

Cryptography in the Bounded Quantum-Storage Model*

Ivan B. Damgård^{†‡} Serge Fehr[§] Louis Salvail^{†‡¶}

Christian Schaffner^{†||}

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Abstract

We initiate the study of two-party cryptographic primitives with unconditional security, assuming that the adversary's *quantum* memory is of bounded size. We show that oblivious transfer and bit commitment can be implemented in this model using protocols where honest parties need no quantum memory, whereas an adversarial player needs quantum memory of size at least $n/2$ in order to break the protocol, where n is the number of qubits transmitted. This is in sharp contrast to the classical bounded-memory model, where we can only tolerate adversaries with memory of size quadratic in honest players' memory size. Our protocols are efficient, non-interactive and can be implemented using today's technology. On the technical side, a new entropic uncertainty relation involving min-entropy is established.

1 Introduction

It is well known that non-trivial 2-party cryptographic primitives cannot be securely implemented if only error-free communication is available and there is no limitation assumed on the computing power and memory of the players. Fundamental examples of such primitives are bit commitment (BC) and oblivious transfer (OT). In BC, a committer C commits himself to

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[†]Basic Research in Computer Science (BRICS), funded by the Danish National Research Foundation, Department of Computer Science, University of Århus, {ivan|salvail|chris}@brics.dk.

[‡]FICS, Foundations in Cryptography and Security, funded by the Danish Natural Sciences Research Council.

[§]Center for Mathematics and Computer Science (CWI), Amsterdam, Netherlands, fehr@cwi.nl

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a choice of a bit b by exchanging information with a verifier V . We want that V does not learn b (we say the commitment is hiding), yet C can later chose to reveal b in a convincing way, i.e., only the value fixed at commitment time will be accepted by V (we say the commitment is binding). In (Rabin) OT, a sender S sends a bit b to a receiver R by executing some protocol in such a way that R receives b with probability $\frac{1}{2}$ and nothing with probability $\frac{1}{2}$, yet S does not learn what was received.

Informally, BC is not possible with unconditional security since hiding means that when 0 is committed, exactly the same information exchange could have happened when committing to a 1. Hence, even if 0 was actually committed to, C could always compute a complete view of the protocol consistent with having committed to 1, and pretend that this was what he had in mind originally. A similar type of argument shows that OT is also impossible in this setting.

One might hope that allowing the protocol to make use of quantum communication would make a difference. Here, information is stored in qubits, i.e., in the state of two-level quantum mechanical systems, such as the polarization state of a single photon. It is well known that quantum information behaves in a way that is fundamentally different from classical information, enabling, for instance, unconditionally secure key exchange between two honest players. However, in the case of two mutually distrusting parties, we are not so fortunate: even with quantum communication, unconditionally secure BC and OT remain impossible [18, 21].

There are, however, several scenarios where these impossibility results do not apply, namely:

- if the computing power of players is bounded,
- if the communication is noisy,
- if the adversary is under some physical limitation, e.g., the size of the available memory is bounded.

The first scenario is the basis of many well known solutions based on plausible but unproven complexity assumptions, such as hardness of factoring or discrete logarithms. The second scenario has been used to construct both BC and OT protocols in various models for the noise [6, 8, 11]. The third scenario is our focus here. In this model, OT and BC can be done using classical communication assuming, however, quite restrictive bounds on the adversary's memory size [3, 12], namely it can be at most quadratic in the memory size of honest players. Such an assumption is on the edge of being realistic, it would clearly be more satisfactory to have a larger separation between the memory size of honest players and that of the adversary. However, this was shown to be impossible [15].

In this paper, we study for the first time what happens if instead we consider protocols where quantum communication is used and we place a

bound on the adversary’s *quantum* memory size. There are two reasons why this may be a good idea: first, if we do not bound the classical memory size, we avoid the impossibility result of [15]. Second, the adversary’s typical goal is to obtain a certain piece of classical information that we want to keep hidden from him. However, if he cannot store all the quantum information that is sent, he must convert some of it to classical information by measuring. This may irreversibly destroy information, and we may be able to arrange it such that the adversary cannot afford to lose information this way, while honest players can.

It turns out that this is indeed possible: we present protocols for both BC and OT in which n qubits are transmitted, where honest players need *no quantum memory*, but where the adversary must store at least $n/2$ qubits to break the protocol. We emphasize that no bound is assumed on the adversary’s computing power, nor on his classical memory. This is clearly much more satisfactory than the classical case, not only from a theoretical point of view, but also in practice: while sending qubits and measuring them immediately as they arrive is well within reach of current technology, storing even a single qubit for more than a fraction of a second is a formidable technological challenge. Furthermore, we show that our protocols also work in a non-ideal setting where we allow the quantum source to be imperfect and the quantum communication to be noisy.

We emphasize that what makes OT and BC possible in our model is not so much the memory bound per se, rather it is the loss of information it implies on the part of the adversary. Indeed, our results also hold if the adversary’s memory device holds an arbitrary number of qubits, but is imperfect in certain ways. This is discussed in more detail in Section 5.

Our protocols are non-interactive, only one party sends information when doing OT, commitment or opening. Furthermore, the commitment protocol has the interesting property that the only message is sent to the committer, i.e., it is possible to commit while only *receiving* information. Such a scheme clearly does not exist without a bound on the committer’s memory, even under computational assumptions and using quantum communication: a corrupt committer could always store (possibly quantumly) all the information sent, until opening time, and only then follow the honest committer’s algorithm to figure out what should be sent to convincingly open a 0 or a 1. Note that in the classical bounded-storage model, it is known how to do time-stamping that is non-interactive in our sense: a player can time-stamp a document while only receiving information [22]. However, no reasonable BC or protocol that time-stamps a bit exist in this model. It is straightforward to see that any such protocol can be broken by an adversary with classical memory of size twice that of an honest player, while our protocol requires no memory for the honest players and remains secure against any adversary not able to store more than half the size of the quantum transmission.

We also note that it has been shown earlier that BC is possible using quantum communication, assuming a different type of physical limitation, namely a bound on the size of coherent measurement that can be implemented [25]. This limitation is incomparable to ours: it does not limit the total size of the memory, instead it limits the number of bits that can be simultaneously operated on to produce a classical result. Our adversary has a limit on the total memory size, but can measure all of it coherently. The protocol from [25] is interactive, and requires a bound on the maximal measurement size that is sub-linear in n .

On the technical side, we derive a new type of uncertainty relation involving the min-entropy of a quantum encoding (Theorem 3.7 and Corollary 3.8). The relation is in a suitable form to apply privacy amplification against quantum adversaries as introduced by Renner and König [23].

2 Preliminaries

2.1 Notation

For a set $I = \{i_1, i_2, \dots, i_\ell\} \subseteq \{1, \dots, n\}$ and a n -bit string $x \in \{0, 1\}^n$, we define $x|_I := x_{i_1}x_{i_2} \cdots x_{i_\ell}$. For $x, y \in \{0, 1\}^n$, $x \cdot y \in \{0, 1\}$ denotes the (standard) in-product of x and y . For a probability distribution Q over n -bit strings and a set $L \subseteq \{0, 1\}^n$, we abbreviate the (overall) probability of L with $Q(L) := \sum_{x \in L} Q(x)$. All logarithms in this paper are to base two. We denote by $h(p)$ the binary entropy function $h(p) := -(p \cdot \log p + (1 - p) \cdot \log(1 - p))$. We denote by $\text{negl}(n)$ any function of n smaller than any polynomial provided n is sufficiently large. For $x \in \{0, 1\}^n$, we write $B^{\delta n}(x)$ for the set of all n -bit strings at Hamming distance at most δn from x . Note that the number of elements in $B^{\delta n}(x)$ is the same for all x , we denote it by $B^{\delta n} := |B^{\delta n}(x)|$. It is well known that $B^{\delta n} \leq 2^{nh(\delta)}$.

The pair $\{|0\rangle, |1\rangle\}$ denotes the computational or rectilinear or “+” basis for the 2-dimensional complex Hilbert space \mathbb{C}^2 . The diagonal or “ \times ” basis is defined as $\{|0\rangle_\times, |1\rangle_\times\}$ where $|0\rangle_\times = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|1\rangle_\times = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$. Measuring a qubit in the + -basis (resp. \times -basis) means applying the measurement described by projectors $|0\rangle\langle 0|$ and $|1\rangle\langle 1|$ (resp. projectors $|0\rangle_\times\langle 0|_\times$ and $|1\rangle_\times\langle 1|_\times$). When the context requires it, we write $|0\rangle_+$ and $|1\rangle_+$ instead of $|0\rangle$ respectively $|1\rangle$; and for any $x \in \{0, 1\}^n$ and $r \in \{+, \times\}$, we write $|x\rangle_r = \bigotimes_{i=1}^n |x_i\rangle_r$. If we want to choose the + or \times -basis according to the bit $b \in \{0, 1\}$, we write $\{+, \times\}_{[b]}$.

