# Inquisitive Semantics and Pragmatics<sup>\*</sup>

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**Abstract.** This paper starts with an informal introduction to inquisitive semantics. After that, we present a formal definition of the semantics, introduce the semantic notions of inquisitiveness and informativeness, and define the semantic categories of questions, assertions, and hybrid sentences.

The focus of the paper will be on the logical pragmatical notions that the semantics gives rise to. We introduce and motivate inquisitive versions of principles of cooperation, which direct a conversation towards enhancement of the common ground. We define a notion of compliance, which judges relatedness of one utterance to another, and a notion of homogeneity, which enables quantitative comparison of compliant moves. We end the paper with an illustration of the cooperative way in which implicatures are established, or cancelled, in inquisitive pragmatics.

Keywords: inquisitiveness, informativeness, common ground, compliance.

## 1 Mission Statement

Traditionally, the meaning of a sentence is identified with its informative content. In inquisitive semantics, a sentence is not only associated with the information it provides, but also with the issues it raises. The notion of meaning embodied by inquisitive semantics directly reflects a primary use of language: the exchange of information in a cooperative process of raising and resolving issues.

The way in which inquisitive semantics enriches the notion of meaning also changes our perspective on logic. The central notion that comes with the semantics is the notion of compliance. Compliance is concerned with what the utterance of a sentence contributes to a conversation, how it is related to what was said before. Just as the standard logical notion of entailment rules the validity of argumentation, the logical notion of compliance rules the coherence of information exchange.

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The way in which inquisitive semantics enriches the notion of meaning will also change our perspective on pragmatics. The main objective of Gricean pragmatics is to explain aspects of interpretation which are not directly dictated by semantic content, in terms of general features of rational human behaviour. Since inquisitive semantics changes the notion of semantic content, pragmatics will change with it. Implicatures will not only arise from the informative content of a sentence, but also from its inquisitive content.

## 2 Getting the Picture

The primary aim of inquisitive semantics is to develop a notion of meaning which directly reflects a primary use of language, which lies in the interactive process of exchanging information in a cooperative conversation.

The classical notion of meaning embodies the informative content of sentences, and thereby reflects the descriptive function of language. Stalnaker (1978) gave this informative notion a dynamic and conversational twist by taking the meaning of a sentence to be its potential to change the common ground, where the common ground is viewed as a body of shared information as it has been established in a conversation.

What is implicit in the picture of meaning that resulted from this 'dynamic turn' is that the goal of cooperative informative discourse is to enhance the common ground. And one can view this against the background of the human need for a common ground to be able to perform coordinated action. Thus, the dynamic notion of meaning reflects the *active* use of language in *changing* information. However, what it does not yet capture is the *interactive* use of language in *exchanging* information. This requires yet another turn, an 'inquisitive turn', leading to a notion of meaning that directly reflects the nature of informative dialogue as a cooperative process of raising and resolving issues.

#### 2.1 **Propositions as Proposals**

We follow the standard practice of referring to the meaning of a sentence as the proposition that it expresses. The classical logical-semantical picture of a proposition is a set of possible worlds, those worlds that are compatible with the information that the sentence provides. The common ground is also standardly pictured as a set of worlds, those worlds that are compatible with the conversational participants' common beliefs and assumptions. The communicative effect of a sentence, then, is to enhance the common ground by excluding certain worlds, namely those worlds in the common ground that are not included in the proposition expressed by the sentence.

Of course, this picture is limited in several ways. First, it only applies to sentences which are used exclusively to provide information. Even in a typical informative dialogue, utterances may serve different purposes as well. Second, the given picture does not take into account that enhancing the common ground is a cooperative process. One speech participant cannot simply change the common ground all by herself. All she can do is *propose* a certain change. Other speech participants may react to such a proposal in several ways. These reactions play a crucial role in the dynamics of conversation.

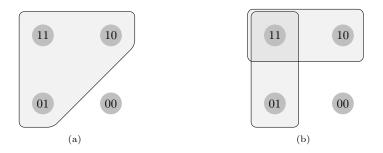
In order to overcome these limitations, inquisitive semantics starts with an altogether different picture. It views propositions as proposals to enhance the common ground. These proposals do not always specify just one way of changing the common ground. They may suggest alternative ways of doing so, among which the responder is then invited to choose.

Formally, a proposition consists of one or more *possibilities*. Each possibility is a set of possible worlds—a set of *indices*, as we will call them—and embodies a possible way to change the common ground. If a proposition consists of two or more possibilities, it is *inquisitive*: it invites the other participants to respond in a way that will lead to a cooperative choice between the proposed alternatives. In this sense, inquisitive propositions raise an issue. They give direction to a dialogue. Purely informative non-inquisitive propositions do not invite other participants to choose between different alternatives. But still, they are proposals. They do not automatically establish a change of the common ground.

Thus, the notion of meaning in inquisitive semantics is directly related to the interactive process of exchanging information. Propositions, conceived of as proposals, give direction to this process. Changes of the common ground come about by mutual agreement among speech participants.

#### 2.2 Two Possibilities for Disjunction

An inquisitive semantics for the language of propositional logic has been specified and studied in detail (cf. Groenendijk, 2009b; Mascarenhas, 2009; Ciardelli and Roelofsen, 2009). The crucial aspect of this semantics is the interpretation of *disjunction*. To see how the inquisitive treatment of disjunction differs from the classical treatment, consider figure 1 below.



**Fig. 1.** (a) the traditional and (b) the inquisitive picture of  $p \lor q$ 

Figure 1(a) depicts the traditional proposition associated with  $p \lor q$ , consisting of all indices in which either p or q, or both, are true (in the picture, 11 is the

index in which both p and q are true, 10 is the index in which only p is true, etcetera). Figure 1(b) depicts the proposition associated with  $p \lor q$  in inquisitive semantics. It consists of two possibilities. One possibility is made up of all indices in which p is true, and the other of all indices in which q is true.

Thus,  $p \lor q$  is inquisitive. It invites a response which is directed at chosing between two alternatives. On the other hand,  $p \lor q$  also proposes to exclude one index, namely the index in which both p and q are false. This illustrates two things: first, that  $p \lor q$  is informative, just as in the classical analysis, and second, that, unlike in the classical analysis, sentences can be informative and inquisitive at the same time. We call such sentences *hybrid* sentences.

#### 2.3 Non-inquisitive Closure and Negation

The classical proposition in figure 1(a) is non-inquisitive: it consists of a single possibility, which is the union of the two possibilities in the inquisitive proposition in figure 1(b). In general, the non-inquisitive proposition that is classically expressed by a sentence  $\varphi$  is expressed in inquisitive semantics by the *non-inquisitive closure*  $!\varphi$  of  $\varphi$ . The proposition expressed by  $!\varphi$  always consists of a single possibility, which is the union of the possibilities for  $\varphi$ . In particular, the proposition depicted in figure 1(a) is expressed by  $!(p \lor q)$ .

The non-inquisitive closure operator, !, is not a basic operator in the language, but is defined in terms of negation. The proposition expressed by a negation  $\neg \varphi$ is taken to contain (at most) one possibility, which consists of all the indices that are not in any of the possibilities for  $\varphi$ , i.e., the indices that are not in the union of the possibilities for  $\varphi$ . Hence the proposition expressed by  $\neg \neg \varphi$  will always contain (at most) one possibility, consisting exactly of the indices that are in the union of the possibilities for  $\varphi$ . Thus, ! $\varphi$  can be defined as  $\neg \neg \varphi$ .

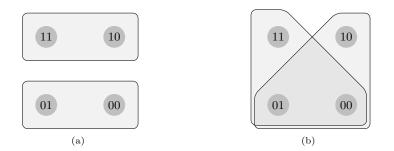
Notice that it follows from this analysis of negation that  $\neg \neg \varphi$  and  $\varphi$  are not fully equivalent. They are from a purely informative perspective in that  $\neg \neg \varphi$  and  $\varphi$  always exclude the same possibility, but whereas  $\varphi$  can be inquisitive,  $\neg \neg \varphi$  never is. That is why  $!\varphi$  is called the non-inquisitive closure of  $\varphi$ .

#### 2.4 Questions

As a consequence of the inquisitive treatment of disjunction, a classical tautology like  $p \lor \neg p$  is associated with two possibilities as well: the possibility that p and the possibility that  $\neg p$ . This means that in inquisitive semantics,  $p \lor \neg p$  can be taken to express the polar question whether p. It turns out that this observation does not only apply to atomic sentences, but also to more complex sentences. So, in general, a non-informative sentence  $\varphi \lor \neg \varphi$  can express a question, adding the possibility that  $\neg \varphi$  as an alternative to the possibility or possibilities for  $\varphi$ , and is therefore abbreviated as  $?\varphi$ .

One important empirical issue that has partly driven the development of inquisitive semantics so far is the analysis of *conditional* questions (e.g., *If Alf comes to the party, will Bea come as well?*). It is problematic for classical analyses of questions (Hamblin, 1973; Karttunen, 1977; Groenendijk and Stokhof,

1984) to predict that the answers to a conditional question  $p \rightarrow ?q$  are  $p \rightarrow q$ (yes, if Alf comes, Bea will come as well) and  $p \rightarrow \neg q$  (no, if Alf comes, Bea won't come) (cf. Velissaratou, 2000; Isaacs and Rawlins, 2008). In inquisitive semantics, this prediction is straightforwardly obtained by the interaction of the inquisitive interpretation of disjunction and the interpretation assigned to conditional sentences. Figure 2 depicts the inquisitive treatment of a polar question, ?p, and a conditional question,  $p \rightarrow ?q$ .



**Fig. 2.** (a) the polar question ?p and (b) the conditional question  $p \rightarrow ?q$ 

#### 2.5 Alternatives

Inquisitive propositions are taken to be sets of *alternative* possibilities. The significance of alternatives is widely recognized in semantics and pragmatics. For instance, sets of alternatives have been argued to play a crucial role in the analysis of focus (cf. Rooth, 1985), and in the treatment of indefinites and disjunction (cf. Kratzer and Shimoyama, 2002; Alonso-Ovalle, 2006).

