Topological interactions in gauge theories

Topological defects

Much of the relevant physics described by gauge theories is non-perturbative in nature; for example features like the spectrum, the confinement phenomenon and the study of topological defects and their physical properties. The research in the my group has in recent years focused on some - up to now - poorly understood non perturbative features of topological defects in gauge theories such as their interactions. In particular, the interplay of topological and non-topological quantum numbers on fluxes and monopoles has been studied, partially motyivated by the search for truly non-abelian dualities. In this context a lot has been learned from the study of non-abelian discrete gauge theories [16]. These feature topological (magnetic flux) sectors which are characterized by certain non-abelian quantum numbers (conjugacy classes to be specific), whereas the allowed dyonic sectors carry electric charges falling into representations of the stabilizer group of the magnetic conjugacy class. This yields an intricate interdependence of admissible electric and magnetic quantum numbers in the semi-classical excitation spectrum of the theory. Important was the discovery that this interplay is a manifestation of an underlying quantum group structure, whose representation theory exactly accounts for the structure just described [14], [11].

It should be added that for the monopoles in phases with residual non-abelian continuous symmetries interesting results have been obtained [12], [9] which show that a similar picture emerges, but the precise mathematical structure has not yet been fully determined. In other words, the algebraic and physical structure reflecting the rather well studied geometry of the moduli space is still an important topic.

Important was the realization that the features mentioned above are quite generic and may lead to unusual physical phenomena.

Topological Interactions

The non-abelian topological nature of defects in phases with non-abelian residual symmetries, implies that these defects will exhibit nontrivial topological interactions with the other (fundamental and/or topological) degrees of freedom in the theory. These interactions are a consequence of the nontrivial connectivity properties of the solution space of a single defect.

From a physical point of view these topological interactions manifest themselves through the possibility of highly nontrivial entanglements and consequently of various scattering phenomena which can be described as non-abelian generalizations of the Ahoronov-Bohm effect [4], [11]. Another consequence is the possibility of exotic, non-abelian quantum statistics for collective excitations in certain phases of the theory, as occurring for example in the fractional quantum Hall effect [5]. Also the peculiar situation of general relativity in 2+1 dimensions

- which as was shown by Witten, is a topological field theory - was studied from this perspective and the quantum symmetry underlying its structure was constructed [10], [8]. The Hopf algebra in question turned out to be the quantum double of the group SU(2). Recently we have shown how gravitational scattering of particles, with and without spin, can be treated consistently within this framework [1].

Broken Hopf-symmetry and confinement

We have mentioned the appearance of quantum symmetries, generally speaking Hopf-algebra's, which have the important feature that they allow one to treat topological (say magnetic) and ordinary (say color-electric) quantum-numbers of (quasi) particles on the same footing. Having states labeled in this universal way, it is interesting to study the question of duality and conceivable condensation phenomena.

We have recently completed the phase classification of the non-abelian (topological) field theories and the many distinct but allowed types of confinement that may occur in such theories. These phases are precisely characterized by the the breaking of the Hopf symmetry through a condensate of certain well defined bosonic order-parameter fields (electric, magnetic or both) in the theory [2]. These findings may be linked to certain duality properties of the Hopf symmetries we have studied [7]. This phenomenon of the breaking of quantum-symmetries appears to be important and generic in 2 dimensional physics and we continue the study of its applications.

Core instabilities of monopoles

Very interesting in the above context of unbroken nonabelian symmetries are phases of theories where different types of topological excitations can coexist, in which case the topological interactions between these will lead to rather exotic physical properties. As we showed long ago, topological excitations (monopoles an instantons) in these theories may exhibit core deformations [15].

A simple but interesting theory of this type is Alice electrodynamics, a theory whose gauge group is the usual U(1) enhanced with a local Z_2 realization of charge conjugation symmetry. One of the exotic properties of the theory is the emergence of the topological concept of "cheshire charge', a nonlocalizable manifestation of electric and/or magnetic charge. We have shown that this elusive concept for certain parameter ranges in the theory manifests itself through a core instability of magnetic monopoles, where the point defect decays into a ring-shaped object carrying non-localisable ymagnetic charge [3]. This is a line of research which we certainly want to continue, as there may have been phases in the early universe featuring these phenomena (SO(10) breaking to SU(5) for example).

Future plans

With a number of students and postdocs, I will continue to explore the rather surprising aspects of gauge theories in 2 and 3 dimensions, in which the interplay between topological and non-topological features leads to unusual physical properties. As these features are only beginning to be uncovered but appear to be generic for many physical systems, we consider it very worthwhile to do so.

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