How the Deontic Issue in the Miners' Puzzle Depends on an Epistemic Issue

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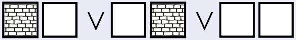
https://www.illc.uva.nl/inquisitivesemantics/

Kolodny and MacFarlane (2010)

The facts

- Miners are in one of two mine shafts.

- We can block either shaft.



- Blocking the correct mine shaft saves all miners.
- Blocking the wrong mine shaft kills all miners.
- Blocking neither mine shaft kills one miner.

Deontic question

(1) Ought shaft A, shaft B, or neither be blocked?
$?(\boxed{\vee}p' \vee \boxed{\vee}q' \vee (\boxed{\vee}\neg p' \wedge \boxed{\vee}\neg q'))$

Notation: $\diamondsuit_{\vee}, \boxed{\vee}$ are deontic, $\diamond, \Box$ epistemic modalities.

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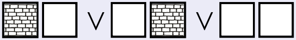
- We can block either shaft. 
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(1) Ought shaft A, shaft B, or neither be blocked?
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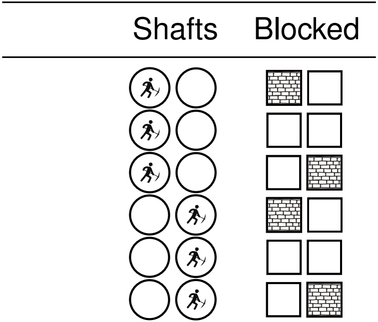
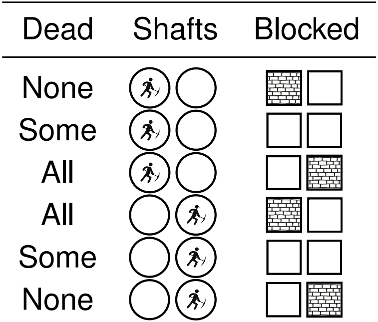
Notation: $\Diamond_v, \boxed{v}$ are deontic, $\Diamond, \Box$ epistemic modalities.

Ontic situation



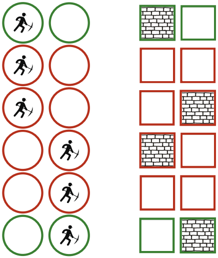
Ontic situation



Deontic situation

Aim: least dead

Dead	Shafts	Blocked
None		
Some		
All		
All		
Some		
None		



(2) Either shaft A or shaft B ought to be blocked. $\Box p' \vee \Box q'$

The puzzle

(3) a. The miners are in shaft A or shaft B.
$p \vee q$

b. If the miners are in shaft A, we ought to block it.
$p \rightarrow \boxed{v}\, p'$

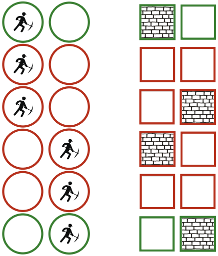
c. If the miners are in shaft B, we ought to block it.
$q \rightarrow \boxed{v}\, q'$

d. Hence, either shaft A or shaft B ought to be blocked.
$\boxed{v}\, p' \vee \boxed{v}\, q'$

Consider the information of the rescuers

Aim: least dead

Dead	Shafts	Blocked
None		
Some		
All		
All		
Some		
None		

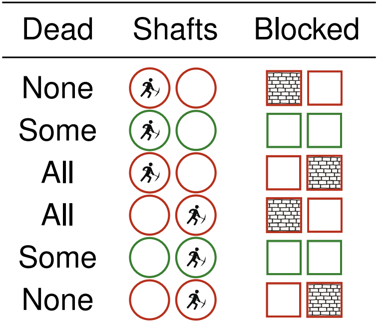


Might kill all miners

(4) Either shaft ought to be blocked. $\boxed{v} p' \vee \boxed{v} q'$

Intuition: neither ought to be blocked. $\boxed{v} \neg p' \wedge \boxed{v} \neg q'$

Deontic situation: miners' location unknown

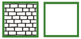
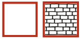


Answer to the deontic question in this epistemic state:

(5) Neither shaft ought to be blocked. $\boxed{\vee}\neg p' \wedge \boxed{\vee}\neg q'$

Deontic situation: miners' location known

Rescuers learn that the miners are in shaft A.