What is new about inquisitive semantics is that it puts the inquisitive aspect of meaning directly at the heart of the notion of semantic content, and does not treat it as a collateral feature. The new conception of propositions as proposals, and the shift to a conversation oriented logic that it brings along, provide philosophical and mathematical foundations for research in the above-mentioned linguistic traditions, and may pave the way for more extensive applications.

#### 2.6 A Hierarchy of Alternativehood

A question that has played a fundamental role in the development of inquisitive semantics is when two or more possibilities should count as alternatives. We have moved from a very strict notion of alternatives as sets of mutually exclusive possibilities (blocks in a partition), via intermediate notions, to a rather weak notion where a set of possibilities counts as a set of alternatives iff it is not the case that one of the possibilities is included in another. This means, in particular, that alternative possibilities may overlap in quite dramatic ways. For instance, not only the possibilities in figure 1 and figure 2 above count as alternatives, but also those in figure 3 below.

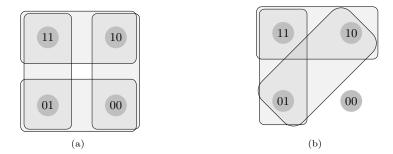


Fig. 3. Alternative possibilities with a high degree of overlap.

In the order in which these pictures are presented in figure 2 and 3, they exemplify increasingly weaker notions of alternativehood. This 'hierarchy' of alternativehood appears to be relevant in several respects. First, as propositions correspond to weaker alternatives, they are less straightforwardly expressible in natural language. Second, a process of 'alternative strengthening' seems to play an important role in a range of semantic and pragmatic phenomena. Informal observations of this kind have been made in the literature from time to time (cf. Zimmermann, 2000). Inquisitive semantics might provide a principled explanation of these phenomena.

## **3** Inquisitive Semantics

In this section we define an inquisitive semantics for a propositional language, which is based on a finite set of propositional variables, and has  $\neg$ ,  $\land$ ,  $\lor$ , and  $\rightarrow$  as its basic logical operators. We add two non-standard operators: ! and ?. ! $\varphi$  is defined as  $\neg \neg \varphi$ , and ? $\varphi$  is defined as  $\varphi \lor \neg \varphi$ . ! $\varphi$  is called the *non-inquisitive* closure of  $\varphi$ , and ? $\varphi$  is called the *non-informative closure* of  $\varphi$ .

#### 3.1 Support, Possibilities, and Propositions

The basic ingredients for the semantics are *indices* and *states*. An index is a binary valuation for the atomic sentences in the language. A state is a non-empty set of indices. We use v as a variable ranging over indices, and  $\sigma, \tau$  as variables ranging over states. The set of all indices is denoted by  $\omega$ , and the set of all states is denoted by S.

The proposition expressed by a sentence  $\varphi$  is defined indirectly, via the notion of *support* (just as, classically, the proposition expressed by a sentence is usually defined indirectly in terms of truth). We read  $\sigma \models \varphi$  as *state*  $\sigma$  *supports*  $\varphi$ . Support is recursively defined as follows.

## Definition 1 (Support).

1.  $\sigma \models p$  iff  $\forall v \in \sigma : v(p) = 1$ 2.  $\sigma \models \neg \varphi$  iff  $\forall \tau \subseteq \sigma : \tau \not\models \varphi$ 3.  $\sigma \models \varphi \lor \psi$  iff  $\sigma \models \varphi$  or  $\sigma \models \psi$ 4.  $\sigma \models \varphi \land \psi$  iff  $\sigma \models \varphi$  and  $\sigma \models \psi$ 5.  $\sigma \models \varphi \rightarrow \psi$  iff  $\forall \tau \subseteq \sigma : if \tau \models \varphi$  then  $\tau \models \psi$ 

It may be worth emphasizing that  $\sigma$  and  $\tau$  are always non-empty. So  $\tau \subseteq \sigma$  implies that  $\tau$  is a non-empty subset of  $\sigma$ . The clauses can be paraphrased as follows:<sup>1</sup>

- 1. A state  $\sigma$  supports an atomic sentence p iff every index in  $\sigma$  makes p true;
- 2. A state  $\sigma$  supports a negation  $\neg \varphi$  iff no substate of  $\sigma$  supports  $\varphi$ ;
- 3. A state  $\sigma$  supports a disjunction iff it supports at least one of the disjuncts;
- 4. A state  $\sigma$  supports a conjunction iff it supports both conjuncts;
- 5. A state  $\sigma$  supports an implication  $\varphi \to \psi$  iff every substate of  $\sigma$  that supports  $\varphi$  also supports  $\psi$ .

In terms of support, we define the possibilities for a sentence, and the proposition expressed by a sentence.

#### Definition 2 (Possibilities and Propositions).

- 1. A possibility for  $\varphi$  is a maximal state supporting  $\varphi$ , that is, a state that supports  $\varphi$  and is not properly included in any other state supporting  $\varphi$ .
- 2. The proposition expressed by  $\varphi$ , denoted  $\lfloor \varphi \rfloor$ , is the set of possibilities for  $\varphi$ .

We will illustrate the behavior of atomic sentences and the logical operators by means of the examples displayed in figure 4—most of which were already discussed informally in section 2. In doing so, it will be useful to distinguish between *classical* and *inquisitive* sentences.

## Definition 3 (Classical and Inquisitive Sentences).

- 1.  $\varphi$  is classical iff  $|\varphi|$  contains at most one possibility;
- 2.  $\varphi$  is inquisitive iff  $|\varphi|$  contains at least two possibilities.

Atoms. The proposition expressed by an atomic sentence p always consists of *exactly one* possibility: the possibility containing *all* indices that make p true. So atomic sentences are always classical.

**Negation.** The proposition expressed by  $\neg \varphi$  always consists of *at most* one possibility. If there are indices that make  $\varphi$  false (classically speaking), then the unique possibility for  $\neg \varphi$  consists of all such indices; if there are *no* indices that make  $\varphi$  false, then there is no possibility for  $\neg \varphi$ . In any case, negated sentences, like atomic sentences, are always classical.

<sup>&</sup>lt;sup>1</sup> Readers familiar with *intuitionistic logic* will notice that the notion of support is very similar to the notion of satisfaction in Kripkean semantics for intuitionistic logic. For an exploration of this connection, see (Mascarenhas, 2009; Ciardelli and Roelofsen, 2009).

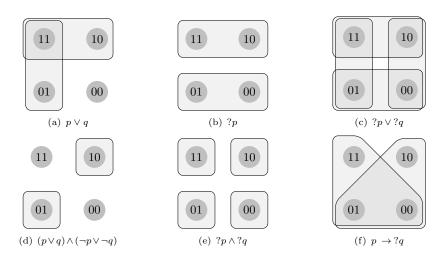


Fig. 4. Some examples of inquisitive sentences.

**Disjunction.** Disjunctions are typically inquisitive. To determine the proposition expressed by a disjunction  $\varphi \lor \psi$  we first collect all states that support  $\varphi$  or  $\psi$ . The maximal elements among these states are the possibilities for  $\varphi \lor \psi$ . Figures 4(a)–4(c) give some examples: a simple disjunction of two atomic sentences  $p \lor q$ , a polar question ?p (recall that ?p is defined as  $p \lor \neg p$ ), and the disjunction of two polar questions  $?p \lor ?q$ .

**Conjunction.** The proposition expressed by a conjunction  $\varphi \wedge \psi$  consists of all maximal states supporting both  $\varphi$  and  $\psi$ . If  $\varphi$  and  $\psi$  are both classical, then conjunction amounts to intersection, just as in the classical setting. If  $\varphi$  and/or  $\psi$  are inquisitive, then the conjunction  $\varphi \wedge \psi$  may be inquisitive as well. Figures 4(d) and 4(e) show what this amounts to for the conjunction of two disjunctions  $(p \lor q) \land (\neg p \lor \neg q)$  and the conjunction of two polar questions  $?p \land ?q$ .

**Implication.** The proposition expressed by  $\varphi \to \psi$  consists of all maximal states  $\sigma$  such that all substates of  $\sigma$  that support  $\varphi$  also support  $\psi$ . If the consequent  $\psi$  is classical, then  $\varphi \to \psi$  behaves just as it does in the classical setting: in this case,  $\lfloor \varphi \to \psi \rfloor$  consists of a single possibility, containing all indices that make  $\psi$  true or  $\varphi$  false. If the consequent  $\psi$  is inquisitive, then  $\varphi \to \psi$  may be inquisitive as well. Figure 4(f) shows what this amounts to for a conditional question  $p \to ?q$ . When  $\varphi$  and  $\psi$  are both inquisitive, then unlike when the consequent is classical, inquisitiveness of the antecedent may have an effect on the interpretation of the implication as a whole. For example, whereas  $!(p \lor q) \to ?r$  is a polar conditional question for which there are two possibilities corresponding to  $(p \lor q) \to r$  and  $(p \lor q) \to \neg r$ , the proposition expressed by  $(p \lor q) \to ?r$  contains two more possibilities which correspond to  $(p \to r) \land (q \to \neg r)$  and  $(p \to \neg r) \land (q \to r)$ . There is much more to say about implication, but that would take us too far

astray from the central concern of this paper (see, for instance, Groenendijk, 2009a; Ciardelli and Roelofsen, 2009).

#### 3.2 Truth-sets and Excluded Possibilities

Besides the proposition expressed by a sentence  $\varphi$  it will also be useful to speak of the *truth-set* of  $\varphi$ , and of the possibility *excluded by*  $\varphi$ .

**Definition 4 (Truth Sets).** The truth-set of  $\varphi$ , denoted by  $|\varphi|$ , is the set of indices where  $\varphi$  is classically true.

In a classical setting, the truth-set of  $\varphi$  is simply the proposition expressed by  $\varphi$ . In the inquisitive setting,  $|\varphi|$  is identical to the *union* of all the possibilities that make up the proposition expressed by  $\varphi$ . In both cases,  $|\varphi|$  embodies the *informative content* of  $\varphi$ : someone who utters  $\varphi$  proposes to eliminate all indices that are not in  $|\varphi|$  from the common ground.