2.2 Quantum Probability Theory

As basis for the security definitions and proofs of our protocols, we are using the formalism introduced in [23], which we briefly summarize here. A *random state* ρ is a random variable, with distribution P_ρ , whose range

is the set of density operators of a fixed Hilbert space. The view of an observer (which is ignorant of the value of ρ) is given by the quantum system described by the density operator $[\rho] := \sum_{\rho} P_{\rho}(\rho)\rho$. In general, for any event \mathcal{E} , we define $[\rho|\mathcal{E}] := \sum_{\rho} P_{\rho|\mathcal{E}}(\rho)\rho$. If ρ is dependent on some classical random variable X , with joint distribution $P_{X\rho}$, we also write ρ_x instead of $[\rho|X = x]$. Note that ρ_x is a density operator (for any fixed x) whereas ρ_X is again a random state. The overall quantum system is then given by $[\{X\} \otimes \rho] = \sum_x P_X(x) \{x\} \otimes \rho_x$, where $\{x\} := |x\rangle\langle x|$ is the *state representation* of x and $\{X\}$ the corresponding random state. Obviously, $[\{X\} \otimes \rho] = [\{X\}] \otimes [\rho]$ if and only if ρ_X is independent of X , where the latter in particular implies that no information on X can be learned by observing only ρ . Furthermore, if $[\{X\} \otimes \rho]$ and $[\{X\}] \otimes [\rho]$ are ε -close in terms of their trace distance $\delta(\rho, \sigma) = \frac{1}{2} \text{tr}(|\rho - \sigma|)$, then the real system $[\{X\} \otimes \rho]$ “behaves” as the ideal system $[\{X\}] \otimes [\rho]$ except with probability ε [23] in that for any evolution of the system no observer can distinguish the real from the ideal one with advantage greater than ε . Henceforth, we use UNIF to denote a random variable with range $\{0, 1\}$, uniformly distributed and independent of anything else, and, as in [23], we use $d(X|\rho)$ as a short hand for $\delta([\{X\} \otimes \rho], [\{\text{UNIF}\}] \otimes [\rho])$.

We consider the notion of the classical *Rényi entropy* $H_{\alpha}(X)$ of order α of a random variable X [24], as well as its generalization to the Rényi entropy $S_{\alpha}(\rho)$ of a state ρ [23]. It holds that $S_{\alpha}([\{X\}]) = H_{\alpha}(X)$ and $S_{\alpha}([\{X\}]) \leq S_{\beta}([\{X\}])$ if $\alpha \geq \beta$. The cases that are relevant for us are the classical *min-entropy* $H_{\infty}(X) = -\log(\max_x P_X(x))$ as well as the *max* and the *collision Von Neumann entropy* $S_0(\rho) = \log(\text{rank}(\rho))$ respectively $S_2(\rho) = -\log(\sum_i \lambda_i^2)$, where $\{\lambda_i\}_i$ are the eigenvalues of ρ .

2.3 Bounded Quantum Storage and Privacy Amplification

All our protocols take place in the *bounded quantum-storage model*, which concretely means the following: the state of an adversarial player may consist of an arbitrary number of qubits, and he may perform arbitrary quantum computation. At a certain point in time though, we say that *the memory bound applies*, which means that all but q of the qubits are measured. After this point, the player is again unbounded in (quantum) memory and computing power. We note that our results also apply to some cases where the adversary’s memory is not bounded but is noisy in certain ways, see Section 5.

An important tool we will use is universal hashing. A class \mathbb{H}_n of hashing functions from $\{0, 1\}^n$ to $\{0, 1\}$ is called *two-universal* if for any pair $x, y \in \{0, 1\}^n$ with $x \neq y$

$$|\{f \in \mathbb{H}_n : f(x) = f(y)\}| \leq \frac{|\mathbb{H}_n|}{2}.$$

Several two-universal classes of hashing functions are such that evaluating and picking a function uniformly and at random in H_n can be done efficiently [4, 27].

Theorem 2.1 ([23]). *Let X be distributed over $\{0, 1\}^n$, and let ρ be a random state of q qubits¹. Let F be the random variable corresponding to the random choice (with uniform distribution and independent from X and ρ) of a member of a two-universal class of hashing functions H_n . Then*

$$d([F(X)|\{F\} \otimes \rho]) \leq \frac{1}{2} 2^{-\frac{1}{2}(S_2(\{X\} \otimes \rho) - S_0(\rho) - 1)} \quad (1)$$

$$\leq \frac{1}{2} 2^{-\frac{1}{2}(H_\infty(X) - q - 1)}. \quad (2)$$

The first inequality (1) is the original theorem from [23], and (2) follows by observing that $S_2(\{X\} \otimes \rho) \geq S_2(\{X\}) = H_2(X) \geq H_\infty(X)$. In this paper, we essentially only use this weaker version of the theorem.

Note that if the rightmost term of (2) is negligible, i.e. say smaller than $2^{-\varepsilon n}$, then this situation is $2^{-\varepsilon n}$ -close to the ideal situation where $F(X)$ is perfectly uniform and independent of ρ and F . In particular, the situations $F(X) = 0$ and $F(X) = 1$ are statistically indistinguishable given ρ and F [17].

The following lemma is a direct consequence of Theorem 2.1. In Section 4, this lemma will be useful for proving the binding condition of our commitment scheme. Recall that for $X \in \{0, 1\}^n$, $B^{\delta n}(X)$ denotes the set of all n -bit strings at Hamming distance at most δn from X and $B^{\delta n} := |B^{\delta n}(X)|$ is the number of such strings.

Lemma 2.2. *Let X be distributed over $\{0, 1\}^n$, let ρ be a random state of q qubits and let \hat{X} be a guess for X given ρ . Then, for all $\delta < \frac{1}{2}$ it holds that*

$$\Pr [\hat{X} \in B^{\delta n}(X)] \leq 2^{-\frac{1}{2}(H_\infty(X) - q - 1) + \log(B^{\delta n})}.$$

In other words, given a quantum memory of q qubits arbitrarily correlated with a classical random variable X , the probability to find \hat{X} at Hamming distance at most δn from X where $nh(\delta) < \frac{1}{2}(H_\infty(X) - q)$ is negligible.

Proof: Here is a strategy to try to bias $F(X)$ when given \hat{X} and $F \in_R H_n$: Sample $X' \in_R B^{\delta n}(\hat{X})$ and output $F(X')$. Note that, using p_{succ} as a short hand for the probability $\Pr [\hat{X} \in B^{\delta n}(X)]$ to be bounded,

$$\Pr [F(X') = F(X)] = \frac{p_{\text{succ}}}{B^{\delta n}} + \left(1 - \frac{p_{\text{succ}}}{B^{\delta n}}\right) \frac{1}{2}$$

¹Remember that ρ can be correlated with X in an arbitrary way. In particular, we can think of ρ as an attempt to store the n -bit string X in q qubits.

$$= \frac{1}{2} + \frac{p_{\text{succ}}}{2 \cdot B^{\delta n}},$$

where the first equality follows from the fact that if $X' \neq X$ then, as H_n is two-universal, $\Pr[F(X) = F(X')] = \frac{1}{2}$. Since the probability of correctly guessing a binary $F(X)$ given F and ρ is always upper bounded by $\frac{1}{2} + d(F(X)|\{F\} \otimes \rho)$, in combination with Theorem 2.1 the above results in

$$\frac{1}{2} + \frac{p_{\text{succ}}}{2 \cdot B^{\delta n}} \leq \frac{1}{2} + \frac{1}{2} 2^{-\frac{1}{2}(H_\infty(X) - q - 1)}$$

and the claim follows immediately. \square

3 Rabin Oblivious Transfer

3.1 The Definition

A protocol for Rabin Oblivious Transfer (ROT) between sender Alice and receiver Bob allows for Alice to send a bit b through an erasure channel to Bob. Each transmission delivers b or an erasure with probability $\frac{1}{2}$. Intuitively, a protocol for ROT is secure if

- the sender Alice gets no information on whether b was received or not, no matter what she does, and
- the receiver Bob gets no information about b with probability at least $\frac{1}{2}$, no matter what he does.

In this paper, we are considering quantum protocols for ROT. This means that while the inputs and outputs of the honest senders are classical, described by random variables, the protocol may contain quantum computation and quantum communication, and the view of a dishonest player is quantum, and is thus described by a random state.

Any such (two-party) protocol is specified by a family $\{(S_n, R_n)\}_{n>0}$ of pairs of interactive quantum circuits (i.e. interacting through a quantum channel). Each pair is indexed by a security parameter $n > 0$, where S_n and R_n denote the circuits for sender Alice and receiver Bob, respectively. In order to simplify the notation, we often omit the index n , leaving the dependency on it implicit.

For the formal definition of the security requirements of a ROT protocol, let us fix the following notation. Let B denote the binary random variable describing S 's input bit b , and let A and B' denote the binary random variables describing R 's two output bits, where the meaning is that A indicates whether the bit was received or not. Furthermore, for a dishonest sender \tilde{S} (respectively \tilde{R}) let $\rho_{\tilde{S}}$ ($\rho_{\tilde{R}}$) denote the random state describing \tilde{S} 's (\tilde{R} 's) view of the protocol. Note that for a fixed candidate protocol for ROT, and for a fixed input distribution P_B , depending on whether we consider

two honest S and R , a dishonest \tilde{S} and an honest R , or an honest S and a dishonest \tilde{R} , the corresponding joint distribution $P_{BAB'}$, $P_{\rho_{\tilde{S}}AB'}$ respectively $P_{B\rho_{\tilde{R}}}$ is uniquely determined.