Dead	Shafts	Blocked
None		
Some		
All		

Answer to the deontic question in this epistemic state:

(6) Shaft A ought to be blocked. $\boxed{v}p'$

Deontic question depends on the epistemic one

Deontic question

(7) Ought shaft A be blocked, or shaft B, or neither?
$?(\boxed{V}p' \vee \boxed{V}q' \vee (\boxed{V}\neg p' \wedge \boxed{V}\neg q'))$

Epistemic question

(8) Is it the case that the miners might be in shaft *A* and they might be in *B*? $?(\Diamond p \wedge \Diamond q)$

a. Yes, they might be in shaft A and they might be in shaft B. $\Diamond p \wedge \Diamond q$

b. No, they must be in shaft A. $\Box p$

c. No, they must be in shaft B. $\Box q$

Deontic question depends on the epistemic one

Deontic question

(7) Ought shaft A be blocked, or shaft B, or neither?
$?(\boxed{v}\, p' \vee \boxed{v}\, q' \vee (\boxed{v} \neg p' \wedge \boxed{v} \neg q'))$

Epistemic question

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a. Yes, they might be in shaft A and they might be in shaft B. $\Diamond p \wedge \Diamond q$
b. No, they must be in shaft A. $\Box p$
c. No, they must be in shaft B. $\Box q$

Deontic question depends on the epistemic one

If the epistemic question is resolved, the deontic one is too.

- If the miners might be in shaft A and they might be in shaft B, then neither shaft ought to be blocked. $(\Diamond p \wedge \Diamond q) \rightarrow (\boxed{\vee}\neg p' \wedge \boxed{\vee}\neg q')$
- If the miners must be shaft A, shaft A ought to be blocked. $\Box p \rightarrow \boxed{\vee} p'$
- If the miners must be shaft B, shaft B ought to be blocked. $\Box q \rightarrow \boxed{\vee} q'$

Conclusion:

Full picture of the deontic information should distinguish epistemic possibilities.

Deontic question depends on the epistemic one

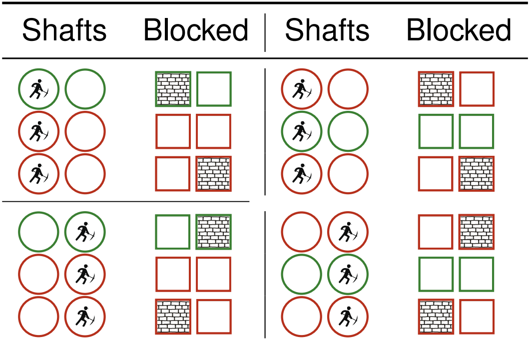
If the epistemic question is resolved, the deontic one is too.

- If the miners might be in shaft A and they might be in shaft B, then neither shaft ought to be blocked. $$(\Diamond p \wedge \Diamond q) \rightarrow (\boxed{v}\neg p' \wedge \boxed{v}\neg q')$$
- If the miners must be shaft A, shaft A ought to be blocked. $\Box p \rightarrow \boxed{v}\, p'$
- If the miners must be shaft B, shaft B ought to be blocked. $\Box q \rightarrow \boxed{v}\, q'$

Conclusion:

Full picture of the deontic information should distinguish epistemic possibilities.

Picture of the question dependency

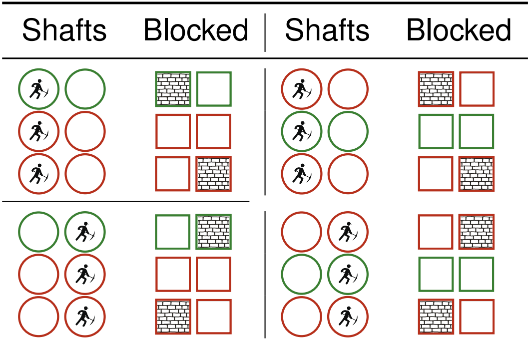


Answer doesn't depend only on the ontic information

(9) If the miners are in shaft A, shaft A ought to be blocked.
$p \rightarrow \boxed{v} p'$ [a]

[a] See von Fintel 2012 for discussion.

Picture of the question dependency



Answer doesn't depend only on the ontic information

(9) If the miners are in shaft A, shaft A ought to be blocked.

$p \rightarrow \boxed{v} p'$[a]

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[a] See von Fintel 2012 for discussion.

Epistemic versus ontic

Answer doesn't depend only on the ontic information

(10) If the miners are in shaft A, shaft A ought to be blocked.
$p \rightarrow \boxed{v} p'$

Conclusion:

The antecedent is taken to be the prejacent of a covert epistemic necessity operator, that contextually relates to the information of the person amenable to the obligation.