#### Definition 5 (Excluded Possibility).

- 1. If  $\omega |\varphi| \neq \emptyset$ , then  $\omega |\varphi|$  is called the possibility excluded by  $\varphi$ ;
- 2. If  $\omega |\varphi| = \emptyset$ , then we say that  $\varphi$  does not exclude any possibility;
- 3. The (singleton- or empty) set of possibilities excluded by  $\varphi$  is denoted by  $\lceil \varphi \rceil$ .

The semantics for  $\neg$ , ?, and ! can be stated in a transparent way in terms of exclusion (recall that  $!\varphi$  was defined as  $\neg\neg\varphi$  and  $?\varphi$  as  $\varphi \lor \neg\varphi$ ).

## Fact 1 ( $\neg$ , ?, and ! in terms of exclusion).

1.  $\lfloor \neg \varphi \rfloor = \lceil \varphi \rceil$ 2.  $\lfloor !\varphi \rfloor = \lceil \neg \varphi \rceil$ 3.  $\lfloor ?\varphi \rfloor = \lfloor \varphi \rfloor \cup \lceil \varphi \rceil$ 

## 3.3 Questions, Assertions, and Hybrids

We already defined a sentence  $\varphi$  to be inquisitive just in case  $\lfloor \varphi \rfloor$  contains at least two possibilities. Uttering an inquisitive sentence is one way of making a significant contribution to a conversation. The other way in which a significant contribution can be made is by being *informative*. A sentence  $\varphi$  is informative iff there is at least one possibility for  $\varphi$ , and also a possibility that  $\varphi$  excludes.

#### Definition 6 (Informative Sentences).

 $\varphi$  is informative iff  $|\varphi|$  and  $[\varphi]$  both contain at least one possibility.

In terms of whether a sentence is inquisitive and/or informative or not, we distinguish the following four semantic categories:

	informative	inquisitive
question	—	+
assertion	+	_
hybrid	+	+
insignificant	_	_

A question is inquisitive and not informative, an *assertion* is informative and not inquisitive, a *hybrid* sentence is both informative and inquisitive, and an *insignificant* sentence is neither informative nor inquisitive. Some examples are provided in figure 5.

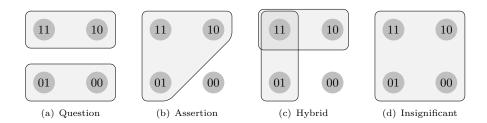


Fig. 5. One example for each of the four semantic categories.

It is a major feature of inquisitive semantics that questions and assertions are not distinguished syntactically, but are characterized semantically, next to hybrids. There is a single syntactic category of sentences in the language. In forming the disjunction of two sentences the semantic category of the resulting sentence can be different from the semantic category of either disjunct. Disjunction can turn two classical sentences into an inquisitive sentence. Negation has the opposite effect, it turns any sentence into a classical sentence.

#### 3.4 Inquisitive Entailment

Classically,  $\varphi$  entails  $\psi$  iff the proposition expressed by  $\varphi$  is contained in the proposition expressed by  $\psi$ . In inquisitive semantics, every possibility for  $\varphi$  must be contained in some possibility for  $\psi$ .

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Definition 7 (Entailment). \varphi \models \psi \quad iff \quad \forall \alpha \in \lfloor \varphi \rfloor : \exists \beta \in \lfloor \psi \rfloor : \alpha \subseteq \beta
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Entailment may also be formulated in terms of support rather than in terms of possibilities. This formulation is analogous to the classical formulation of entailment in terms of truth.

# Fact 2 (Entailment in terms of support).

 $\varphi \models \psi \quad \textit{iff} \quad \textit{every state that supports } \varphi \ \textit{also supports } \psi.$ 

It is immediately clear from the definition of entailment and the interpretation of implication that the two notions are, as usual, closely related:

#### Fact 3 (Entailment and Implication). $\varphi \models \psi \quad iff \quad \models \varphi \rightarrow \psi$

If  $\psi$  is classical, then inquisitive entailment boils down to classical entailment:

**Fact 4.** If  $\psi$  is classical, then  $\varphi \models \psi$  iff  $\varphi$  classically entails  $\psi$ .

If  $\psi$  is classical, then there is at most a single possibility for  $\psi$ , which equals  $|\psi|$ . Then,  $\varphi$  entails  $\psi$  iff every possibility for  $\varphi$  is included in  $|\psi|$ . This is the case iff the union of all the possibilities for  $\varphi$  is included in  $|\psi|$ , that is, iff  $|\varphi| \subseteq |\psi|$ .

**Fact 5.** For every sentence  $\varphi$ ,  $\varphi \models !\varphi$ .

Every possibility for  $\varphi$  is included in the single possibility for  $!\varphi$ . The reverse, however, does not always hold. In particular, it does not hold if  $\varphi$  is inquisitive. For instance, the hybrid disjunction  $p \lor q$  entails its non-inquisitive closure  $!(p \lor q)$ , but the reverse does not hold (this can easily be seen by inspecting figure 1).

**Fact 6.** For every sentence  $\varphi$ ,  $\varphi \models ?\varphi$  and  $\neg \varphi \models ?\varphi$ .

The non-informative closure  $?\varphi$  of a sentence  $\varphi$  is entailed by  $\varphi$  itself, by its negation  $\neg \varphi$ , and therefore also by its non-inquisitive closure  $!\varphi$ . But, whereas classically *any* sentence  $\psi$  entails  $?\varphi$  (that is,  $\varphi \lor \neg \varphi$ ), this does not hold inquisitively. For instance,  $p \not\models ?q$ .

If an assertion  $!\varphi$  entails a question  $?\psi$ , then  $!\varphi$  completely resolves the issue raised by  $?\psi$ . To some extent this means that  $!\varphi \models ?\psi$  characterizes answerhood. We say to some extent since it only characterizes complete and not partial answerhood, and it is not very 'precise' in characterizing complete answerhood in that it allows for over-informative answers: if  $!\varphi \models ?\psi$  and  $!\chi \models !\varphi$ , then also  $!\chi \models ?\psi$ .

For some questions, but not for all, we can characterize precise and partial answerhood in terms of entailment by saying that  $!\varphi$  is an answer to  $?\psi$  iff  $?\psi \models ?!\varphi$ . The intuition here is that  $!\varphi$  is an answer to  $?\psi$  just in case the polar question  $?!\varphi$  behind  $!\varphi$  is a subquestion of  $?\psi$  (cf. Groenendijk, 1999; ten Cate and Shan, 2007).

This characterization gives correct results as long as we are dealing with sentences that satisfy the strong notion of alternativehood where every two possibilities mutually exclude each other. However, given the weak notion of alternativehood adopted in inquisitive semantics,  $?\psi \models ?!\varphi$  does not give us a general characterization of answerhood, and neither does  $?\varphi \models ?\psi$  give us a general characterization of subquestionhood.

Problems arise as soon as we consider questions with *overlapping* possibilities. Conditional questions and alternative questions are questions of this kind. First, consider a conditional question  $p \to ?q$  (If Alf goes to the party, will Bea go as well?). We certainly want  $p \to q$  to count as an answer to this question, but  $p \to ?q \not\models ?(p \to q)$ . This can easily be seen by inspecting the propositions expressed by  $p \to ?q$  and  $?(p \to q)$ , depicted in figure 6(a) and 6(b). In fact, entailment goes in the other way in this case:  $?(p \to q) \models p \to ?q$ .

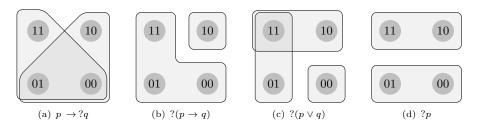


Fig. 6. Questions with overlapping possibilities are problematic for characterizations of answerhood and subquestionhood in terms of entailment.

Similarly, we certainly want p to count as an answer to the alternative question  $?(p \lor q)$  (*Does* ALF or BEA go to the party?). But  $?(p \lor q) \not\models ?p$ , as can be seen by comparing figure 6(c) and 6(d).

This does not mean that there is anything wrong with the entailment relation as such. It does what it should do: provide a characterization of meaninginclusion. As noted above, entailment between an assertion and a question means that the assertion fully resolves the issue raised by the question, and entailment between two questions  $?\varphi$  and  $?\psi$  means that the issue raised by  $?\psi$  is fully resolved whenever the issue raised by  $?\varphi$  is.

At the same time, given that entailment does not lead to a general notion of answerhood and subquestionhood, we surely are in need of a logical notion that does characterize these relations. The notion of *compliance*, to be defined in section 5, will—among other things—fulfil this role.

## 4 Inquisitive Information Exchange

The previous two sections were concerned with inquisitive semantics. We now turn to pragmatics. We will be concerned with a particular type of conversation a particular type of 'language game' if you want—which is geared towards the exchange of information. The present section offers an informal analysis of the regulative principles that we take to underlie the behavior of the participants of this type of conversation. Section 5 will develop the logical tools that are necessary to capture these principles, and section 6 will illustrate how inquisitive pragmatics can be used to explain certain well-known, but ill-understood pragmatic inferences.

It will be clear that our analysis is very much in the spirit of (Grice, 1975). However, there are also important differences between our framework and that of Grice. Inquisitiveness is, of course, the main source of these differences.

#### 4.1 Maintain Your Information State!

To analyze the exchange of information through conversation, we first of all assign an *information state* to each conversational participant. The simplest and

most standard way to model such a state is as a set of indices, where each index represents a possible way the world may be according to the beholder of that state. We take states to be *non-empty* sets of indices. That is, we assume information states to be consistent.

We think of the information state of each participant as embodying what that participant *takes himself to know*<sup>2</sup>, and we assume that every participant is *aware* of what he takes himself to know.

To *update* a state with new information is to *eliminate indices* from it. The amount of information increases as the number of indices in a state decreases.

We assume that, in the process of exchanging information, all participants in a conversation *maintain their own information state*, and assume each other to do so. This means that each participant makes sure that, at all times, his information state does indeed embody what he takes himself to know. In particular, no participant will update his state with information that is inconsistent with what he already takes himself to know. Doing so would eliminate all indices from his information state. This should never happen. Every participant must be sure to maintain a consistent state.