Definition 3.1. *A two-party (quantum) protocol (S, R) is a (statistically) secure ROT if the following holds.*

Correctness: *For honest S and R*

$$\Pr[B = B'|A = 1] \geq 1 - \text{negl}(n).$$

Receiver-Privacy: *For any \tilde{S}*

$$d(A|\rho_{\tilde{S}}) \leq \text{negl}(n).$$

Sender-Privacy: *For any \tilde{R} there exists an event \mathcal{E} with $P[\mathcal{E}] \geq \frac{1}{2} - \text{negl}(n)$ such that*

$$\delta([B \otimes \rho_{\tilde{R}}|\mathcal{E}], [B] \otimes [\rho_{\tilde{R}}|\mathcal{E}]) \leq \text{negl}(n).$$

*If any of the above trace distances equals 0, then the corresponding property is said to hold **perfectly**. If one of the properties only holds with respect to a restricted class \mathfrak{S} of \tilde{S} 's respectively \mathfrak{R} of \tilde{R} 's, then this property is said to hold and the protocol is said to be secure **against** \mathfrak{S} respectively \mathfrak{R} .*

Receiver-privacy requires that the joint quantum state is essentially the same as when A is uniformly distributed and independent of the sender's view, and sender-privacy requires that there exists some event which occurs with probability at least $\frac{1}{2}$ (the event that the receiver does not receive the bit) and under which the joint quantum state is essentially the same as when B is distributed (according to P_B) independently of the receiver's view.

We warn the reader that the above definition does not guarantee that the ROT protocol is equivalent to an "ideal black-box implementation" of ROT, so it does not guarantee universal composability, for instance. One main reason for this is that, unlike the classical case [7], receiver-privacy as we define it does not guarantee that the input bit b is determined after the execution of ROT. In other words, \tilde{S} is not necessarily bound to her input. In fact, this is not surprising, since our model places no limitations whatsoever on the sender. If \tilde{S} was indeed bound to her input, a straight-forward reduction would allow us to build from ROT a statistically hiding commitment scheme where the ROT sender is the committer. But since the sender is unbounded, she can always break the binding property using essentially the standard attack against unconditionally secure quantum bit commitment [18, 21].

A more rigorous definition of Oblivious Transfer is therefore required in order to allow for composability. Moreover, we see from the above that

satisfying such a definition will require some limitation to be placed on the sender, such as a memory bound. This would, for instance, allow using the commitment scheme we present later in this paper with the ROT sender in the role of committer. These issue will be further addressed in a forthcoming paper [9].

3.2 The Protocol

We introduce a quantum protocol for ROT that will be shown perfectly receiver-private (against any sender) and statistically sender-private against any quantum memory-bounded receiver. Our protocol exhibits some similarity with quantum conjugate coding introduced by Wiesner [28].

The protocol is very simple (see Figure 1): **S** picks $x \in_R \{0, 1\}^n$ and sends to **R** n qubits in state either $|x\rangle_+$ or $|x\rangle_\times$ each chosen with probability $\frac{1}{2}$. **R** then measures all received qubits either in the rectilinear or in the diagonal basis. With probability $\frac{1}{2}$, **R** picked the right basis and gets x , while any $\tilde{\mathbf{R}}$ that is forced to measure part of the state (due to a memory bound) can only have full information on x in case the $+$ -basis was used *or* in case the \times -basis was used (but not in both cases). Privacy amplification based on any two-universal class of hashing functions \mathbf{H}_n is then used to destroy partial information. (In order to avoid aborting, we specify that if a dishonest $\tilde{\mathbf{S}}$ refuses to participate, or sends data in incorrect format, then **R** samples its output bits a and b' both at random in $\{0, 1\}$.)

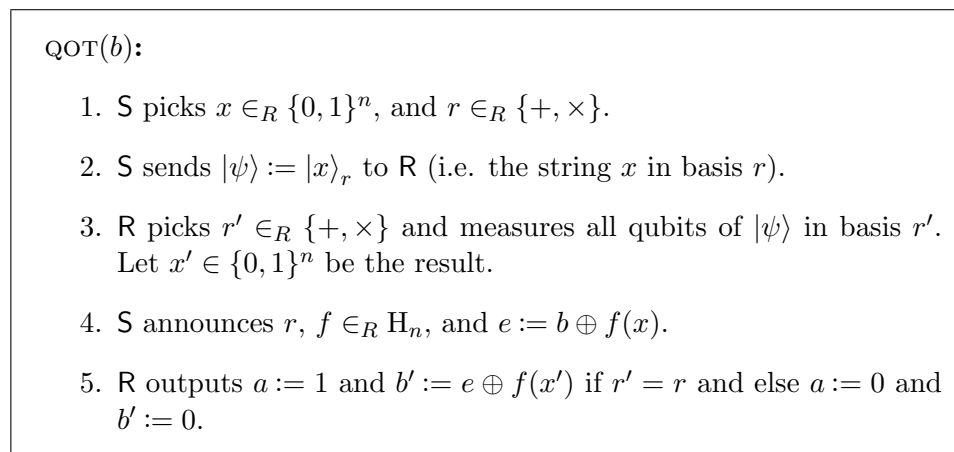


Figure 1. Protocol for Rabin QOT

As we shall see in Section 3.5, the security of the QOT protocol against receivers with bounded-size quantum memory holds as long as the bound applies before Step 4 is reached. An equivalent protocol is obtained by purifying the sender's actions. Although QOT is easy to implement, the purified or EPR-based version [16] depicted in Figure 2 is easier to prove secure. A similar approach was taken in the Shor-Preskill proof of security

for the BB84 quantum key distribution scheme [26].

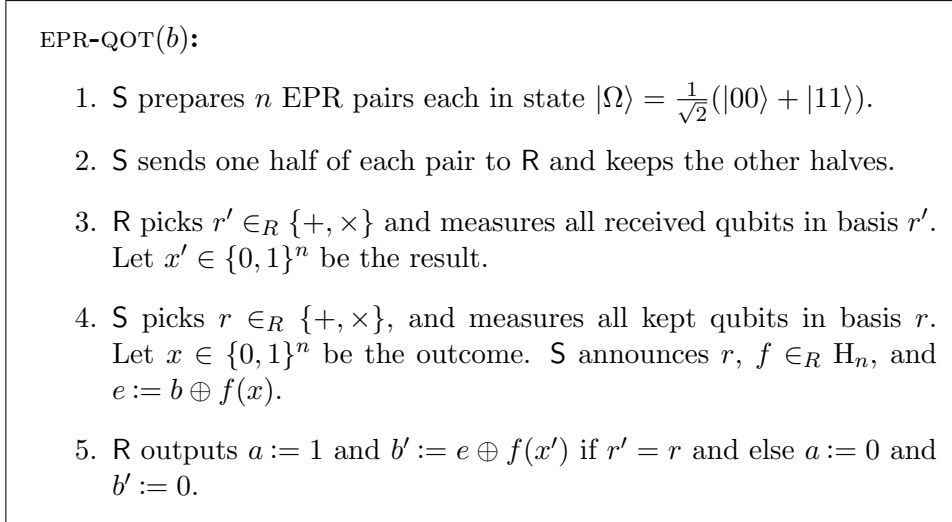


Figure 2. Protocol for EPR-based Rabin QOT

Notice that while QOT requires no quantum memory for honest players, quantum memory for S seems to be required in EPR-QOT. The following Lemma shows the strict equivalence between QOT and EPR-QOT.

Lemma 3.2. *QOT is secure if and only if EPR-QOT is secure.*

Proof: The proof follows easily after observing that S's choices of r and f , together with the measurements all commute with R's actions. Therefore, they can be performed right after Step 1 with no change for R's view. Modifying EPR-QOT that way results in QOT. \square

Note that for a dishonest receiver it is not only irrelevant whether he tries to attack QOT or EPR-QOT, but in fact there is no difference in the two protocols from his point of view.

Lemma 3.3. *EPR-QOT is perfectly receiver-private.*

Proof: It is obvious that no information about whether R has received the bit is leaked to any sender \tilde{S} , since R does not send anything, i.e. EPR-QOT is non-interactive! \square

3.3 Modeling Dishonest Receivers

We model dishonest receivers in QOT respectively EPR-QOT under the assumption that the maximum size of their quantum storage is bounded. These adversaries are only required to have bounded quantum storage when they reach Step 4 in (EPR-)QOT. Before that, the adversary can store and

carry out quantum computations involving any number of qubits. Apart from the restriction on the size of the quantum memory available to the adversary, no other assumption is made. In particular, the adversary is not assumed to be computationally bounded and the size of its classical memory is not restricted.

Definition 3.4. *The set \mathfrak{R}_γ denotes all possible quantum dishonest receivers $\{\tilde{R}_n\}_{n>0}$ in QOT or EPR-QOT where for each $n > 0$, \tilde{R}_n has quantum memory of size at most γn when Step 4 is reached.*

In general, the adversary \tilde{R} is allowed to perform any quantum computation compressing the n qubits received from S into a quantum register M of size at most γn when Step 4 is reached. More precisely, the compression function is implemented by some unitary transform C acting upon the quantum state received and an ancilla of arbitrary size. The compression is performed by a measurement that we assume in the computational basis without loss of generality. Before starting Step 4, the adversary first applies a unitary transform C :

$$2^{-n/2} \sum_{x \in \{0,1\}^n} |x\rangle \otimes C|x\rangle|0\rangle \mapsto 2^{-n/2} \sum_{x \in \{0,1\}^n} |x\rangle \otimes \sum_y \alpha_{x,y} |\varphi_{x,y}\rangle^M |y\rangle^Y,$$

where for all x , $\sum_y |\alpha_{x,y}|^2 = 1$. Then, a measurement in the computational basis is applied to register Y providing classical outcome y . The result is a quantum state in register M of size γn qubits. Ignoring the value of y to ease the notation, the re-normalized state of the system is now in its most general form when Step 4 in EPR-QOT is reached:

$$|\psi\rangle = \sum_{x \in \{0,1\}^n} \alpha_x |x\rangle \otimes |\varphi_x\rangle^M,$$

where $\sum_x |\alpha_x|^2 = 1$.