Deontic information models[1]

Ingredients of a deontic information model M

- **worlds** is a non empty set
- **states** powerset of the set of worlds
- **facts** maps atoms to $\{0, 1\}$ in each world
- **e-state** assigns an (information) state to each world
- **v-state** assign a (violation) state to each world

e-state(w)
the information state in *w* of a contextually given agent.

v-state(w)
the set of worlds where a rule that holds in *w* is violated.

[1] We're drawing on Aher and Groenendijk 2015 and Ciardelli and Roelofsen 2015.

Standard constraints on e-state:

- $\forall w, v \in$ **worlds**: if $v \in$ **e-state**(w), then **e-state**$(v) =$ **e-state**(w) (Introspection)
- $\forall w \in$ **worlds**: $w \in$ **e-state**(w). (Trust)
- Introspection and Trust guarantee that **e-state** induces a partition on **worlds**.

Constraint on v-state:

- $\forall w, v \in$ **worlds**: **v-state**$(w) =$ **v-state**(v) (Indisputability)
- Indisputability guarantees that **v-state** rigidly characterizes a set of worlds: **bad** $= \{v \in$ **worlds** $\mid \exists w : v \in$ **v-state**$(w)\}$

Standard constraints on e-state:

- $\forall w, v \in \textbf{worlds}:$ if $v \in \textbf{e-state}(w)$, then $\textbf{e-state}(v) = \textbf{e-state}(w)$ (Introspection)
- $\forall w \in \textbf{worlds}: w \in \textbf{e-state}(w)$. (Trust)
- Introspection and Trust guarantee that **e-state** induces a partition on **worlds**.

Constraint on v-state:

- $\forall w, v \in \textbf{worlds}: \textbf{v-state}(w) = \textbf{v-state}(v)$ (Indisputability)
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Picture of the model for the miners' puzzle

	pq	$p'q'$		pq	$p'q'$
w_1	10	10	w_7	10	10
w_2	10	00	w_8	10	00
w_3	10	01	w_9	10	01
w_4	01	01	w_{10}	01	01
w_5	01	00	w_{11}	01	00
w_6	01	10	w_{12}	01	10

SEMANTICS[2]

- The semantics is information-based (not truth-based).
- We define by simultaneous recursion:
  - State σ in M supports φ $M,\sigma \models^{+} \varphi$
  - State σ rejects φ $M,\sigma \models^{-} \varphi$
  - State σ in M dismisses a supposition of φ $M,\sigma \models^{\circ} \varphi$
- We only present those elements of the semantic clauses that are immediately relevant here.

[2]We draw upon Groenendijk and Roelofsen 2015.

Support and suppositionality

Dismissal in the inconsistent state.

- Basic feature concerning dismissal:
  - The inconsistent state, $\emptyset$, does not support or reject any sentence, it suppositionally dismisses every sentence.

Support in a model.

- Notation convention:
  - $M \models^{+} \varphi \ := \ M, \mathbf{worlds}_M \models^{+} \varphi$
- A model M supports φ if the state consisting of all worlds in M supports φ.

Alternatives and Inquisitiveness.

Alternatives

- The (support) alternatives for φ, $\mathrm{alt}^{+}(\varphi)$ is the set of maximal states that support φ.
- The rejection alternatives for φ, $\mathrm{alt}^{-}(\varphi)$ is the set of maximal states that reject φ.

Inquisitiveness

- φ is inquisitive if there is more than one (support) alternative for φ.
- If there's only one (support) alternative for φ we denote it by $|\varphi|$.
- Inquisitiveness plays a role with phrasing the issues facing the rescuers.

Atomic sentences

Notational convention:

- $\forall_{\exists}$ represents universal quantification with existential import.

Clauses for atomic sentences:

- $M, \sigma \models^{+} p$ iff $\forall_{\exists} w \in \sigma : w(p) = 1$.
- $M, \sigma \models^{-} p$ iff $\forall_{\exists} w \in \sigma : w(p) = 0$.
- $M, \sigma \models^{\circ} p$ iff $\sigma = \emptyset$

Support alternatives for p and q'

	pq	$p'q'$		pq	$p'q'$
w_1	10	10	w_7	10	10
w_2	10	00	w_8	10	00
w_3	10	01	w_9	10	01
w_4	01	01	w_{10}	01	01
w_5	01	00	w_{11}	01	00
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$\text{alt}^+(p)$, $|p|$

	pq	$p'q'$		pq	$p'q'$
w_1	10	10	w_7	10	10
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w_5	01	00	w_{11}	01	00
w_6	01	10	w_{12}	01	10

$\text{alt}^+(q')$, $|q'|$

(11) a. The miners are in shaft *A*. p

b. Shaft *B* is blocked. q'