#### 4.2 Trust and be Truthful!

When entering a conversation with the purpose of exchanging information, you should be prepared to *trust* the quality of the information communicated by others, although you are aware of the fact that just like your own information, the information of others might eventually turn out to be incorrect. Adopting new information might deteriorate your state. But that's all in the game of information exchange. Don't enter the game if you don't dare to take a risk.

We will not, and cannot, set limits to how far trust should go, except that the requirement to maintain your own information state dictates that you will never go as far as to trust information that goes against what you already take yourself to know.

For a proper balance between the threat of deterioration and the prospect of enhancement of your state, your trust should be met by *truthfulness* on part of the others. That is, you should be able to expect that other participants only communicate information that they take themselves to know.

Fortunately, this delicate balance holds for everyone involved in a conversation. So, to make a long story short, in the end, to trust and be truthful is the only rational thing for everyone to do (cf. Lewis, 1969).

<sup>&</sup>lt;sup>2</sup> One important simplification here is that we do not distinguish between different types of information, such as *direct information* obtained by observation, *indirect information* obtained by reasoning, *information from hearsay*, obtained by conversation, etc. The significance of these distinctions is reflected by the fact that in many languages, sentences are obligatorily marked by so-called *evidentials* in order to communicate what kind of information is involved (cf. Murray, 2009).

#### 4.3 Maintain the Common Ground!

In line with standard practices, we take the *common ground of a conversation* to be the body of information that has been established by the conversation so far (cf. Stalnaker, 1978). The common ground is modeled as an information state, only it is not *private* to one of the participants, but *public* to all of them.

The *initial* common ground of a conversation is  $\omega$ , the state in which no information has been established yet. Every time an informative sentence is uttered, the common ground is updated with the information provided, unless one of the participants objects to this update (for instance, because the given information is inconsistent with his own information state). In this way, the common ground gradually comes to embody more and more information.

The information embodied by the common ground is supposed to be *common* information. That is, if some piece of information is supported by the common ground, then it should also be supported by the state of every individual participant. In terms of indices, if an index has been eliminated from the common ground, then it should also be absent from each individual information state. Certainly, this is true for the initial common ground  $\omega$ . It is the common responsibility of all participants that it remains true throughout the conversation.

It follows from this requirement that all participants should be truthful. If one participant would untruthfully convey some piece of information, and none of the other participants would protest, then the common ground would be updated with the information provided, which is not supported by the state of the untruthful participant. Hence, the common ground would no longer embody common information. Some indices would be removed from the common ground, but remain present in the information state of the untruthful participant.<sup>3</sup>

If all participants are required to maintain the common ground, it also follows that, if one participant conveys a certain piece of information, then every other participant should either update his own state with the information provided, or publicly announce that he is not willing to do so. For suppose that one participant would not update his state with the information provided, and would also refrain from publicly announcing his unwillingness to do so. Then the common ground would be updated with the given piece of information, but the state of the unwilling participant would not be. As a result, the common ground would no longer embody common information. Some indices would be eliminated from the common ground, but remain present in the information state of the unwilling participant.

In particular, this means that if the information provided by one participant is inconsistent with the state of another participant, then that other participant should publicly announce that he is unwilling to update with the information provided (updating would force him to give up his own state).

<sup>&</sup>lt;sup>3</sup> Note that the untruthful participant cannot update his state with the given information, because if he did, the state would no longer be in accordance with what he takes himself to know.

#### 4.4 The Internal Common Ground

We will sometimes refer to the common ground of a conversation as the *exter-nal* common ground, in order to explicitly distinguish it from another notion of common information which we will refer to as the *internal* common ground. The internal common ground is the *union* of all the individual information states. Thus, the internal common ground is itself a state, embodying what every participant in the conversation takes himself to know.<sup>4</sup>

The internal common ground differs from the external common ground in that it usually contains common information of which the participants are *not aware* that they have it in common. Such 'unconscious' common information is rather useless in that, for instance, it cannot form a basis for coordinated action. On the other hand, the common information that is embodied by the external common ground has been established by the conversation. Every participant will therefore be aware that this information is indeed common information.

Figure 7 illustrates how the external common ground, the internal common ground, and the individual participants' information states are related. If the external common ground is carefully maintained, then every individual state will be contained in it. In this case, the internal common ground, which is the union of all individual states, will also be contained in the external common ground. Each individual participant is aware of the boundaries of his own information state and those of the external common ground. In general, however, he will be in the dark about the information states of the other participants and about the internal common ground.

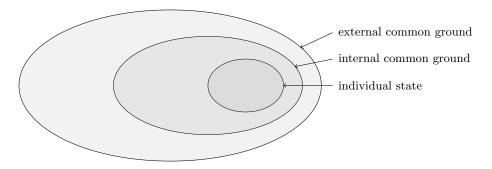


Fig. 7. An individual information state, the internal, and the external common ground.

<sup>&</sup>lt;sup>4</sup> Gerbrandy (1999) discusses the distinction between the external and internal common ground in detail. He shows that the two notions can only be appropriately connected if (i) information states do not contain 'higher order information', that is, information of one participant about the information states of others; and (ii) operations on states do not involve revision of information.

#### 4.5 Enhance the Common Ground!

The main purpose of a conversation—at least, of the type of conversation we are interested in here—is to exchange information, typically in order to (i) satisfy the informational needs of some of the individual participants, and/or (ii) to establish common information that is needed for coordinated action. Whatever the external goals of the conversation may be (personal information needs and/or a basis for coordinated action), the participants will always have to cooperatively *enhance the common ground* in order to achieve these goals.

Ideally, each conversational move either sets a new goal, or contributes to achieving goals that have been set previously. To set a new goal is to raise a new issue. Any move that does not set a new goal should contribute to enhancing the common ground in such a way that at least one of the existing issues may be resolved. One way to do so is to directly provide information that (partially) resolves one of the issues. If this is not possible, however, a conversational move may still make a significant contribution, namely by replacing one of the existing issues by an easier to answer sub-issue. That is, one way to contribute to achieving a given goal is to set an appropriate sub-goal.

In order to achieve a basis for coordinated action, it is not only necessary to establish common information, but also to ascertain that all participants are *aware* that this information is indeed common information. This can only be achieved by enhancing the external common ground.

In order to satisfy the personal information needs of some particular participant, it would in principle suffice to enhance the information state of that participant. However, in order to achieve this in a coordinated fashion, it is again crucial that the external common ground is enhanced. This is the only information state that is publicly accessible and manipulable by all participants.

The principles discussed above are all subservient to the desire to enhance the common ground. Clearly, the common ground cannot be enhanced if it is not maintained properly, and we have already seen that maintainance requires truthfulness. Trust is also essential, because if the participants refuse to trust each other, enhancement of the common ground is clearly impossible. Finally, a minimal requirement for cooperative information exchange is that every participant maintains his own information state. Thus, the fundamental driving force behind all these principles is the joint desire to enhance the common ground.

## 5 Inquisitive Logic and Conversation

The previous section provided an informal analysis of the regulative principles that play a role in conversations which are geared towards exchanging information. We now turn to the central logical notions of inquisitive pragmatics, which are intended to capture the essential features of these regulative principles.

#### 5.1 Basic Logical Notions

The first step is to relativize possibilities and propositions to information states.

#### Definition 8 (Relative Possibilities and Propositions).

- 1. A possibility for  $\varphi$  in  $\sigma$  is a maximal substate of  $\sigma$  supporting  $\varphi$ .
- 2. The proposition expressed by  $\varphi$  in  $\sigma$ , denoted by  $\sigma[\varphi]$ , is the set of possibilities for  $\varphi$  in  $\sigma$ .
- 3. We say that  $\varphi$  excludes a possibility in  $\sigma$  iff the union of all the possibilities for  $\varphi$  in  $\sigma$  is not identical to  $\sigma$  itself.

Next, we define relative notions of inquisitiveness and informativeness (the corresponding 'absolute' notions were introduced in section 3).

#### Definition 9 (Relative Inquisitiveness and Informativeness).

- 1.  $\varphi$  is inquisitive in  $\sigma$  iff there are at least two possibilities for  $\varphi$  in  $\sigma$ ;
- 2.  $\varphi$  is informative in  $\sigma$  iff there is a possibility for  $\varphi$  in  $\sigma$  and  $\varphi$  excludes a possibility in  $\sigma$ .

When we take the notions of inquisitiveness and informativeness defined here relative to  $\omega$ , they coincide with the absolute notions defined in section 3.

Notice that  $\varphi$  must satisfy two conditions in order to be informative in  $\sigma$ . It will be useful to be able to refer to these two conditions separately. Therefore, we introduce the notions of *acceptability* and *eliminativity*.

#### Definition 10 (Acceptability and Eliminativity).

- 1.  $\varphi$  is acceptable in  $\sigma$  iff there is at least one possibility for  $\varphi$  in  $\sigma$ ;
- 2.  $\varphi$  is eliminative in  $\sigma$  iff  $\varphi$  excludes a possibility in  $\sigma$ .

Informativity can then be formulated in terms of acceptability and eliminativity.

#### Fact 7 (Informativity, Acceptability, and Eliminativity).

 $\varphi$  is informative in  $\sigma$  iff  $\varphi$  is both acceptable in  $\sigma$  and eliminative in  $\sigma$ .

Similarly, support can be formulated in terms of inquisitivity and eliminativity.

#### Fact 8 (Support, Inquisitivity, and Eliminativity).

 $\varphi$  is supported by  $\sigma$  iff  $\varphi$  is neither inquisitive in  $\sigma$  nor eliminative in  $\sigma$ .

Finally, support and acceptability allow us to distinguish between two kinds of insignificant sentences.

#### Definition 11 (Contradictions and Tautologies).

- 1.  $\varphi$  is a contradiction iff it is unacceptable in  $\omega$ ;
- 2.  $\varphi$  is a tautology iff it is supported by  $\omega$ ;

Having established these basic logical notions, we are now ready to start formalizing the conversational principles discussed in section 4.

