \psi)] = [\exists!x\phi][\psi]$,
   else it is undefined
2. Let $s' = \cup_{d \in D} s[x/d], s'' = s'[\phi]$.
   Precondition:
   $s'' \neq \emptyset$ and $\forall i'' \in s'', \exists i \in s, \exists! i' \in s' \colon i \leq i' \leq i''$

Now the precondition for $\imath x(Qx, Px)$ to be defined in s amounts to: s allows $\exists!xPx$.

The definition for anaphoric use can be adapted in a similar way. Whether thins really can/should be done in this way remains to be seen.

Handout 8

Was never handed out.

17 Definite descriptions, presuppositions, and accommodation

17.1 Adjusting the definitions

Definition 39 (Semantics, absolute use)

1. Provided that the precondition in 2 is fulfilled,
   $s[\imath x(\phi, \psi)] = \cup_{d \in D}(s[x/d][\phi])[\psi]$,
   else it is undefined
2. Precondition
   $\forall i \in s, \exists! d \in D$ such that i subsists in $s[x/d][\phi]$
3. Accommodating the precondition in s:
   $s^* = \{i \in s \mid \exists! d \in D$ such that i subsists in $s[x/d][\phi]\}$

Definition 40 (Auxiliary notions)

1. Let $i = \langle v, n, r, g, w \rangle$.
   $Q_i = \{d \in D \mid \exists m < n \colon g(m) = d\}$
2. $s[x/d]_Q = \{i[x/d] \mid i \in s \text{ and } d \in Q_i\}$

Definition 41 (Semantics, anaphoric use)

1. Provided that the precondition in 2 is fulfilled,
   $s[\imath x(\phi, \psi)] = \cup_{d \in D}(s[x/d]_Q[\phi])[\psi]$,
   else it is undefined
2. Precondition
   $\forall i \in s, \exists! d \in D \colon i$ subsists in $s[x/d]_Q[\phi]$
3. Acommodating the precondition in s:
   $s^* = \{i \in s \mid \exists! d \in D \colon i$ subsists in $s[x/d]_Q[\phi]\}$

17.2 Putting accommodation in the definition

In definition 1.1 on the handout entitled 'Definite decriptions continued', accommodation is built into the definition of the interpretation of definite descriptions. This means that if the existence, the uniqueness, or the contents of the restriction ϕ is not supported by s, but *is* consistent with s, then interpretation does not fail, but s is simply updated with that information.

In terms of the definitions stated above, it means that in case the state s^* that results after updating s with the precondition is not the absurd state (in which case undefinednes would result), we simply replace s in the update clause by s^*, and proceed from there.

This is not the right idea about accommodation. Consider $s[\neg\imath x(Px, Qx)]$. Suppose your state does not support that there is a unique object that is P. Say, in some world w $w(P) = \emptyset$, in others it is not. Using the accommodating definition, after $s[\imath xPx, Qx)]$ possibilities with that w will have been eliminated. But this means that if we apply negation, then in the end result the world w with no P in it returns. So, in the resulting state you end up with something like: either there is no P, or there is one, but it doesn't have the property Q. (Notice that this works in the same way if the negation is internal rather than eternal.)

This is wrong. If you assume that presuppositions can be accommodated, then also after the negation $\neg\imath x(Px, Qx)$, you should have updated with there being a unique thing with the property P. The world w should have been eliminated.

In other words, accommodation should not be build into the definition, then you get the wrong results under negation.

17.3 Accommodation

If you are in an updating process, and a precondition fails, update the original s at the very beginning of the process with the precondition. (Replace$s$ by s^*) Restart the process. If you end up in the absurd state, remove the most global * and start again. Keep going on like like this, until no stars can be removed anymore, and what comes out of tht iss the final result.