Negation

Clauses for negation

- $M, \sigma \models^{+} \neg\varphi$ iff $M, \sigma \models^{-} \varphi$.
- $M, \sigma \models^{-} \neg\varphi$ iff $M, \sigma \models^{+} \varphi$.
- $M, \sigma \models^{\circ} \neg\varphi$ iff $M, \sigma \models^{\circ} \varphi$

Fact (Double negation)

$\neg\neg\varphi \equiv \varphi$

Fact (Rejection = Support of negation)

$\text{alt}^{-}(\varphi) = \text{alt}^{+}(\neg\varphi)$

Support of q' = rejection of $\neg q'$

	pq	$p'q'$		pq	$p'q'$
w_1	10	10	w_7	10	10
w_2	10	00	w_8	10	00
w_3	10	01	w_9	10	01
w_4	01	01	w_{10}	01	01
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$\text{alt}^+(q')$

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w_4	01	01	w_{10}	01	01
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w_6	01	10	w_{12}	01	10

$\text{alt}^-(\neg q')$

$$\text{alt}^+(q') = \text{alt}^-(\neg q')$$

Supposability in suppositional inquisitive semantics

Supposability

Let $\alpha \in \text{alt}(\varphi)$, α is supposable in σ, $\sigma \lhd \alpha$ iff for all τ in between α and $\sigma \cap \alpha$: $\tau \models^{+} \varphi$

In all following examples, supposability boils down to consistency:

$\sigma \lhd \alpha$ iff $\sigma \cap \alpha \neq \emptyset$

Contextual epistemic possibility[3]

Support clause

$M,\sigma \models^{+} \Diamond\varphi$ iff $\forall_{\exists} w \in \sigma : \exists^{a} \alpha \in \text{alt}^{+}(\varphi) : \textbf{e-state}(w) \lhd \alpha$.

In every e-state compatible with σ some support-alternative for φ is supposable.

[a] Universal quantification over alternatives semantically captures free choice effects but then necessity no longer follows as a natural dual.

Relevant Example:

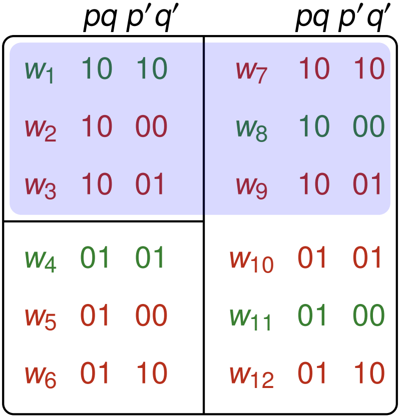
(12) The miners might be in shaft A. $\Diamond p$

Support of (12) boils down to:

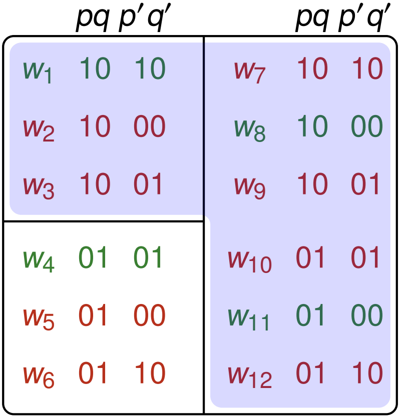
$M,\sigma \models^{+} \Diamond p$ iff $\forall_{\exists} w : \textbf{e-state}(w) \cap |p| \neq \emptyset$.

[3] See Aher and Groenendijk 2015.

Contextual epistemic possibility in the model



$|p|$



$\text{alt}^{+}(\Diamond p)$

$M \models^{+} \Diamond p$ iff $\forall w : \textbf{e-state}(w) \cap |p| \neq \emptyset$.

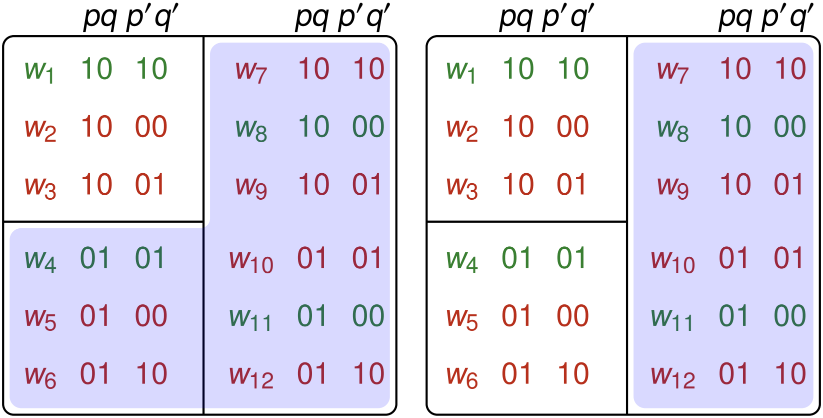
$M \not\models^{+} \Diamond p$

Conjunction

Support clause:

$$M, \sigma \models^{+} \varphi \wedge \psi \text{ iff } M, \sigma \models^{+} \varphi \text{ and } M, \sigma \models^{+} \psi$$