3.4 Uncertainty Relation

We first prove a general uncertainty result and derive from that a corollary that plays the crucial role in the security proof of EPR-QOT and thus of QOT. The uncertainty result concerns the situation where the sender holds an arbitrary quantum register of n qubits. He may measure them in either the $+$ or the \times basis. We are interested in the distribution of both these measurement results, and we want to claim that they cannot *both* be “very far from uniform”. One way to express this is to say that a distribution is very non-uniform if one can identify a subset of outcomes that has much higher probability than for a uniform choice. Intuitively, the theorem below says that such sets cannot be found for both of the sender’s measurements.

Theorem 3.5. *Let the density matrix ρ^A describe the state of a n -qubit register A . Let $Q^+(\cdot)$ and $Q^\times(\cdot)$ be the respective distributions of the outcome when register A is measured in the $+$ -basis respectively the \times -basis. Then, for any two sets $L^+ \subset \{0, 1\}^n$ and $L^\times \subset \{0, 1\}^n$ it holds that*

$$Q^+(L^+) + Q^\times(L^\times) \leq \left(1 + \sqrt{2^{-n}|L^+||L^\times|}\right)^2.$$

Proof: We can purify register A by adding a register B , such that the state of the composite system is pure. It can then be written as $|\psi\rangle^{AB} = \sum_{x \in \{0,1\}^n} \alpha_x |x\rangle^A |\varphi_x\rangle^B$ for some complex amplitudes α_x and normalized state vectors $|\varphi_x\rangle$.

Clearly, $Q^+(x) = |\alpha_x|^2$. To give a more explicit form of the distribution Q^\times , we apply the Hadamard transformation to register A :

$$(H^{\otimes n} \otimes \mathbb{1}^B)|\psi\rangle = \sum_{z \in \{0,1\}^n} |z\rangle \otimes \sum_{x \in \{0,1\}^n} 2^{-\frac{n}{2}} (-1)^{x \cdot z} \alpha_x |\varphi_x\rangle$$

and obtain

$$Q^\times(z) = \left| \sum_{x \in \{0,1\}^n} 2^{-\frac{n}{2}} (-1)^{x \cdot z} \alpha_x |\varphi_x\rangle \right|^2.$$

Let \bar{L}^+ denote the complement of L^+ and p its probability $Q^+(\bar{L}^+)$. We can now split the sum in $Q^\times(z)$ in the following way:

$$\begin{aligned} Q^\times(z) &= \left| \sum_{x \in \{0,1\}^n} 2^{-\frac{n}{2}} (-1)^{x \cdot z} \alpha_x |\varphi_x\rangle \right|^2 \\ &= \left| \sqrt{p} \sum_{x \in \bar{L}^+} 2^{-\frac{n}{2}} (-1)^{x \cdot z} \frac{\alpha_x}{\sqrt{p}} |\varphi_x\rangle + \sum_{x \in L^+} 2^{-\frac{n}{2}} (-1)^{x \cdot z} \alpha_x |\varphi_x\rangle \right|^2 \\ &= \left| \sqrt{p} \cdot \zeta_z |v_z\rangle + \sum_{x \in L^+} 2^{-\frac{n}{2}} (-1)^{x \cdot z} \alpha_x |\varphi_x\rangle \right|^2 \end{aligned}$$

where $|v_z\rangle$ is defined as follows: For the normalized state $|v\rangle := \sum_{x \in \bar{L}^+} \frac{\alpha_x}{\sqrt{p}} |x\rangle |\varphi_x\rangle$, $\zeta_z |v_z\rangle$ is the z -component of the state $H^{\otimes n}|v\rangle = \sum_z \zeta_z |z\rangle \otimes |v_z\rangle$. It therefore holds that $\sum_z |\zeta_z|^2 = 1$.

To upper-bound the amplitudes provided by the sum over L^+ , we notice that the amplitude is maximized when all unit vectors $|\varphi_x\rangle$ point in the same direction and when $(-1)^{x \cdot z} \alpha_x = |\alpha_x|$. More formally,

$$\left| \sum_{x \in L^+} 2^{-\frac{n}{2}} (-1)^{x \cdot z} \alpha_x |\varphi_x\rangle \right| \leq 2^{-\frac{n}{2}} \sum_{x \in L^+} |\alpha_x|$$

$$\begin{aligned}
&\leq 2^{-\frac{n}{2}} \sqrt{|L^+|} \sqrt{\sum_{x \in L^+} |\alpha_x|^2} & (3) \\
&\leq 2^{-\frac{n}{2}} \sqrt{|L^+|},
\end{aligned}$$

where (3) is obtained from the Cauchy-Schwarz inequality. Using ℓ^+ and ℓ^\times as shorthands for $|L^+|$ respectively $|L^\times|$, we conclude that

$$\begin{aligned}
Q^\times(L^\times) &= \sum_{z \in L^\times} Q^\times(z) \\
&\leq \sum_{z \in L^\times} \left(|\sqrt{p} \cdot \zeta_z| v_z + 2^{-\frac{n}{2}} \sqrt{\ell^+} \right)^2 \\
&\leq p \sum_{z \in L^\times} |\zeta_z|^2 + 2 \cdot 2^{-\frac{n}{2}} \sqrt{\ell^+} \sum_{z \in L^\times} |\zeta_z| + \ell^\times \cdot 2^{-n} \ell^+ \\
&\leq p + 2 \cdot 2^{-\frac{n}{2}} \sqrt{\ell^+} \sqrt{\ell^\times \sum_{z \in L^\times} |\zeta_z|^2} + 2^{-n} \ell^+ \ell^\times & (4) \\
&\leq p + 2\sqrt{2^{-n} \ell^+ \ell^\times} + 2^{-n} \ell^+ \ell^\times \\
&= 1 - Q^+(L^+) + 2\sqrt{2^{-n} \ell^+ \ell^\times} + 2^{-n} \ell^+ \ell^\times. & (5)
\end{aligned}$$

Inequality (4) follows again from Cauchy-Schwarz while in (5), we use the definition of p . The claim of the proposition follows after re-arranging the terms. \square

This theorem yields a meaningful bound as long as $|L^+| \cdot |L^\times| < (\sqrt{2} - 1)^2 \cdot 2^n$, e.g. if L^+ and L^\times both contain less than $2^{n/2}$ elements. If for $r \in \{+, \times\}$, L^r contains only the n -bit string with the maximal probability of Q^r , we obtain as a corollary a slightly weaker version of a known relation (see (9) in [19]).

Corollary 3.6. *Let q_∞^+ and q_∞^\times be the maximal probabilities of the distributions Q^+ and Q^\times from above. It then holds that $q_\infty^+ \cdot q_\infty^\times \leq \frac{1}{4}(1+c)^4$ where $c = 2^{-n/2}$.*

Theorem 3.5 can be generalized to more than two mutually unbiased bases. We call different sets $\mathcal{B}^0, \mathcal{B}^1, \dots, \mathcal{B}^N$ of bases of the complex Hilbert space \mathbb{C}^{2^n} *mutually unbiased*, if for all $i \neq j \in \{0, \dots, N\}$, it holds that

$$\forall |\varphi\rangle \in \mathcal{B}^i \forall |\psi\rangle \in \mathcal{B}^j : |\langle \varphi | \psi \rangle|^2 = 2^{-n}.$$

Theorem 3.7. *Let the density matrix ρ^A describe the state of a n -qubit register A and let $\mathcal{B}^0, \mathcal{B}^1, \dots, \mathcal{B}^N$ be mutually unbiased bases of register*

A. Let $Q^0(\cdot), Q^1(\cdot), \dots, Q^N(\cdot)$ be the distributions of the outcome when register A is measured in bases $\mathcal{B}^0, \mathcal{B}^1, \dots, \mathcal{B}^N$, respectively. Then, for any sets $L^0, L^1, \dots, L^N \subset \{0, 1\}^n$, it holds that

$$\sum_{i=0}^N Q^i(L^i) \leq 1 - \binom{N+1}{2} + \sum_{0 \leq j < k \leq N} \left(1 + \sqrt{2^{-n} |L^j| |L^k|}\right)^2.$$

Proof: Like in the proof of Theorem 3.5, we can purify register A by adding a register B . The composite state can then be written as $|\psi\rangle^{AB} = \sum_{x \in \{0,1\}^n} \alpha_x |x\rangle^A |\varphi_x\rangle^B$ for some complex amplitudes α_x and normalized state vectors $|\varphi_x\rangle$.

We prove the statement by induction over N : For $N = 1$, by applying an appropriate unitary transform to the whole system, we can assume without loss of generality that \mathcal{B}^0 is the standard $+$ -basis.

Let us denote by T the matrix of the basis change from \mathcal{B}^0 to \mathcal{B}^1 . As the inner product between states $|\phi\rangle \in \mathcal{B}^0$ and $|\phi'\rangle \in \mathcal{B}^1$ is always $|\langle \phi | \phi' \rangle| = 2^{-n/2}$, it follows that all entries of T are complex numbers of the form $2^{-n/2} \cdot e^{i\lambda}$ for real $\lambda \in \mathbb{R}$.