#### 5.2 Significance, Sincerity, and Transparency

A first, basic requirement is that every conversational move be *significant* with respect to the current common ground. There are essentially two ways in which a sentence can be significant: by being informative or by being inquisitive. This is captured by the following maxim:

**Definition 12 (Significance Maxim).** Any sentence that is uttered in a conversation should be informative or inquisitive w.r.t. the current common ground.

A second requirement is that every conversational move should be *sincere*. This requirement can be divided into two sub-requirements. On the one hand, if an utterance  $\varphi$  is eliminative, than the indices that it proposes to eliminate should all be indices that the speaker himself already considers impossible. Formally,  $\varphi$  should *not* be eliminative in the information state of the speaker. We will refer to this requirement as *informative sincerity*.

On the other hand, if  $\varphi$  is inquisitive with respect to the current common ground, then, under normal circumstances, it is expected to be inquisitive in the information state of the speaker. In particular, if  $\varphi$  is a question, then the speaker is, under normal circumstances, expected *not* to know the complete answer to that question. We will refer to this requirement as *inquisitive sincerity*.<sup>5</sup>

**Definition 13 (Sincerity Maxim).** Let  $\varphi$  be a sentence uttered by a speaker with state  $\varsigma$ , given a common ground  $\sigma$ . Then:

1.	$\varphi$ should not be eliminative in $\varsigma$	[Informative Sincerity]
2.	if $\varphi$ is inquisitive in $\sigma$ , then it should be inquisitive in	$\varsigma$ . [Inquisitive Sincerity]

Notice that informative sincerity corresponds to truthfulness. Inquisitive sincerity follows from the desire to enhance the common ground. To see this, consider a sentence  $\varphi$  which is inquisitive with respect to the common ground  $\sigma$ , and which is uttered by a speaker s with information state  $\varsigma$ . Suppose that s is not inquisitively sincere in uttering  $\varphi$ . This means that  $\varphi$  is not inquisitive in  $\varsigma$ . In other words,  $\varsigma$  supports an assertion  $!\psi$  which completely resolves the issue raised by  $\varphi$ .

Now, consider the contribution that  $\varphi$  makes toward enhancing the common ground  $\sigma$ . By assumption,  $\varphi$  is inquisitive in  $\sigma$ . Thus, (part of) the contribution of  $\varphi$  is to set a new goal or sub-goal. But this goal could just as well have been achieved directly by uttering  $!\psi$ . Moreover, if  $\varphi$  is informative in  $\sigma$  (as well as inquisitive), the information it provides would also be provided by  $!\psi$ . So, overall,  $!\psi$  would make a more substantial contribution toward enhancing the common

<sup>&</sup>lt;sup>5</sup> It should be emphasized that inquisitive sincerity cannot be assumed in all circumstances. For instance, if  $\varphi$  is a rhetorical question or an exam question, it is *not* supposed to be inquisitive in the information state of the speaker. The same is true when, for reasons of politeness, a speaker chooses to ask a suggestive question rather than to state the answer to that question directly.

ground than  $\varphi$  does. More generally, we may conclude that a conversational participant who strives to enhance the common ground will never utter a sentence that is inquisitive in the common ground but not in her own information state.

We now turn to a third requirement, which is that all participants should react to a given utterance in such a way that both the common ground and every individual information state is properly maintained. In particular, unacceptability should be publicly announced, and utterances to which no objection is made should be absorbed into the common ground and into every individual information state. We will call this requirement *transparency*.

**Definition 14 (Transparency Maxim).** Let  $\varphi$  be a sentence, and let r be a hearer with information state  $\varrho$ . Then:

- 1. If  $\varphi$  is unacceptable in  $\varrho$ , r should publicly announce this, and as a consequence,  $\varphi$  should neither be absorbed into the common ground nor into any individual information state.
- 2. If no participant makes any objections,  $\varphi$  should be absorbed into the common ground and into every individual information state.<sup>6</sup>

Significance and sincerity are both *speaker-oriented* requirements, while transparency is *hearer-oriented*. Together, significance, sincerity, and transparency cover several aspects of the conversational principles discussed in section 4. Informative sincerity covers truthfulness, transparency partly covers the maintenance principles, while significance and inquisitive sincerity partly cover the enhancement principle. This is summarized in the table below:

	Maxim	Principle
Speaker-oriented	Informative sincerity	Enhance! Be truthful! Enhance!
Hearer-oriented	Transparency	Maintain!

Significance, sincerity, and transparency are absolute *qualitative* requirements. In the next two sections, we will introduce the logical notion of *compliance*, which is intended to capture *relatedness*, and the logical notion of *homogeneity*, which is intended to capture *quantitative* preferences for one sentence over another.

#### 5.3 Compliance

Just as entailment traditionally judges whether an argument is valid, the logical notion of *compliance* judges whether a certain conversational move is *related* to the foregoing discourse. We will see that compliance embodies a rather strict notion of relatedness. Before stating the formal definition, however, let us first discuss the basic logico-pragmatical intuitions behind it.

<sup>&</sup>lt;sup>6</sup> We have not been, and will not be, completely explicit about what it means for  $\varphi$  to be *absorbed* into the common ground or into an individual information state. See (Groenendijk, 2008) for more detailed discussion of this issue.

**Basic intuitions.** Consider a situation where a sentence  $\varphi$  is a *response* to an *initiative*  $\psi$ . We are mainly interested in the case where the initiative  $\psi$  is inquisitive, and hence proposes several alternatives. In this case, we consider  $\varphi$  to be an optimally compliant response just in case it picks out exactly one of the alternatives proposed by  $\psi$ . Such an optimally compliant response is an assertion  $\varphi$  such that the unique possibility  $\alpha$  for  $\varphi$  equals one of the possibilities for  $\psi$ :  $\lfloor \varphi \rfloor = \{\alpha\}$  and  $\alpha \in \lfloor \psi \rfloor$ . Of course, the responder is not supposed to choose randomly, but in accordance with the sincerity requirement, which means that his state  $\varsigma$  must be included in the alternative  $\alpha$ :  $\varsigma \subseteq \alpha$ .

If the state of the responder is not included in any of the alternatives proposed by  $\psi$ , such an optimally compliant response is not possible. However, it may still be possible in this case to give a compliant informative response, not by picking out *one* of the alternatives proposed by  $\psi$ , but by selecting some of them, and excluding others. The informative content of such a response must correspond with the union of some but not all of the alternatives proposed by  $\psi$ . That is,  $|\varphi|$  must coincide with the union of a proper non-empty subset of  $|\psi|$ .

If such an informative compliant response cannot be given without breaching the sincerity requirement, it may still be possible to make a significant compliant move, namely by responding with an inquisitive sentence, replacing the issue raised by  $\psi$  with an easier to answer sub-issue. The rationale behind such an inquisitive move is that, if part of the original issue posed by  $\psi$  were resolved, it might become possible to subsequently resolve the remaining issue as well.

Summing up, there are basically two ways in which  $\varphi$  may be compliant with  $\psi$ :

- (a)  $\varphi$  may partially *resolve* the issue raised by  $\psi$ ;
- (b)  $\varphi$  may *replace* the issue raised by  $\psi$  by an easier to answer sub-issue.

Combinations are also possible:  $\varphi$  may partially resolve the issue raised by  $\psi$  and at the same time replace the remaining issue with an easier to answer sub-issue.

**Compliance and over-informative answers.** As mentioned above, compliance embodies a rather strict notion of relatedness. In particular, it does not allow for over-informative answers. For instance, p and  $\neg p$  are compliant responses to ?p, but  $p \land q$  is not. More generally, if  $\psi$  is an inquisitive initiative, and  $\varphi$  and  $\chi$  are two assertive responses such that the unique possibility for  $\varphi$ coincides with one of the possibilities for  $\psi$ , and the unique possibility for  $\chi$  is properly included in the one for  $\varphi$ , then  $\varphi$  is regarded as optimally compliant, while  $\chi$  is not consistered to be compliant at all, because it is over-informative.

**Compliance and enhancing the common ground.** The fact that compliance discards over-informative responses may seem arbitrary at first sight, but it is in fact motivated by our general conversational principles. In the scenario just considered,  $\varphi$  and  $\chi$  both have the potential to enhance the common ground in such a way that the issue raised by  $\psi$  is resolved. However,  $\chi$ , by being overinformative, runs a higher risk of being unacceptable in the information state of one of the participants, and therefore of being rejected. This higher risk is completely unnecessary, given that the information provided by  $\varphi$  is sufficient to resolve the given issue. These considerations are captured by the following definition:<sup>7</sup>

**Definition 15 (Compliance).**  $\varphi$  is compliant with  $\psi$ ,  $\varphi \propto \psi$ , iff

- 1. every possibility in  $|\varphi|$  is the union of a set of possibilities in  $|\psi|$
- 2. every possibility in  $|\psi|$  restricted to  $|\varphi|$  is contained in a possibility in  $|\varphi|$

Here, the *restriction* of  $\alpha \in \lfloor \psi \rfloor$  to  $|\varphi|$  is defined to be the intersection  $\alpha \cap |\varphi|$ . To explain the workings of the definition, we will consider the case where  $\psi$  is an insignificant sentence, an assertion, a question, and a hybrid one by one.

If  $\psi$  is a contradiction, the first clause can only be met if  $\varphi$  is a contradiction as well. The second clause is trivially met in this case. Similarly, if  $\psi$  is a tautology, the first clause can only be met if  $\varphi$  is a tautology as well, and the second clause is also satisfied in this case. Thus, if  $\psi$  is insignificant,  $\varphi$  is compliant with  $\psi$  just in case  $\varphi$  and  $\psi$  are equivalent.

**Fact 9.** If  $\psi$  is insignificant, then  $\varphi \propto \psi$  iff  $|\varphi| = |\psi|$ .

Next, consider the case where  $\psi$  is an assertion. Then the first clause says that every possibility for  $\varphi$  should coincide with the unique possibility for  $\psi$ . This can only be the case if  $\varphi$  is equivalent to  $\psi$ . In this case, the second clause is trivially met. Thus, the only way to compliantly respond to an assertion is to confirm it.<sup>8</sup>

**Fact 10.** If  $\psi$  is an assertion, then  $\varphi \propto \psi$  iff  $|\varphi| = |\psi|$ .

If  $\psi$  is a question and  $\varphi$  is an assertion, then the first clause in the definition of compliance requires that  $|\varphi|$  coincides with the union of a set of possibilities for  $\psi$ . The second clause is trivially met in this case. Such an assertion provides information that is fully dedicated to partially resolving the issue raised by the question, and does not provide any information that is not directly related to the issue. Recall that at the end of section 3 we criticized the notion of entailment for not delivering a notion of 'precise' (partial) answerhood. This is precisely what compliance of assertions to questions characterizes.