Conjunction of possible miner locations



$\text{alt}^+(\Diamond q)$ $\quad$ $\text{alt}^+(\Diamond p \wedge \Diamond q)$

$M \models^+ \Diamond q$ iff $\forall w$: **e-state**$(w) \cap |q| \neq \emptyset$.

$M \not\models^+ \Diamond p \wedge \Diamond q$

Obligation[4]

Support clause

$M, \sigma \models^{+} \boxed{v}\varphi$ iff $\forall_{\exists} \alpha \in \text{alt}^{-}(\varphi): \sigma \lhd \alpha$ and $\sigma \cap \alpha \subseteq$ **bad.**

Every reject alternative for φ is supposable in σ and when we suppose it, all remaining worlds are violation worlds

(13) Shaft B ought not to be blocked. $\boxed{v}\neg q'$

Support for (13) in the whole model boils down to:

$M \models^{+} \boxed{v}\neg q'$ iff $|q'| \neq \emptyset$ and $|q'| \subseteq$ **bad**

[4] The definition follows Aher 2013 and Aher and Groenendijk 2015.

The obligation not to block shaft B in the model

	pq	$p'q'$		pq	$p'q'$
w_1	10	10	w_7	10	10
w_2	10	00	w_8	10	00
w_3	10	01	w_9	10	01
w_4	01	01	w_{10}	01	01
w_5	01	00	w_{11}	01	00
w_6	01	10	w_{12}	01	10

$|q'|$

	pq	$p'q'$		pq	$p'q'$
w_1	10	10	w_7	10	10
w_2	10	00	w_8	10	00
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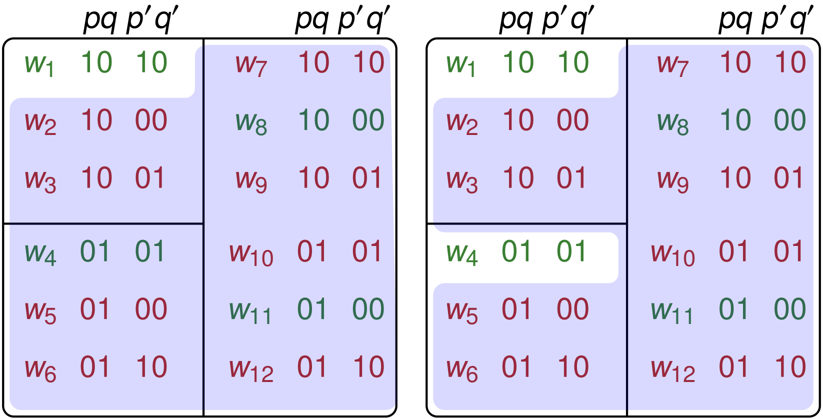
w_5	01	00	w_{11}	01	00
w_6	01	10	w_{12}	01	10

$\mathrm{alt}^+(\boxed{\vee}\neg q')$

$M \models^+ \boxed{\vee}\neg q'$ iff $|q'| \neq \emptyset$ and $|q'| \subseteq$ **bad**

$M \not\models^+ \boxed{\vee}\neg q'$

Obligation to block neither



$\text{alt}^+(\boxed{v}\neg p')$ $\quad$ $\text{alt}^+(\boxed{v}\neg p' \wedge \boxed{v}\neg q')$

$$M \not\models^+ \boxed{v}\neg p'$$

$$M \not\models^+ \boxed{v}\neg p' \wedge \boxed{v}\neg q'$$

Implication

Support clause:

$M, \sigma \models^+ \varphi \rightarrow \psi$ iff $\forall_\exists \alpha \in \text{alt}^+(\varphi)$: $\sigma \triangleleft \alpha$ and $M, \sigma \cap \alpha \models^+ \psi$.

Every support alternative α for φ is supposable in σ, and when we suppose it, then ψ is supported.