It is easy to verify that the same proof as for Theorem 3.5 applies after replacing the Hadamard transform $H^{\otimes n}$ on the sender's part by T and using the above observation about the entries of T .

For the induction step from N to $N + 1$, we define $p := Q^0(\bar{L}^0)$, $|v\rangle := \sum_{x \in \bar{L}^0} \frac{\alpha_x}{\sqrt{p}} |x\rangle |\varphi_x\rangle$, and let $|\zeta_z^j|v_z^j\rangle$ be the z -component of the state $|v\rangle$ transformed into basis \mathcal{B}^j . As in the proof of Theorem 3.5, using ℓ_i as a short hand for $|L^i|$, it follows:

$$\begin{aligned} \sum_{i=1}^N Q^i(L^i) &= \sum_{i=1}^N \sum_{z \in L^i} Q^i(z) \\ &\leq \sum_{i=1}^N \sum_{z \in L^i} \left(\sqrt{p} |\zeta_z^i|v_z^i\rangle| + 2^{-n/2} \sqrt{\ell_0} \right)^2 \\ &\leq p \cdot \sum_{i=1}^N \sum_{z \in L^i} |\zeta_z^i|^2 + \sum_{i=1}^N \left(2 \cdot \sqrt{2^{-n} \ell_0 \ell_i} + 2^{-n} \ell_0 \ell_i \right) \\ &\leq p \cdot \sum_{i=1}^N P^i(L^i) + \sum_{i=1}^N \left(1 + \sqrt{2^{-n} \ell_0 \ell_i} \right)^2 - N \end{aligned}$$

where the distributions P^i are obtained by measuring register A of the normalized state $|v\rangle$ in the mutually unbiased bases $\mathcal{B}^1, \mathcal{B}^2, \dots, \mathcal{B}^N$. We apply the induction hypothesis to the sum of $P^i(L^i)$:

$$\sum_{i=1}^N Q^i(L^i) \leq p \cdot \sum_{i=1}^N P^i(L^i) + \sum_{i=1}^N \left(1 + \sqrt{2^{-n} \ell_0 \ell_i} \right)^2 - N$$

$$\begin{aligned}
&\leq [1 - Q^0(L^0)] \left[\sum_{1 \leq j < k \leq N} \left(1 + \sqrt{2^{-n} \ell_j \ell_k}\right)^2 + 1 - \binom{N}{2} \right] \\
&\quad + \sum_{i=1}^N \left(1 - \sqrt{2^{-n} \ell_0 \ell_i}\right)^2 - N \\
&\leq -Q^0(L^0) + 1 - \binom{N+1}{2} + \sum_{0 \leq j < k \leq N} \left(1 + \sqrt{2^{-n} \ell_j \ell_k}\right)^2
\end{aligned}$$

where the last inequality follows by observing that the term in the right bracket is at least 1 and rearranging the terms. This completes the induction step and the proof of the proposition. \square

Analogous to Corollary 3.6, we derive an uncertainty relation about the sum of the min-entropies of up to $2^{\frac{n}{4}}$ distributions.

Corollary 3.8. *For an $\varepsilon > 0$, let $0 < N < 2^{(\frac{1}{4}-\varepsilon)n}$. For $i = 0, \dots, N$, let H_∞^i be the min-entropies of the distributions Q^i from the theorem above. Then,*

$$\sum_{i=0}^N H_\infty^i \geq (N+1)(\log(N+1) - \text{negl}(n)).$$

Proof: For $i = 0, \dots, N$, we denote by q_∞^i the maximal probability of Q^i and let L^i be the set containing only the n -bit string x with this maximal probability q_∞^i . Theorem 3.7 together with the assumption about N assures $\sum_{i=0}^N q_\infty^i \leq 1 + \text{negl}(n)$. By the inequality of the geometric and arithmetic mean follows:

$$\begin{aligned}
\sum_{i=0}^N H_\infty^i &= -\log \prod_{i=0}^N q_\infty^i \geq -\log \left(\frac{1 + \text{negl}(n)}{N+1} \right)^{N+1} \\
&= (N+1)(\log(N+1) - \text{negl}(n)).
\end{aligned}$$

\square

3.5 Security Against Dishonest Receivers

In this section, we show that EPR-QOT is secure against any dishonest receiver having access to a quantum storage device of size strictly smaller than half the number of qubits received at Step 2.

In our setting, we use Theorem 3.5 to lower-bound the overall probability of strings with small probabilities in the following sense. For $0 \leq \gamma + \kappa \leq 1$, define

$$\begin{aligned}
S^+ &:= \{x \in \{0, 1\}^n : Q^+(x) \leq 2^{-(\gamma+\kappa)n}\} \quad \text{and} \\
S^\times &:= \{z \in \{0, 1\}^n : Q^\times(z) \leq 2^{-(\gamma+\kappa)n}\}
\end{aligned}$$

to be the sets of strings with small probabilities and denote by $L^+ := \overline{S^+}$ and $L^\times := \overline{S^\times}$ their complements. (Here's the mnemonic: S for the strings with *S*mall probabilities, L for *L*arge.) Note that for all $x \in L^+$, we have that $Q^+(x) > 2^{-(\gamma+\kappa)n}$ and therefore $|L^+| < 2^{(\gamma+\kappa)n}$. Analogously, we have $|L^\times| < 2^{(\gamma+\kappa)n}$. For the ease of notation, we abbreviate the probabilities that strings with small probabilities occur as follows: $q^+ := Q^+(S^+)$ and $q^\times := Q^\times(S^\times)$. The next corollary now immediately follows from Theorem 3.5.

Corollary 3.9. *Let $\gamma + \kappa < \frac{1}{2}$. For the probability distributions Q^+ , Q^\times and the sets S^+ , S^\times defined above, we have*

$$q^+ + q^\times = Q^+(S^+) + Q^\times(S^\times) \geq 1 - \text{negl}(n).$$

Theorem 3.10. *For all $\gamma < \frac{1}{2}$, QOT is secure against \mathfrak{R}_γ .*

Proof: After Lemmata 3.2 and 3.3, it remains to show that EPR-QOT is sender-private against \mathfrak{R}_γ . Since $\gamma < \frac{1}{2}$, we can find $\kappa > 0$ with $\gamma + \kappa < \frac{1}{2}$. Consider a dishonest receiver in EPR-QOT \tilde{R} with quantum memory of size γn .

Using the notation from Section 3.1, we show that there exists an event \mathcal{E} such that $P[\mathcal{E}] \geq \frac{1}{2} - \text{negl}(n)$ as well as $\delta([\{B\} \otimes \rho_{\tilde{R}} | \mathcal{E}], [\{B\}] \otimes [\rho_{\tilde{R}} | \mathcal{E}]) \leq \text{negl}(n)$, as required by the sender-privacy condition of Definition 3.1. Let X denote the random variable describing the outcome x of S 's measurement (in basis r) in Step 4 of EPR-QOT. We implicitly understand the distribution of X to be conditioned on the classical outcome y of the measurement \tilde{R} performs when the memory bound applies, as described in Section 3.3. We define \mathcal{E} to be the event $X \in S^r$. Note that \mathcal{E} is independent of B and thus $[B | \mathcal{E}] = [B]$. Furthermore, due to the uniform choice of r , and using Corollary 3.9, $P[\mathcal{E}] = \frac{1}{2}(q^+ + q^\times) \geq \frac{1}{2} - \text{negl}(n)$.

In order to show the second condition, we have to show that whenever \mathcal{E} occurs, the dishonest receiver cannot distinguish the situation where $B = 0$ is sent from the one where $B = 1$ is sent. As the bit B is masked by the output of the hash function $F(X)$ in Step 4 of EPR-QOT (where the random variable F represents the random choice for f), this is equivalent to distinguish between $F(X) = 0$ and $F(X) = 1$. This situation is exactly suited for applying Theorem 2.1, which says that $F(X) = 0$ is indistinguishable from $F(X) = 1$ whenever the right-hand side of (2) is negligible.

In the case $r = +$, we have

$$\begin{aligned} H_\infty(X | X \in S^+) &= -\log \left(\max_{x \in S^+} \frac{Q^+(x)}{q^+} \right) \\ &\geq -\log \left(\frac{2^{-(\gamma+\kappa)n}}{q^+} \right) = \gamma n + \kappa n + \log(q^+). \end{aligned} \quad (6)$$

If $q^+ \geq 2^{-\frac{\kappa}{2}n}$ then $H_\infty(X | X \in S^+) \geq \gamma n + \frac{\kappa}{2}n$ and indeed the right-hand side of (2) decreases exponentially when conditioning on $X \in S^+$. The corresponding holds for the case $r = \times$.