**Fact 11.** If  $\psi$  is a question and  $\varphi$  an assertion, then  $\varphi \propto \psi$  iff  $|\varphi|$  coincides with the union of a set of possibilities for  $\psi$ .

If  $\varphi$  and  $\psi$  are both questions, then the first clause requires that  $\varphi$  is related to  $\psi$  in the sense that every complete answer to  $\varphi$  is at least a partial answer to  $\psi$ . In this case the second clause has work to do as well. However, since  $\varphi$  is assumed

<sup>&</sup>lt;sup>7</sup> This definition of compliance is slightly different from the one in (?). See appendix A for discussion and a correspondence result.

<sup>&</sup>lt;sup>8</sup> Recall that common ground maintenance requires a dialogue participant to explicitly reject an assertion  $\psi$  in case it is unacceptable in his own information state. Such a move is strictly speaking not compliant. However, it *is* compliant with respect to 'the question behind  $\psi$ ', which is, in the simplest case,  $\psi$ . For detailed discussion of this issue, see (Groenendijk, 2008).

to be a question, and since questions are not informative, the second clause can be simplified in this case: the restriction of the possibilities for  $\psi$  to  $|\varphi|$  does not have any effect, because  $|\varphi| = \omega$ . Hence, the second clause simply requires that every possibility for  $\psi$  is contained in a possibility for  $\varphi$  (i.e., that  $\psi$  entails  $\varphi$ ). This constraint prevents  $\varphi$  from being more difficult to answer than  $\psi$ .

We illustrate this with an example. Consider the case where  $\psi \equiv ?p \lor ?q$  and  $\varphi \equiv ?p$ . The propositions expressed by these sentences are depicted in figure 8.

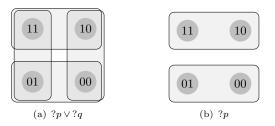


Fig. 8. Two non-compliant questions.

Intuitively,  $p \lor ?q$  is a choice question. To resolve it, one may either provide an answer to the question ?p or to the question ?q. Thus, there are four possibilities, each corresponding to an optimally compliant response:  $p, \neg p, q$  and  $\neg q$ . The question ?p is more demanding: there are only two possibilities and thus only two optimally compliant responses, p and  $\neg p$ . Hence, ?p is more difficult to answer than  $?p \lor ?q$ , and should therefore not count as compliant with it. This is not taken care of by the first clause in the definition of compliance, since every possibility for ?p is also a possibility for  $?p \lor ?q$ . So the second clause is essential in this case: it says that ?p is not compliant with  $?p \lor ?q$  because two of the possibilities for  $?p \lor ?q$  are not contained in any possibility for ?p. The fact that these possibilities are, as it were, 'ignored' by ?p is the reason that ?p is more difficult to answer than  $?p \lor ?q$ .

Recall that at the end of section 2, we criticized the notion of entailment for not delivering a satisfactory notion of subquestionhood. The difference with compliance, which does give the right characterization, lies in the first clause of the definition, which requires that the two questions are *related*. As we saw above, entailment only covers (the simplified version of) the second clause.

**Fact 12.** If both  $\psi$  and  $\varphi$  are questions, then  $\varphi \propto \psi$  iff

- 1. every possibility in  $|\varphi|$  is the union of a set of possibilities in  $|\psi|$
- 2. every possibility in  $\lfloor \psi \rfloor$  is contained in a possibility in  $\lfloor \varphi \rfloor$

The second clause in the definition of compliance only plays a role in case both  $\varphi$  and  $\psi$  are inquisitive. Moreover, the restriction of the possibilities for  $\psi$  to  $|\varphi|$  can only play a role if  $|\varphi| \subset |\psi|$ , which is possible only if  $\varphi$  is informative. Thus, the second clause can only play a role in its unsimplified form if  $\varphi$  is

both inquisitive and informative, i.e., hybrid. If  $\varphi$  is hybrid, just as when  $\varphi$  is a question, the second clause forbids that a possibility for  $\psi$  is ignored by  $\varphi$ . But now it also applies to cases where a possibility for  $\psi$  is partly excluded by  $\varphi$ . The part that remains should then be fully included in one of the possibilities for  $\varphi$ .

As an example where this condition applies, consider  $p \lor q$  as a response to  $p \lor q \lor r$ . One of the possibilities for  $p \lor q \lor r$ , namely |r|, is ignored by  $p \lor q$ : the restriction of |r| to  $|p \lor q|$  is not contained in any possibility for  $p \lor q$ . Again, this reflects the fact that the issue raised by  $p \lor q$  is more difficult to resolve than the issue raised by  $p \lor q \lor r$ .

A general characterization of what the second clause says, then, is that  $\varphi$  may only remove possibilities for  $\psi$  by providing information. A possibility for  $\psi$  must either be excluded altogether, or it must be preserved: its restriction to  $|\varphi|$  must be contained in some possibility for  $\varphi$ .

#### 5.4 Homogeneity: Say More, Ask Less!

There may be several possible compliant responses to a given initiative. Among these compliant responses, some may be preferable over others. The main point of this section—as is foretold by its title—is that there is a general preference for *more informative*, and *less inquisitive* responses. To make this more precise, let us first define comparative notions of informativeness and inquisitiveness.

#### Definition 16 (Comparative Informativeness and Inquisitiveness).

- 1.  $\varphi$  is at least as informative as  $\psi$  iff in every state where  $\psi$  is eliminative,  $\varphi$  is eliminative as well.
- 2.  $\varphi$  is at most as inquisitive as  $\psi$  iff in every state where  $\psi$  is not inquisitive,  $\varphi$  is not inquisitive either.

Note that although the first clause deals with comparative informativeness, it is (and should be) formulated in terms of eliminativity. If it were formulated in terms of informativity, it would give very counter-intuitive results. Suppose that we defined  $\varphi$  to be at least as informative as  $\psi$  iff in every state where  $\psi$  is informative,  $\varphi$  is informative as well. Then, for instance,  $p \wedge q$  would not count as more informative than p. To see this consider the state  $|\neg q|$ . In this state, p is informative, but  $p \wedge q$  is not, because it is unacceptable in  $|\neg q|$ . More generally, for any non-tautological sentence  $\chi$ , it would be impossible to find a formula that is more informative than  $\chi$ . This is clearly undesirable. Thus, in order to measure comparative informativeness, the acceptability aspect of informativeness must be left out of consideration—the only relevant feature is eliminativity.

Now let us motivate the general preference for more informative and less inquisitive responses to a given initiative. In each case, we will provide a general argument, and a concrete example. Say More! Consider an inquisitive initiative  $\psi$  and two compliant assertive responses  $\varphi$  and  $\chi$ , such that  $\varphi$  is more informative than  $\chi$ . This means that  $\varphi$  rules out more of the possibilities proposed by  $\psi$  than  $\chi$  does. In this sense,  $\varphi$  more fully resolves the issue raised by  $\psi$ , and thus makes a more substantial contribution to enhancing the common ground than  $\chi$  does. Therefore,  $\varphi$  is preferred over  $\chi$ .

To illustrate this with a concrete example, consider a conversation between two people, A and B. Suppose A utters  $?p \land ?q$ . This sentence expresses a proposition consisting of four possibilities (see figure 4). Now, consider q and  $p \rightarrow q$ , which are both compliant responses to A's initiative. q is more informative than  $p \rightarrow q$ . In particular, q rules out two of the possibilities proposed by  $?p \land ?q$ , while  $p \rightarrow q$  only rules out one of these possibilities. Therefore, q is preferred.

However, q is not yet optimal. An even more informative compliant response is  $p \wedge q$ . This response picks out exactly one of the possibilities proposed by  $p \wedge p \wedge q$ , and thus fully resolves the issue. In general, one compliant response is preferred over another if it more fully resolves the given issue.

Ask Less! Now consider an initiative  $\psi$  and an *inquisitive* compliant response  $\varphi$ . In this case,  $\varphi$  raises a sub-issue, which addresses the original issue in an indirect way. The hope is that the sub-issue may be resolved first, and that, subsequently, there will be a better chance of resolving the original issue as well. Now, this strategy will only work if it is indeed possible to resolve the sub-issue first. And this is more likely to be the case if  $\varphi$  is *less inquisitive*. This is why less inquisitive responses are generally preferred over more inquisitive responses.

To illustrate this, consider again the example sketched above. As before, suppose that A raises an issue by uttering  $?p \land ?q$  (see figure 4). But now suppose that B is not able to resolve this issue directly. Then he may try to resolve it indirectly by raising a sub-issue. Consider the following two sentences that B may utter in this situation: ?q and  $p \rightarrow ?q$  (see again figure 4). Now, it is very unlikely that A will have an answer to ?q, given that he has just asked  $?p \land ?q$  himself. On the other hand, it is not so unlikely that A will have an answer to  $p \rightarrow ?q$ . Intuitively, this question is weaker than ?q, it merely asks whether or not p and q are related in a certain way. Thus, it is much more advisable for B to ask  $p \rightarrow ?q$  than to ask ?q. Both ?q and  $p \rightarrow ?q$  are compliant with the original question. But  $p \rightarrow ?q$  is preferred because it is less inquisitive.

These considerations lead to the following definitions:

#### Definition 17 (Homogeneity).

 $\varphi$  is at least as homogeneous as  $\chi$ ,  $\varphi \succeq \chi$  iff  $\varphi$  is at least as informative and at most as inquisitive as  $\chi$ .

#### Definition 18 (Comparative Compliance).

 $\varphi$  is a more compliant response to  $\psi$  than  $\chi$  iff  $\varphi$  and  $\chi$  are both compliant responses to  $\psi$ , and  $\varphi$  is more homogeneous than  $\chi$ .