Relevant example

(14) If the miners might be in shaft *A* and they might be in shaft *B*, then neither shaft ought to be blocked.
$(\Diamond p \wedge \Diamond q) \rightarrow (\boxed{v}\neg p' \wedge \boxed{v}\neg q')$

For this example the clause boils down to:

$M \models^+ (\Diamond p \wedge \Diamond q) \rightarrow (\boxed{v}\neg p' \wedge \boxed{v}\neg q')$
iff $|\Diamond p \wedge \Diamond q| \cap |p'| \cap |q'| \neq \emptyset$ and $|\Diamond p \wedge \Diamond q| \cap |p'| \cap |q'| \subseteq \textbf{bad}$.

Implication

Support clause:

$M, \sigma \models^{+} \varphi \to \psi$ iff $\forall_{\exists} \alpha \in \mathrm{alt}^{+}(\varphi)$: $\sigma \lhd \alpha$ and $M, \sigma \cap \alpha \models^{+} \psi$.

Every support alternative α for φ is supposable in σ, and when we suppose it, then ψ is supported.

Relevant example

(14) If the miners might be in shaft A and they might be in shaft B, then neither shaft ought to be blocked.
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Implication

Support clause:

$M, \sigma \models^{+} \varphi \rightarrow \psi$ iff $\forall_{\exists} \alpha \in \mathrm{alt}^{+}(\varphi): \sigma \triangleleft \alpha$ and $M, \sigma \cap \alpha \models^{+} \psi$.

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Relevant example

(14) If the miners might be in shaft *A* and they might be in shaft *B*, then neither shaft ought to be blocked.
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iff $|\Diamond p \wedge \Diamond q| \cap |p'| \cap |q'| \neq \emptyset$ and $|\Diamond p \wedge \Diamond q| \cap |p'| \cap |q'| \subseteq$ **bad**.

Implying an obligation to block neither

	pq	$p'q'$		pq	$p'q'$
w_1	10	10	w_7	10	10
w_2	10	00	w_8	10	00
w_3	10	01	w_9	10	01
w_4	01	01	w_{10}	01	01
w_5	01	00	w_{11}	01	00
w_6	01	10	w_{12}	01	10

$\mathrm{alt}^+(\Diamond p \wedge \Diamond q)$

	pq	$p'q'$		pq	$p'q'$
w_1	10	10	w_7	10	10
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w_3	10	01	w_9	10	01
w_4	01	01	w_{10}	01	01
w_5	01	00	w_{11}	01	00
w_6	01	10	w_{12}	01	10

$\mathrm{alt}^+(\boxed{\mathrm{v}}\neg p' \wedge \boxed{\mathrm{v}}\neg q')$

$|\Diamond p \wedge \Diamond q| \cap |p'| \cap |q'| = \{w_7, w_9, w_{10}, w_{12}\}$

$M \models^+ (\Diamond p \wedge \Diamond q) \rightarrow (\boxed{\mathrm{v}}\neg p' \wedge \boxed{\mathrm{v}}\neg q')$

Contextual epistemic necessity[5]

Support clause:

$$M,\sigma \models^{+} \Box\varphi \text{ iff } \exists\alpha \in \mathrm{alt}^{+}(\varphi) : \exists w \in \sigma : \textbf{e-state}(w) \lhd \alpha;$$
$$\forall\beta \in \mathrm{alt}^{-}(\varphi) : \forall w \in \sigma : \textbf{e-state}(w) \ntriangleleft \beta.$$

Some support-alternative for φ is supposable in some e-state compatible with σ; and
no rejection-alternative for φ is supposable in any e-state compatible with σ.

[5] See Aher and Groenendijk 2015.

Contextual epistemic necessity

Relevant example:

(15) The miners must be in shaft A. $\Box p$

Support in the model for $\Box p$ boils down to:

$$M \models^{+} \Box p \text{ iff } \exists w: \textbf{e-state}(w) \cap |p| \neq \emptyset; \text{ and}$$
$$\forall w: \textbf{e-state}(w) \cap |\neg p| = \emptyset.$$

Necessity of miners being in a shaft

	pq	$p'q'$		pq	$p'q'$
w_1	10	10	w_7	10	10
w_2	10	00	w_8	10	00
w_3	10	01	w_9	10	01
w_4	01	01	w_{10}	01	01
w_5	01	00	w_{11}	01	00
w_6	01	10	w_{12}	01	10

$|p|, |\neg p|$

	pq	$p'q'$		pq	$p'q'$
w_1	10	10	w_7	10	10
w_2	10	00	w_8	10	00
w_3	10	01	w_9	10	01
w_4	01	01	w_{10}	01	01
w_5	01	00	w_{11}	01	00
w_6	01	10	w_{12}	01	10

$|\Box p|$

$M \models^+ \Box p$ iff $\exists w$: **e-state**$(w) \cap |p| \neq \emptyset$; and
$\forall w$: **e-state**$(w) \cap |\neg p| = \emptyset$.