Finally, if $q^+ < 2^{-\frac{\kappa}{2}n}$ (or similarly $q^\times < 2^{-\frac{\kappa}{2}n}$) then instead of as above we define \mathcal{E} as the *empty event* if $r = +$ and as the event $X \in S^\times$ if $r = \times$. It follows that $P[\mathcal{E}] = \frac{1}{2} \cdot q^\times \geq \frac{1}{2} - \text{negl}(n)$ as well as $H_\infty(X|\mathcal{E}) = H_\infty(X|X \in S^\times) \geq \gamma n + \kappa n + \log(q^\times) \geq \gamma n + \frac{\kappa}{2}n$ (for n large enough), both by Corollary 3.9 and the bound on q^+ . \square

3.6 On the Necessity of Privacy Amplification

In this section, we show that randomized privacy amplification seems to be needed for protocol QOT to be secure. It is tempting to believe that the sender could use the xor $\bigoplus_i x_i$ in order to mask the bit b , rather than $f(x)$ for a randomly sampled $f \in \mathbb{H}_n$. This would reduce the communication complexity as well as the number of random coins needed. However, we argue in this section that this is not secure (against an adversary as we model it). Indeed, somewhat surprisingly, this variant can be broken by a dishonest receiver that has *no quantum memory at all* (but that can do coherent measurements on pairs of qubits).

Clearly, a dishonest receiver can break the modified scheme QOT and learn the bit b with probability 1 if he can compute $\bigoplus_i x_i$ with probability 1. Note that, using the equivalence between QOT and EPR-QOT, x_i can be understood as the outcome of the measurement in either the $+$ - or the \times -basis, performed by the sender on one part of an EPR pair while the other has been handed over to the receiver. The following proposition shows that indeed the receiver can learn $\bigoplus_i x_i$ by a suitable measurement of his parts of the EPR pairs. Concretely, he measures the qubits he receives pair-wise by a suitable measurement which allows him to learn the xor of the two corresponding x_i 's, no matter what the basis is (and he needs to store one single qubit in case n is odd). This obviously allows him to learn the xor of all x_i 's in all cases.

Proposition 3.11. *Consider two EPR pairs, i.e., $|\psi\rangle = \frac{1}{2} \sum_x |x\rangle^S |x\rangle^R$ where x ranges over $\{0, 1\}^2$. Let $r \in \{+, \times\}$, and let x_1 and x_2 be the result when measuring the two qubits in register S in basis r . There exists a fixed measurement for register R so that the outcome together with r uniquely determines $x_1 \oplus x_2$.*

Proof: The measurement that does the job is the *Bell measurement*, i.e., the measurement in the Bell basis $\{|\Phi^+\rangle, |\Psi^+\rangle, |\Phi^-\rangle, |\Psi^-\rangle\}$. Recall,

$$\begin{aligned} |\Phi^+\rangle &= \frac{1}{\sqrt{2}}(|00\rangle_+ + |11\rangle_+) = \frac{1}{\sqrt{2}}(|00\rangle_\times + |11\rangle_\times) \\ |\Psi^+\rangle &= \frac{1}{\sqrt{2}}(|01\rangle_+ + |10\rangle_+) = \frac{1}{\sqrt{2}}(|00\rangle_\times - |11\rangle_\times) \\ |\Phi^-\rangle &= \frac{1}{\sqrt{2}}(|00\rangle_+ - |11\rangle_+) = \frac{1}{\sqrt{2}}(|01\rangle_\times + |10\rangle_\times) \end{aligned}$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle_+ - |10\rangle_+) = \frac{1}{\sqrt{2}}(|10\rangle_\times - |01\rangle_\times).$$

Due to the special form of the Bell basis, when register R is measured and, as a consequence, one of the four Bell states is observed, the state in register S collapses to that *same* Bell state. Indeed, when doing the basis transformation, all cross-products cancel each other out. It now follows by inspection that knowledge of the Bell state and the basis r allows to predict the xor of the two bits observed when measuring the Bell state in basis r . For instance, for the Bell state $|\Psi^+\rangle$, the xor is 1 if $r = +$ and it is 0 if $r = \times$. \square

Note that from the above proof one can see that the receiver's attack, respectively his measurement on each pair of qubits, can be understood as teleporting one of the two (entangled) qubits from the receiver to the sender using the other as EPR pair (but the receiver does not send the outcome of his measurement to the sender, but keeps it in order to predict the xor).

Clearly, the same strategy also works against any fixed linear function. Therefore, the only hope for doing deterministic privacy amplification is by using a non-linear function; but whether it is possible at all is not known to us.

3.7 Weakening the Assumptions

Observe that QOT requires error-free quantum communication, in that a transmitted bit b , that is encoded by the sender and measured by the receiver using the same basis, is always received as b . And it requires a perfect quantum source which on request produces *one* qubit in the right state, e.g. *one* photon with the right polarization. Indeed, in case of noisy quantum communication, an honest receiver in QOT is likely to receive an incorrect bit, and the sender-privacy of QOT is vulnerable to imperfect sources that once in while transmit more than one qubit in the same state: a malicious receiver \tilde{R} can easily determine the basis $r \in \{+, \times\}$ and measure all the following qubits in the right basis. However, current technology only allows to approximate the behavior of single-photon sources and of noise-free quantum communication. It would be preferable to find a variant of QOT that allows to weaken the technological requirements put upon the honest parties.

In this section, we present such a protocol based on BB84 states [1], BB84-QOT (see Figure 3). The security proof follows essentially by adapting the security analysis of QOT in a rather straightforward way, as will be discussed later.

Let us consider a quantum channel with an error probability $\phi < \frac{1}{2}$, i.e., ϕ denotes the probability that a transmitted bit b , that is encoded by the sender and measured by the receiver using the same basis, is received as

$1 - b$. In order not to have the security rely on any level of noise, we assume the error probability to be zero when considering a *dishonest* receiver. Also, let us consider a quantum source which produces two or more qubits (in the same state), rather than just one, with probability $\eta < 1 - \phi$. We call this the (ϕ, η) -weak quantum model.

In order to deal with noisy quantum communication, we need to do error-correction without giving the adversary too much information. Techniques to solve this problem are known as *information reconciliation* (e.g. [2]) or as *secure sketches* [13]. Let $x \in \{0, 1\}^\ell$ be an arbitrary string, and let $x' \in \{0, 1\}^\ell$ be the result of flipping every bit in x (independently) with probability ϕ . It is well known that learning the syndrome $S(x)$ of x , with respect to an efficiently decodable linear error-correcting code C of length ℓ with minimal distance $d = (\phi + \varepsilon)\ell$ where $\varepsilon > 0$, allows to recover x from x' , except with negligible probability in ℓ (e.g. [20, 5, 13]). Furthermore, it is known from coding theory that (for large enough ℓ) such a code can be chosen with rate R arbitrary close to (but smaller than) $1 - h(\phi)$, i.e., such that the syndrome length s is bounded by $s < (h(\phi) + \varepsilon)\ell$ where $\varepsilon > 0$ (see e.g. [5] and the reference therein).

Regarding the loss of information, we can analyze privacy amplification in a similar way as before, just by adding the syndrome $S(x)$ to the random state ρ . Using that $S_0(\{|S(X)\} \otimes \rho) \leq q + s$, Theorem 2.1 then reads as

$$d(F(X) | \{|F\} \otimes \{|S(X)\} \otimes \rho) \leq \frac{1}{2} 2^{-\frac{1}{2}(H_\infty(X) - q - s - 1)}. \quad (7)$$

Consider the protocol BB84-QOT in the (ϕ, η) -weak quantum model shown in Figure 3. The protocol uses an efficiently decodable linear code C_ℓ , parameterized in $\ell \in \mathbb{N}$, with codeword length ℓ , minimal distance $d = (\phi + \varepsilon)\ell$, and rate $R = 1 - h(\phi) - \varepsilon$ for some small $\varepsilon > 0$. Let S_ℓ be the corresponding syndrome function. Like before, the memory bound in BB84-QOT applies before Step 4.

By the above mentioned properties of the code C_ℓ , it is obvious that R receives the correct bit b if $r' = r$, except with negligible probability. (The error probability is negligible in ℓ , but by Bernstein's law of large numbers, ℓ is linear in n except with negligible probability.) Also, since there is no communication from R to S, BB84-QOT is clearly receiver-private. Similar as for protocol QOT, in order to argue about sender-privacy we compare BB84-QOT with a purified version shown in Figure 4. BB84-EPR-QOT runs in the $(\phi, 0)$ -weak quantum model, and the imperfectness of the quantum source assumed in BB84-QOT is simulated by S in BB84-EPR-QOT so that there is no difference from R's point of view.

The security equivalence between BB84-QOT (in the (ϕ, η) -weak quantum model) and BB84-EPR-QOT (in the $(\phi, 0)$ -weak quantum model) is omitted here as it follows essentially along the same lines as in Section 3.2.

BB84-QOT(b):

1. S picks $x \in_R \{0, 1\}^n$ and $\theta \in_R \{+, \times\}^n$.
2. S sends x_i in the corresponding bases $|x_1\rangle_{\theta_1}, \dots, |x_n\rangle_{\theta_n}$ to R.
3. R picks $r' \in_R \{+, \times\}$ and measures all qubits in basis r' . Let $x' \in \{0, 1\}^n$ be the result.
4. S picks $r \in_R \{+, \times\}$, sets $I := \{i : \theta_i = \{+, \times\}_{[r]}\}$ and $\ell := |I|$, and announces $r, I, \text{syn} := S_\ell(x|_I), f \in_R \mathbb{H}_\ell$, and $e := b \oplus f(x|_I)$.
5. R recovers $x|_I$ from $x'|_I$ and syn , and outputs $a := 1$ and $b' := e \oplus f(x|_I)$ if $r' = r$ and else $a := 0$ and $b' := 0$.