#### Definition 19 (Compliance Maxim). Be as compliant as you can!

Fact 13 provides a characterization of most and least compliant responses.

#### Fact 13 (Ultimate Compliance).

- 1.  $\varphi$  is a least compliant response to  $\psi$  iff  $\varphi$  is equivalent to  $\psi$ .
- 2.  $\varphi$  is a most compliant response to  $\psi$  iff there is a single possibility  $\alpha$  for  $\varphi$ , and  $\alpha$  is a possibility for  $\psi$  as well.
- If ψ is a question, φ is a most compliant non-informative response to ψ iff φ is a polar sub-question of ψ.

Fact 14 establishes the most essential features of homogeneity, and its relation with entailment.

## Fact 14 (Homogeneity and Entailment).

1. If  $\varphi \succeq \psi$ , then  $!\varphi \models !\psi$ 2.  $!\varphi \succeq !\psi$  iff  $!\varphi \models !\psi$ 3.  $?\varphi \succeq ?\psi$  iff  $?\psi \models ?\varphi$ 4. If  $!\varphi \equiv !\psi$ , then  $\varphi \succeq \psi$  iff  $\psi \models \varphi$ 5.  $!\varphi \succeq ?\psi$ 6.  $\bot \succeq \varphi$ 7.  $\top \succeq ?\varphi$ 

Item 1 says that, if  $\varphi$  is at least as homogeneous as  $\psi$ , then  $!\varphi$  entails  $!\psi$ , which is just to say that  $\varphi$  is at least as informative as  $\psi$ . Item 2 and 3 say that, as far as non-inquisitive sentences and non-informative sentences are concerned, homogeneity coincides with entailment. Item 4 says that, in case  $\varphi$  and  $\psi$  provide exactly the same information, homogeneity is the 'converse' of entailment. For, if  $\varphi$  and  $\psi$  provide exactly the same information, then  $\varphi$  is at least a homogeneous as  $\psi$  iff  $\varphi$  is at most as inquisitive as  $\psi$ , which in turn holds iff  $\psi \models \varphi$ . Item 5 says that any non-inquisitive sentence is at least as homogeneous as any non-informative sentence. A related fact is that any assertion is strictly more homogeneous than any question. Finally, item 6 says that contradictions are the most homogeneous of all sentences, and item 7 says that tautologies are the most homogeneous of all non-informative sentences.

Fact 15 establishes the main connection between homogeneity and compliance.

#### Fact 15 (Compliance Implies Homogeneity).

If  $\varphi$  is compliant with  $\psi$ , then  $\varphi$  is at least as homogeneous as  $\psi$ .

*Proof.* Suppose that  $\varphi$  is compliant with  $\psi$ . Then, according to the first clause of the definition of compliance, every possibility for  $\varphi$  is the union of a set of possibilities for  $\psi$ . But then we must have that  $|\varphi| \subseteq |\psi|$ , which means that  $\varphi$  is at least as informative as  $\psi$ .

It remains to be shown that  $\varphi$  is also at most as inquisitive as  $\psi$ . Suppose, towards a contradiction, that  $\varphi$  is strictly more inquisitive than  $\psi$ . Then there must be a state  $\sigma$  such that:

 $-\varphi$  is inquisitive in  $\sigma$ :  $\sigma[\varphi]$  contains at least two possibilities;  $-\psi$  is not inquisitive in  $\sigma$ :  $\sigma[\psi]$  contains at most one possibility.

We assumed  $\varphi$  to be compliant with  $\psi$ . This means that  $|\varphi| \subseteq |\psi|$ , and therefore  $\sigma \cap |\varphi| \subseteq \sigma \cap |\psi|$ .  $\sigma \lfloor \varphi \rfloor$  contains at least two possibilities, so  $\sigma \cap |\varphi|$  cannot be empty. Hence,  $\sigma \cap |\psi|$  cannot be empty either, which means that  $\sigma \lfloor \psi \rfloor$  contains at least one possibility. We already knew that  $\sigma \lfloor \psi \rfloor$  contains at most one possibility, so now we know that it contains *exactly* one possibility:  $\sigma \cap |\psi|$ .

Now consider the possibilities for  $\psi$  in general, not restricted to  $\sigma$ . There must be at least one possibility for  $\psi$  that contains  $\sigma \cap |\psi|$ . Call this possibility  $\alpha$ . We will show that the restriction of  $\alpha$  to  $|\varphi|$  is not contained in any possibility for  $\varphi$ , which contradicts the assumption that  $\varphi$  is compliant with  $\psi$ , and thus establishes the fact that compliance implies homogeneity.

We have that  $\sigma \cap |\varphi| \subseteq \sigma \cap |\psi|$ , and therefore  $\sigma \cap |\varphi| \subseteq \alpha$ . But we also have that  $\sigma \cap |\varphi| \subseteq |\varphi|$ . It follows that  $\sigma \cap |\varphi| \subseteq \alpha \cap |\varphi|$ .  $\sigma \cap |\varphi|$  cannot be contained in any possibility for  $\varphi$ , because  $\sigma \lfloor \varphi \rfloor$  contains at least two possibilities. Thus,  $\alpha \cap |\varphi|$  cannot be contained in any possibility for  $\varphi$  either.  $\Box$ 

We end this section with three short remarks. First, our maxims, like the Gricean maxims, are *regulative* principles that guide behavior in a certain type of conversation. They are not hard rules that people adhere to without exception. In calling our notion of relatedness *compliance*, we wanted to stress this point: sometimes you can't be compliant, and have every reason to give a non-compliant response instead. To give one example: in response to the question *Will Alf go to the party?*, the counter-question *Will Bea go?* is not compliant. But it may well be a good thing to ask in case a positive answer to your counter-question would make it possible for you to come back with *Then Alf goes as well*.

Second, homogeneity on the one hand and compliance and sincerity on the other can be thought of as 'opposing forces'. Homogeneity gives preference to more informative responses, while compliance does not allow over-informativity and informative sincerity does not allow responses that are eliminative in the responder's own information state. So compliance and informative sincerity impose, as it were, an upper bound on the informative aspect of homogeneity. On the other hand, homogeneity also gives preference to *less* inquisitive responses. But inquisitive sincerity requires that, if a sentence is inquisitive at all (in the common ground), then it should also be inquisitive in the responder's own information state. So inquisitive sincerity imposes a lower bound on the inquisitive aspect of homogeneity.

Third, the fact that over-informative answers do not count as compliant responses was motivated by the observation that such answers always run an unnecessary *risk* of being unacceptable in the information state of one of the participants. Of course, any response that is informative with respect to the common ground incurs such a risk. However, apart from the fact that the risk associated with an over-informative answer is partly *unnecessary*, there is also a difference between the expected acceptability of compliant answers and the expected acceptability of over-informative answers. This is illustrated by the following two conversations:

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- (1) a. A: Is Alf going to the party?
  - b. B: Yes, Alf and Bea are going.
  - c. A: That cannot be. I know that Bea is not going.
  - a. A: Is Alf going to the party?
  - b. B: Yes, Alf is going.

(2)

c. A: That cannot be. I know that Alf is not going.

The over-informative answer in (1-b) runs the risk of being unacceptable in A's information state. If it is indeed unacceptable, A will react as in (1-c), and B's response will not be absorbed into the common ground.

The compliant answer in (2-b), however, does not run any risk of being unacceptable, assuming that A's question in (2-a) was inquisitively sincere. For, under this assumption, B's response *must* be acceptable in A's information state. This explains the oddness of A's objection in (2-c). In general, someone who asks a question is very unlikely to reject a compliant response to that question. Thus, at least in a conversation with just two participants, compliant responses incur a very low risk of being unacceptable.

## 6 Inquisitive Implicatures of Alternative Questions

The inquisitive maxims of Significance, Sincerity, Transparency, and Compliance play much the same role as the maxims of Quality, Quantity, and Relation in Gricean pragmatics. In particular, they give rise to conversational *implicatures*. We will illustrate this by means of a small case-study, showing that certain well-known, but ill-understood phenomena involving *alternative questions* can be explained in terms of inquisitive conversational implicatures.

## 6.1 Not Neither

Consider the alternative question in (3), where SMALLCAPS are used to indicate intonational emphasis.

- (3) Will ALF or BEA go to the party?
  - a. Alf will go to the party.
  - b. #Neither Alf nor Bea will go.

We take it that (3), with the indicated intonation pattern, is translated into our propositional language as  $(p \lor q)$ . The proposition expressed by this sentence is depicted in figure 9(a). There are three possibilities for  $(p \lor q)$ , namely |p|, |q|, and  $|\neg p \land \neg q|$ . This means that (3-a) is correctly predicted to be an optimally compliant response. However, the *neither*-response in (3-b) is also predicted to be optimally compliant, whereas in reality this response is unexpected, and should be marked as such, for example by using an interjection like: *Well, actually...* We use the # sign to indicate that (3-b) requires such conversational marking.

We will propose a pragmatic explanation for the observation that the *nei*ther-response in (3-b) is an unexpected reaction to (3) and needs to be marked

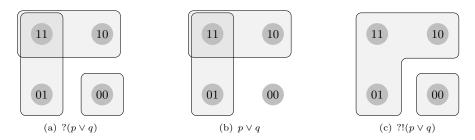


Fig. 9. Alternative question, hybrid disjunction, and polar disjunctive question.

as such. The explanation will be based on the idea that a sentence can come with certain pragmatic *suggestions* that go beyond its proper semantic content. Just as conversational implicatures, suggestions can be cancelled. Only, in inquisitive pragmatics cancelation is a cooperative affair: the required conversational marking of the response in (3-b) signals that the responder proposes to cancel a suggestion made by the utterance of the initiator. If this is the right analysis of interjections like *Well, actually...*, it remains to be explained why the alternative question in (3) suggests that the *neither*-response in (3-b) is unexpected.

Before we turn to that, we first consider a slightly different example, namely the disjunction in (4).

- (4) ALF OF BEA will go to the party.
  - a. (Yes,) Alf will go to the party.
  - b. No, neither Alf nor Bea will go.