$M \not\models^+ \Box p$

Contextual and non-contextual necessity

	pq	$p'q'$		pq	$p'q'$
w_1	10	10	w_7	10	10
w_2	10	00	w_8	10	00
w_3	10	01	w_9	10	01
w_4	01	01	w_{10}	01	01
w_5	01	00	w_{11}	01	00
w_6	01	10	w_{12}	01	10

Non-contextual $|\Box p|$ is $|p|$

	pq	$p'q'$		pq	$p'q'$
w_1	10	10	w_7	10	10
w_2	10	00	w_8	10	00
w_3	10	01	w_9	10	01
w_4	01	01	w_{10}	01	01
w_5	01	00	w_{11}	01	00
w_6	01	10	w_{12}	01	10

Contextual $|\Box p|$

One consequence of adopting contextual necessity:

$M \not\models^+ p \to \boxed{\vee} p'$ but $M \models^+ \Box p \to \boxed{\vee} p'$

Obligation contingent on p

Example

(16) If the miners are in shaft A, then it ought to be blocked. $p \rightarrow \boxed{v} p'$

Support for the formula (16) in the model boils down to:

$$M \models^{+} p \rightarrow \boxed{v} p' \text{ iff } |p| \cap |\neg p'| \neq \emptyset \text{ and}$$
$$|p| \cap |\neg p'| \subseteq \textbf{bad}.$$

Obligation contingent on p

	pq	p′q′		pq	p′q′
w_1	10	10	w_7	10	10
w_2	10	00	w_8	10	00
w_3	10	01	w_9	10	01
w_4	01	01	w_{10}	01	01
w_5	01	00	w_{11}	01	00
w_6	01	10	w_{12}	01	10

$|p|$

	pq	p′q′		pq	p′q′
w_1	10	10	w_7	10	10
w_2	10	00	w_8	10	00
w_3	10	01	w_9	10	01
w_4	01	01	w_{10}	01	01
w_5	01	00	w_{11}	01	00
w_6	01	10	w_{12}	01	10

$|\neg p'|$

$M \models^+ p \rightarrow \boxed{v} p'$ iff $|p| \cap |\neg p'| \neq \emptyset$ and
$|p| \cap |\neg p'| \subseteq$ **bad**.

As $|p| \cap |\neg p'| = \{w_2, w_3, w_8, w_9\}$, $M \not\models^+ p \rightarrow \boxed{v} p'$

Obligation contingent on epistemic necessity

Example

(17) If the miners must be in shaft A, then it ought to be blocked. $\Box p \rightarrow \boxed{v}\, p'$

Support for the formula (17) in the model boils down to:

$$M \models^{+} \Box p \rightarrow \boxed{v}\, p' \;\text{ iff }\; |\Box p| \cap |\neg p'| \neq \emptyset \text{ and } |\Box p| \cap |\neg p'| \subseteq \textbf{bad}.$$

Obligation contingent on epistemic necessity

	pq	p′q′		pq	p′q′
w_1	10	10	w_7	10	10
w_2	10	00	w_8	10	00
w_3	10	01	w_9	10	01
w_4	01	01	w_{10}	01	01
w_5	01	00	w_{11}	01	00
w_6	01	10	w_{12}	01	10

$|\Box p|$

	pq	p′q′		pq	p′q′
w_1	10	10	w_7	10	10
w_2	10	00	w_8	10	00
w_3	10	01	w_9	10	01
w_4	01	01	w_{10}	01	01
w_5	01	00	w_{11}	01	00
w_6	01	10	w_{12}	01	10

$|\neg p'|$

$M \models^{+} \Box p \rightarrow \boxed{\mathrm{v}}\, p'$ iff $|\Box p| \cap |\neg p'| \neq \emptyset$ and
$|\Box p| \cap |\neg p'| \subseteq$ **bad**.