Figure 3. Protocol for the BB84 version of Rabin QOT

Theorem 3.12. *In the (ϕ, η) -weak quantum model, BB84-QOT is secure against \mathfrak{R}_γ for any $\gamma < \frac{1-\eta}{4} - \frac{h(\phi)}{2}$ (if parameter ε is chosen small enough).*

Proof Sketch: It remains to show that BB84-EPR-QOT is sender-private against \mathfrak{B}_γ (in the $(\phi, 0)$ -weak quantum model). The reasoning goes exactly along the lines of the proof of Theorem 3.10, except that we restrict our attention to those i 's which are in J . By Bernstein's law of large numbers, ℓ lies within $(1 \pm \varepsilon)n/2$ and $|J|$ within $(1 - \eta \pm \varepsilon)n/2$ except with negligible probability. In order to make the proof easier to read, we assume that $\ell = n/2$ and $|J| = (1 - \eta)n/2$, and we also treat the ε occurring in the rate of the code C_ℓ as zero. For the full proof, we simply need to carry the ε 's along, and then choose it small enough at the end of the proof.

Write $n' = |J| = (1 - \eta)n/2$, and let γ' be such that $\gamma n = \gamma' n'$, i.e., $\gamma' = 2\gamma/(1 - \eta)$. Let S^+ and S^\times be defined as in Section 3.5, but with respect to n' and γ' (and some $\kappa < \frac{1}{2} - \gamma'$). It then follows as in the proof of Theorem 3.10 that

$$\begin{aligned} H_\infty(X|_J|X|_J \in S^+) &\geq \gamma' n' + \kappa n' + \log(q^+) \\ &= \gamma n + \kappa(1 - \eta)n/2 + \log(q^+). \end{aligned}$$

Similar as in the proof of Theorem 3.10, one can make a case distinction on q^+ (whether $q^+ \geq 2^{-\varepsilon n}$ or $q^+ < 2^{-\varepsilon n}$), and in both cases argue that the min-entropy in question is larger than $\gamma n + \kappa(1 - \eta)n/2$ (\pm some εn 's). By (2), it remains to argue that this is larger than $q + s = \gamma n + h(\phi)n/2$, i.e.,

$$\kappa(1 - \eta) > h(\phi),$$

where κ has to satisfy

$$\kappa < \frac{1}{2} - \gamma' = \frac{1}{2} - 2\gamma/(1 - \eta).$$

BB84-EPR-QOT(b):

1. S prepares n EPR pairs each in state $|\Omega\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. Additionally, S initializes $I'_+ := \emptyset$ and $I'_\times := \emptyset$.
2. For every $i \in \{1, \dots, n\}$, S does the following. With probability $1 - \eta$ S sends one half of the i -th pair to R and keeps the other half. While with probability η S picks $\theta_i \in_R \{+, \times\}$, replaces I'_{θ_i} by $I'_{\theta_i} \cup \{i\}$ and sends two or more qubits in the same state $|x_i\rangle_{\theta_i}$ to R where $x_i \in_R \{0, 1\}$.
3. R picks $r' \in_R \{+, \times\}$ and measures all received qubits in basis r' . Let $x' \in \{0, 1\}^n$ be the result.
4. S picks a random index set $J \subset_R \{1, \dots, n\} \setminus (I'_+ \cup I'_\times)$. Then, it picks $r \in_R \{+, \times\}$, sets $I := J \cup I'_r$ and $\ell := |I|$, and for each $i \in J$ it measures the corresponding qubit in basis r . Let x_i be the corresponding outcome, and let $x|_I$ be the collection of all x_i 's with $i \in I$. S announces $r, I, \text{syn} = S_\ell(x|_I), f \in_R \mathbb{H}_\ell$, and $e = b \oplus f(x|_I)$.
5. R recovers $x|_I$ from $x'|_I$ and syn , and outputs $a := 1$ and $b' := e \oplus f(x|_I)$, if $r' = r$ and else $a := 0$ and $b' := 0$.

Figure 4. Protocol for EPR-based Rabin QOT, BB84 version

This can obviously be achieved (by choosing κ appropriately) if and only if the claimed bound on γ holds. \square

4 Quantum Commitment Scheme

In this section, we present a BC scheme from a committer C with bounded quantum memory to an unbounded receiver V. The scheme is peculiar since in order to commit to a bit, the committer does not send anything. During the committing stage information only goes from V to C. The security analysis of the scheme uses similar techniques as the analysis of EPR-QOT.

4.1 The Protocol

The objective of this section is to present a bounded quantum-memory BC scheme COMM (see Figure 5). Intuitively, a commitment to a bit b is made by measuring random BB84-states in basis $\{+, \times\}_{[b]}$.

As for the OT-protocol of Section 3.2, we present an equivalent EPR-version of the protocol that is easier to analyze (see Figure 6).

COMM(b):

1. V picks $x \in_R \{0, 1\}^n$ and $r \in_R \{+, \times\}^n$.
2. V sends x_i in the corresponding bases $|x_1\rangle_{r_1}, |x_2\rangle_{r_2}, \dots, |x_n\rangle_{r_n}$ to C .
3. C commits to the bit b by measuring all qubits in basis $\{+, \times\}_{[b]}$. Let $x' \in \{0, 1\}^n$ be the result.
4. To open the commitment, C sends b and x' to V .
5. V verifies that $x_i = x'_i$ for those i where $r_i = \{+, \times\}_{[b]}$. V accepts if and only if this is the case.

Figure 5. Protocol for quantum commitment

Lemma 4.1. *COMM is secure if and only if EPR-COMM is secure.*

Proof: The proof uses similar reasoning as the one for Lemma 3.2. First, it clearly makes no difference, if we change Step 5 to the following:

- 5'. V chooses the subset I , measures all qubits with index in I in basis $\{+, \times\}_{[b]}$ and all qubits not in I in basis $\{+, \times\}_{[1-b]}$. V verifies that $x_i = x'_i$ for all $i \in I$ and accepts if and only if this is the case.

Finally, we can observe that the view of C does not change if V would have done his choice of I and his measurement already in Step 1. Doing the measurements at this point means that the qubits to be sent to C collapse to a state that is distributed identically to the state prepared in the original scheme. The EPR-version is therefore equivalent to the original commitment scheme from C 's point of view. \square

It is clear that EPR-COMM is hiding, i.e., that the commit phase reveals no information on the committed bit, since no information is transmitted to V at all. Hence we have

Lemma 4.2. *EPR-COMM is perfectly hiding.*

4.2 Modeling Dishonest Committers

A dishonest committer \tilde{C} with bounded memory of at most γn qubits in EPR-COMM can be modeled very similarly to the dishonest OT-receiver \tilde{R} from Section 3.3: \tilde{C} consists first of a circuit acting on all n qubits received, then of a measurement of all but at most γn qubits, and finally of a circuit that takes the following input: a bit b that \tilde{C} will attempt to open, the γn qubits in memory, and some ancilla in a fixed state. The output is a string $x' \in \{0, 1\}^n$ to be sent to V at the opening stage.

EPR-COMM(b):

1. V prepares n EPR pairs each in state $|\Omega\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.
2. V sends one half of each pair to C and keeps the other halves.
3. C commits to the bit b by measuring all received qubits in basis $\{+, \times\}_{[b]}$. Let $x' \in \{0, 1\}^n$ be the result.
4. To open the commitment, C sends b and x' to V .
5. V measures all his qubits in basis $\{+, \times\}_{[b]}$ and obtains $x \in \{0, 1\}^n$. He chooses a random subset $I \subseteq \{1, \dots, n\}$. V verifies that $x_i = x'_i$ for all $i \in I$ and accepts if and only if this is the case.

Figure 6. Protocol for EPR-based quantum commitment

Definition 4.3. We define \mathfrak{C}_γ to be the class of all committers $\{\tilde{C}_n\}_{n>0}$ in COMM or EPR-COMM that, at the start of the opening phase (i.e. at Step 4), have a quantum memory of size at most γn qubits.

We adopt the binding condition for quantum BC from [14]:

Definition 4.4. A (quantum) BC scheme is **(statistically) binding** against \mathfrak{C} if for all $\{\tilde{C}_n\}_{n>0} \in \mathfrak{C}$, the probability $p_b(n)$ that \tilde{C}_n opens $b \in \{0, 1\}$ with success satisfies

$$p_0(n) + p_1(n) \leq 1 + \text{negl}(n).$$

In the next section, we show that EPR-COMM is binding against \mathfrak{C}_γ for any $\gamma < \frac{1}{2}$.

Note that the binding condition given here in Definition 4.4 is weaker than the classical one, where one would require that a bit b exists such that $p_b(n)$ is negligible. In the context of *quantum* bit commitment, this weaker definition is typically justified by the argument that this is the best that can be achieved for a general quantum adversary who can always commit to 0 and 1 in superposition. However, an adversary with bounded quantum storage cannot necessarily maintain a commitment in superposition since the memory compression may force a collapse. Indeed, in upcoming work, we show that commitment schemes exist satisfying the stronger binding condition in the bounded quantum-storage model [9]. While the weaker condition is sufficient for many applications, the stronger one seems to be necessary in some cases (see the conclusion).

4.3 Security Proof of the Commitment Scheme

Note that the first three steps of EPR-QOT and EPR-COMM (i.e. before the memory bound applies) are exactly the same! This allows us to reuse Corollary 3.9 and the analysis of Section 3.5 to prove the binding property of EPR-COMM.