We take it that (4), with the indicated intonation pattern, is translated into our logical language as the hybrid disjunction  $p \lor q$ . The proposition expressed by this sentence, which we have seen before, is depicted in figure 9(b).

Notice that the *neither*-response in (4-b) is preceded by an interjection, No, which signals rejection. What we want to point out with this example, however, is that there is a difference between canceling a pragmatic suggestion (which is what happens in (3-b)), and rejecting a proposal that is embodied by the semantic content of a sentence (which is what happens in (4-b)). Cancelation applies to pragmatic suggestions; rejection applies to semantic content. Having introduced this distinction, we now turn to the pragmatic explanation of why (3) suggests that (3-b) is unexpected. Homogeneity will play a key role.

The crucial observation is that the alternative question in (3), translated as  $?(p \lor q)$ , is more inquisitive, and therefore less homogeneous, than the polar question  $?!(p \lor q)$ , which we take to be the translation of the English polar question in (5).

- (5) Will Alf or Bea go to the party?
  - a. (Yes.) Alf or Bea will go.
  - b. (No.) Neither Alf nor Bea will go.

Notice that in English the polar question in (5) is distinguished from the alternative question in (3) by intonation.

The proposition expressed by the polar question  $?!(p \lor q)$  is depicted in figure 9(c). There are two possibilities for  $?!(p \lor q)$ , namely  $|!(p \lor q)|$  and  $|\neg(p \lor q)|$ , which are expressed by the optimally compliant responses in (5-a) and (5-b).

When comparing the polar question in (5) with the alternative question in (3) there are two things to note. First, the *neither*-response in (5-b) does not need any conversational marking, whereas it did as a reaction to (3). Second, the positive answer in (5-a) is a completely natural reaction to (5), whereas it would be decidedly odd as a response to (3).

From a logical-semantical perspective this last observation is unexpected.  $!(p \lor q)$  counts as a compliant response to  $?(p \lor q)$ , because the single possibility for  $!(p \lor q)$  is the union of two of the three possibilities for  $?(p \lor q)$ . From a semantic perspective, then,  $!(p \lor q)$  is a compliant response to  $?(p \lor q)$ ; it partially resolves the issue, by excluding the possibility that  $\neg p \land \neg q$ . But the pragmatic explanation for the oddity of (5-a) as a response to (3) is exactly the same as the explanation why (3-b) needs to be conversationally marked as a response to (3). Namely, if (3) already suggests that the *neither*-response is unexpected, then saying that at least one of the two will go is utterly redundant.

As mentioned above, the key to the explanation of the unexpectedness of the *neither*-answer is the observation that the polar question  $?!(p \lor q)$  is less inquisitive, and thus more homogeneous, than the alternative question  $?(p \lor q)$ . From the perspective of homogeneity, the polar question is therefore preferred over the alternative question. This means that, if the initiator asks the alternative question, there must be a specific reason why she did not ask the polar question, which is in principle preferred. And there is only one potential reason, namely that the polar question is not inquisitive in her own information state. If this is the case, uttering the polar question would not be inquisitively sincere. The quantitative preference is overruled by a qualitative restraint.

Thus, we are led to the conclusion that the polar question is not inquisitive in the initiator's information state, while the alternative question is. But this can only be the case if the initiator already assumes that at least one of Alf and Bea will go to the party. The *neither*-response is inconsistent with this assumption, and this is why it is unexpected.

#### 6.2 Not Both

We stay with the alternative question, repeated in (6), but now consider two additional responses, (6-c) and (6-d).

- (6) Will ALF or BEA go to the party?
  - a. Alf will go to the party.
  - b. #Neither Alf nor Bea will go.
  - c. #Both Alf and Bea will go.
  - d. Only Alf will go, Bea will not go.

A first observation is that (6) does not only suggest that the *neither*-response in (6-b) is unexpected, but also that the *both*-response in (6-c) is unexpected. Both the *neither*- and the *both*-response require conversational marking with an interjection like *Well*, *actually*....

A related observation is that the response in (6-d) needs no conversational marking. In fact, it functions in much the same way as the optimally compliant response in (6-a), despite the fact that it is, semantically speaking, an over-informative response to the given alternative question.

If indeed the alternative question suggests that the *both*-response is unexpected, then we have the basis for an explanation of the fact that the answer in (6-a) is pragmatically strengthened to what is explicitly expressed in (6-d). Responding with (6-a) indicates *acceptance* of the suggestion that Alf and Bea will not both go to the party. Otherwise, the responder should have opted for the more homogeneous response in (6-c). So, the alternative question in (6) and the response in (6-a) *together* generate the implicature that only Alf will go. The implicature is established in a cooperative fashion. The only difference between (6-a) and (6-d) is that in the latter the implicature is explicated. And because (6) already suggests that Alf and Bea will not both go to the party, the over-informativeness of (6-d) does not require any conversational marking.

What remains to be explained is why (6) suggests that the *both*-response is unexpected. In this case, homogeneity and compliance both play a key role. The crucial observation is that, although  $p \wedge q$  is more homogeneous than either por q, only the latter two are compliant responses to  $?(p \vee q)$ . There must be a reason why the initiator excluded the more homogeneous response  $p \wedge q$  from being compliant. And the only potential reason is that  $p \wedge q$  is *unacceptable* in her own information state. Thus, we are led to the conclusion that the initiator assumes that Alf and Bea will not both go to the party. The *both*-response is inconsistent with this assumption, and this is why it is unexpected.

## 7 Mission Accomplished?

We would consider our mission accomplished, if we managed to convince you, first of all, that it is possible to extend classical semantics in an easy and conservative way, leading to a new notion of meaning, that puts informativeness and inquisitiveness on equal footing.

Secondly, we also hope to have convinced you that the new view on semantics is of logical interest, that the new logical notions that inquisitive semantics gives rise to, most notably homogeneity and compliance, are just as interesting to study from a logical perspective as the classical notion of entailment.

Thirdly, and most importantly, we hope to have indicated that adding inquisitiveness to semantic content, leads to an interesting new perspective on pragmatics.

## A Compliance Old and New

In this appendix, we compare the definition of compliance given in section 5.3 with the original definition of compliance from (?):

**Definition 20 (Original Compliance).**  $\varphi$  is compliant with  $\psi$  iff

- 2. every possibility in  $\lfloor \varphi \rfloor$  is the union of a set of possibilities in  $\lfloor \psi \rfloor$
- 3.  $\varphi$  is at least as homogeneous as  $\psi$

For convenience, the new definition of compliance is repeated below:

**Definition 21** (New Compliance).  $\varphi$  is compliant with  $\psi$  iff

- 1. every possibility in  $|\varphi|$  is the union of a set of possibilities in  $|\psi|$
- 2. every possibility in  $\lfloor \psi \rfloor$  restricted to  $|\varphi|$  is contained in a possibility in  $\lfloor \varphi \rfloor$

Notice that the first and third clause of the original definition have dissappeared. Compliance is no longer defined in terms of significance and homogeneity. The reason for this is that compliance, significance, and homogeneity have independent motivation. Compliance is supposed to capture *relatedness*, while significance is a *qualitative* notion, and homogeneity is concerned with *quantitative* preferences. This distinction is blurred if one of the notions is defined in terms of the others. To avoid this, compliance is no longer defined in terms of significance and homogeneity, and each of the three notions has been motivated independently.

This said, however, the two definitions give essentially the same results (modulo significance). In stating and proving this fact, we will use n-compliance and o-compliance to refer to the new and the old notion of compliance, respectively.

# Theorem 1 (Old Compliance = Significance + New Compliance).

 $\varphi$  is o-compliant with  $\psi$  iff  $\varphi$  is significant and  $\varphi$  is n-compliant with  $\psi$ .

The theorem follows from fact 15 and the following lemma, where we refer to the second clause of the new definition as the *restriction clause*.

#### Lemma 1 (Homogeneity implies the Restriction Clause).

If  $\varphi$  is at least as homogeneous as  $\psi$ , then the restriction clause is satisfied.

*Proof.* We will show something slightly stronger:

If the restriction clause is not satisfied then  $\varphi$  is not at most as inquisitive as  $\psi$ .

Suppose that the restriction clause is not satisfied. This means that there is a possibility for  $\psi$ , call it  $\alpha$ , such that  $\alpha \cap |\varphi|$  is not contained in any possibility for  $\varphi$ .  $\alpha \cap |\varphi|$  is a set of indices, a state. For convenience, let's call this state  $\sigma$ . We will show that  $\varphi$  is inquisitive in  $\sigma$ , while  $\psi$  is not. This means that  $\varphi$  is not as most as inquisitive as  $\psi$ , which is exactly what we need to establish.

<sup>1.</sup>  $\varphi$  is significant

 $\psi$  is not inquisitive in  $\sigma$ , since  $\sigma$  is contained  $\alpha$ , which is a possibility for  $\psi$ . On the other hand,  $\varphi$  *is* inquisitive in  $\sigma$ , because (i)  $\forall v \in \sigma : v \models \varphi$  (this follows from the fact that  $\sigma \subseteq |\varphi|$ ), and (ii)  $\sigma \not\models \varphi$  (this follows from the fact that  $\sigma$  is not contained in any possibility for  $\varphi$ ). Given these two observations, we can construct two possibilities for  $\varphi$  in  $\sigma$ . Start with an arbitrary index  $v \in \sigma$ . We know that  $\{v\} \models \varphi$ . So we can add indices to  $\{v\}$  until we reach a maximal substate of  $\sigma$  supporting  $\varphi$ . This state, call it  $\beta$ , is the first possibility for  $\varphi$  in  $\sigma$ . We know that  $\{v'\} \models \varphi$ , so we can again extend  $\{v'\}$  to a maximal subset  $\beta'$  of  $\sigma$  supporting  $\varphi$ . We have that  $\beta' \neq \beta$  since  $v' \in \beta'$  but  $v' \notin \beta$ . So  $\beta$  and  $\beta'$  are two distinct possibilities for  $\varphi$  in  $\sigma$ . Hence,  $\varphi$  is inquisitive in  $\sigma$ .

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