As $|\Box p| \cap |\neg p'| = \{w_2, w_3\}$, $M \models^{+} \Box p \rightarrow \boxed{\mathrm{v}}\, p'$

Deontic question depends on the epistemic one

Deontic question

(18) Ought shaft A be blocked, or shaft B, or neither?
$?(\Box p' \vee \Box q; \vee (\Box \neg p' \wedge \Box \neg q'))$

Epistemic question

(19) Is it the case that the miners might be in shaft A and they might be in *B*? $?(\Diamond p \wedge \Diamond q)$

(18) depends on (19), so when (19) is resolved, (18) is too

Inquisitive disjunction and questions

Support clause:

$M, \sigma \models^+ \varphi \vee \psi$ iff $M, \sigma \models^+ \varphi$ or $M, \sigma \models^+ \psi$

There can be more than one alternative for disjunction, so a disjunction can be inquisitive.

Notation convention for questions:

$?\varphi := \varphi \vee \neg\varphi$

Questions in inquisitive semantics

In inquisitive semantics, $p \vee \neg p$ i.e., $?p$, isn't a tautology. It isn't informative, it's inquisitive. For example, $M \not\models^+ p \vee \neg p$.

Equivalence fact about deontic and epistemic questions in the model

Deontic question

$$?(\boxed{v}p' \vee \boxed{v}q; \vee(\boxed{v}\neg p' \wedge \boxed{v}\neg q')) \equiv_M \boxed{v}p' \vee \boxed{v}q' \vee (\boxed{v}\neg p' \wedge \boxed{v}\neg q')$$

Epistemic question

$$?(\Diamond p \wedge \Diamond q) \equiv_M \Box p \vee \Box q \vee (\Diamond p \wedge \Diamond q)$$

Equivalence fact about deontic and epistemic questions in the model

Deontic question

$$?(\boxed{v}\, p' \vee \boxed{v}\, q; \vee (\boxed{v} \neg p' \wedge \boxed{v} \neg q')) \equiv_M \boxed{v}\, p' \vee \boxed{v}\, q' \vee (\boxed{v} \neg p' \wedge \boxed{v} \neg q')$$

Epistemic question

$$?(\Diamond p \wedge \Diamond q) \equiv_M \Box p \vee \Box q \vee (\Diamond p \wedge \Diamond q)$$

Question dependency and entailment[6]

Definition of entailment follows implication:

$\varphi \models_M \psi$ iff $\forall_\exists \alpha \in \text{alt}^+(\varphi): M, \alpha \models^+ \psi$

Question dependence and entailment

A question depends on another if the latter entails the former.

Does the deontic question depend on the epistemic question?

$\Box p \vee \Box q \vee (\Diamond p \wedge \Diamond q) \models_M \boxed{v} p' \vee \boxed{v} q' \vee (\boxed{v} \neg p' \wedge \boxed{v} \neg q')$

[6] On question dependency and entailment see Ciardelli 2014.

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Question dependence and entailment

A question depends on another if the latter entails the former.

Does the deontic question depend on the epistemic question?

$\Box p \vee \Box q \vee (\Diamond p \wedge \Diamond q) \models_M \boxed{\vee} p' \vee \boxed{\vee} q' \vee (\boxed{\vee} \neg p' \wedge \boxed{\vee} \neg q')$

[6] On question dependency and entailment see Ciardelli 2014.

Deontic question depends on the epistemic one

$M, |\Box p| \models^{+} \boxed{v}\, p'$

$M, |\Box q| \models^{+} \boxed{v}\, q'$

$M, |\Diamond p \wedge \Diamond q| \models^{+} \boxed{v}(\neg p' \wedge \neg q')$

$M \models^{+} \Box p \rightarrow \boxed{v}\, p'$

$M \models^{+} \Box q \rightarrow \boxed{v}\, q'$

$M \models^{+} (\Diamond p \wedge \Diamond q) \rightarrow (\boxed{v}\neg p' \wedge \boxed{v}\neg q')$

	pq	$p'q'$		pq	$p'q'$
w_1	10	10	w_7	10	10
w_2	10	00	w_8	10	00
w_3	10	01	w_9	10	01
w_4	01	01	w_{10}	01	01
w_5	01	00	w_{11}	01	00
w_6	01	10	w_{12}	01	10

$\mathrm{alt}^{+}(\Box p \vee \Box q \vee (\Diamond p \wedge \Diamond q))$

$\varphi \models_M \psi$ iff $\forall_{\exists} \alpha \in \mathrm{alt}^{+}(\varphi): M, \alpha \models^{+} \psi$

$\Box p \vee \Box q \vee (\Diamond p \wedge \Diamond q) \models_M \boxed{v}\, p' \vee \boxed{v}\, q; \vee (\boxed{v}\neg p' \wedge \boxed{v}\neg q')$

The end (Or is it?)

Thank you for listening

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https://www.illc.uva.nl/inquisitivesemantics/

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