Theorem 4.5. *For any $\gamma < \frac{1}{2}$, COMM is perfectly hiding and statistically binding against \mathfrak{C}_γ .*

The proof is given below. It boils down to showing that essentially $p_0(n) \leq 1 - q^+$ and $p_1(n) \leq 1 - q^\times$. The binding property then follows immediately from Corollary 3.9. The intuition behind $p_0(n) \leq 1 - q^+ = 1 - Q^+(S^+)$ is that a committer has only a fair chance in opening to 0 if x measured in $+$ basis has a large probability, i.e., $x \notin S^+$. The following proof makes this intuition precise by choosing the ε and δ 's correctly.

Proof: It remains to show that EPR-COMM is binding against \mathfrak{C}_γ . Let $\kappa > 0$ be such that $\gamma + \kappa < \frac{1}{2}$. For the parameters κ and γ considered here, define Q^+ , S^+ and q^+ as well as Q^\times , S^\times and q^\times as in Section 3.5. Furthermore, let $0 < \delta < \frac{1}{2}$ be such that $h(\delta) < \kappa/2$, where h is the binary entropy function, and choose $\varepsilon > 0$ small enough such that $h(\delta) < (\kappa - \varepsilon)/2$. This guarantees that $B^{\delta n} \leq 2^{(\kappa - \varepsilon)n/2}$ for all (sufficiently large) n . For every n we distinguish between the following two cases. If $q^+ \geq 2^{-\varepsilon n/2}$ then

$$H_\infty(X|X \in S^+) \geq \gamma n + \kappa n + \log(q^+) \geq \gamma n + \left(\kappa - \frac{\varepsilon}{2}\right)n$$

where the first inequality is argued as in (6). Applying Lemma 2.2, it follows that any guess \hat{X} for X satisfies

$$\Pr[\hat{X} \in B^{\delta n}(X) | X \in S^+] \leq 2^{-\frac{1}{2}(H_\infty(X|X \in S^+) - \gamma n - 1) + \log(B^{\delta n})} \leq 2^{-\frac{\varepsilon}{4}n + \frac{1}{2}}.$$

However, if $\hat{X} \notin B^{\delta n}(X)$ then sampling a random subset of the positions will detect an error except with probability not bigger than $2^{-\delta n}$. Hence,

$$\begin{aligned} p_0(n) &= (1 - q^+) \cdot p_{0|X \notin S^+} + q^+ \cdot p_{0|X \in S^+} \\ &\leq 1 - q^+ + q^+ \cdot (2^{-\delta n}(1 - 2^{-\frac{\varepsilon}{4}n + \frac{1}{2}}) + 2^{-\frac{\varepsilon}{4}n + \frac{1}{2}}). \end{aligned}$$

If on the other hand $q^+ < 2^{-\varepsilon n/2}$ then trivially

$$p_0(n) \leq 1 = 1 - q^+ + q^+ < 1 - q^+ + 2^{-\varepsilon n/2}.$$

In any case we have $p_0(n) \leq 1 - q^+ + \text{negl}(n)$.

Analogously, we derive $p_1(n) \leq 1 - q^\times + \text{negl}(n)$ and conclude that

$$p_0(n) + p_1(n) \leq 2 - q^+ - q^\times + \text{negl}(n) \leq 1 + \text{negl}(n), \quad (8)$$

where (8) is obtained from Corollary 3.9. \square

4.4 Weakening the Assumptions

As argued earlier, assuming that a party can produce single qubits (with probability 1) is not reasonable given current technology. Also the assumption that there is no noise on the quantum channel is impractical. It can be shown that a straightforward modification of COMM remains secure in the (ϕ, η) -weak quantum model as introduced in Section 3.7, with $\phi < \frac{1}{2}$ and $\eta < 1 - \phi$.

Let COMM' be the modification of COMM where in Step 5 V accepts if and only if $x_i = x'_i$ for all *but about a ϕ -fraction* of the i where $r_i = \{+, \times\}_{[b]}$. More precisely, for all but a $(\phi + \varepsilon)$ -fraction, where $\varepsilon > 0$ is sufficiently small.

Theorem 4.6. *In the (ϕ, η) -weak quantum model, COMM' is perfectly hiding and it is binding against \mathfrak{C}_γ for any γ satisfying $\gamma < \frac{1}{2}(1 - \eta) - 2h(\phi)$.*

Proof Sketch: Using Bernstein's law of large numbers, one can argue that for *honest* C and V, the opening of a commitment is accepted except with negligible probability. The hiding property holds using the same reasoning as in Lemma 4.2. And the binding property can be argued essentially along the lines of Theorem 4.5, with the following modifications. Let J denote the set of indices i where V succeeds in sending a single qubit. We restrict the analysis to those i 's which are in J . By Bernstein's law of large numbers, the cardinality of J is about $(1 - \eta)n$ (meaning within $(1 - \eta \pm \varepsilon)n$), except with negligible probability. Thus, restricting to these i 's has the same effect as replacing γ by $\gamma/(1 - \eta)$ (neglecting the $\pm \varepsilon$ to simplify notation). Assuming that \tilde{C} knows every x_i for $i \notin J$, for all x_i 's with $i \in J$ he has to be able to guess all but about a $\phi/(1 - \eta)$ -fraction correctly, in order to be successful in the opening. However, \tilde{C} succeeds with only negligible probability if

$$\phi/(1 - \eta) < \delta.$$

Additionally, δ must be such that

$$h(\delta) < \frac{\kappa}{2} \quad \text{with} \quad \frac{\gamma}{1 - \eta} + \kappa < \frac{1}{2}.$$

Both restrictions on δ hold (respectively can be achieved by choosing κ appropriately) if

$$2h\left(\frac{\phi}{1 - \eta}\right) + \frac{\gamma}{1 - \eta} < \frac{1}{2}.$$

Using the fact that $h(\nu p) \leq \nu h(p)$ for any $\nu \geq 1$ and $0 \leq p \leq \frac{1}{2}$ such that $\nu p \leq 1$, this is clearly satisfied if $2h(\phi) + \gamma < \frac{1}{2}(1 - \eta)$. This proves the claim. \square

5 Generalizing the Memory Model

The bounded quantum-storage model limits the number of physical qubits the adversary’s memory can contain. A more realistic model would rather address the noise process the adversary’s memory undergoes. For instance, it is not hard to build a very large, but unreliable memory device containing a large number of qubits. It is reasonable to expect that our protocols remain secure also in a scenario where the adversary’s memory is of arbitrary size, but where some quantum operation (modeling noise) is applied to it. Inequality (1) of the Privacy Amplification Theorem 2.1 allows us to apply our constructions to slightly more general memory models. In particular, all our protocols that are secure against adversaries with memory of no more than γn qubits are also secure against any noise model that reduces the rank of the mixed state $[\rho]$, held by the adversary, to at most $2^{\gamma n}$.

An example of a noise process resulting in a reduction of $S_0([\rho])$ is an erasure channel. Assuming the n initial qubits are each erased with probability larger than $1 - \gamma$ when the memory bound applies, it holds except with negligible probability in n that $S_0([\rho]) < \gamma n$. The same applies if the noise process is modelled by a depolarizing channel with error probability $p = 1 - \gamma$. Such a depolarizing channel replaces each qubit by a random one with probability p and does nothing with probability $1 - p$.

The technique we have developed does not allow to deal with depolarizing channels with $p < 1 - \gamma$ although one would expect that some $0 < p < 1 - \gamma$ should be sufficient to ensure privacy against such adversaries. The reason being that not knowing the positions where the errors occurred should make it more difficult for the adversary than when the noise process is modelled by an erasure channel. However, it seems that our uncertainty relations (i.e. Theorems 3.5 and 3.7) are not strong enough to address this case. Generalizing the bounded quantum-storage model to more realistic noisy-memory models is an interesting open question.

6 Conclusion And Further Research

We have shown how to construct ROT and BC securely in the bounded quantum-storage model. Our protocols require no quantum memory for honest players and remain secure provided the adversary has only access to quantum memory of size bounded by a large fraction of all qubits transmitted. Such a gap between the amount of storage required for honest players and adversaries is not achievable by classical means. All our protocols are non-interactive and can be implemented using current technology.

In this paper, we only considered ROT of one bit per invocation. Our technique can easily be extended to deal with string ROT, essentially by using a class of two-universal functions with range $\{0, 1\}^{\ell n}$ rather than $\{0, 1\}$,

for some ℓ with $\gamma + \ell < \frac{1}{2}$ (respectively $< \frac{1-\eta}{4} - \frac{h(\phi)}{2}$ for BB84-QOT).

Although other flavors of OTs can be constructed from ROT using standard reductions, a more direct approach would give a better ratio between storage-bound and communication-complexity. More general security definitions allowing for better composition (such as universal composability) briefly discussed at the end of Section 3.1 also deserve to be studied. Recent extensions have shown that a 1-2 OT protocol built along the lines of BB84-QOT is secure against adversaries with bounded quantum memory [9]. Interestingly, the techniques used are quite different from the ones of this paper (which appear to fail in case of 1-2 OT), and they additionally allow to analyse and prove secure the bit commitment scheme COMM with respect to the stronger security definition, as discussed in Section 4.2.

COMM can easily be transformed into a *string* commitment scheme simply by committing bitwise, at the cost of a corresponding blow-up of the communication complexity. In order to prove this string commitment secure, though, it is necessary that COMM is secure with respect to the stronger security definition.

How to construct and in particular prove secure a more efficient string commitment scheme is still an open problem. Furthermore, it is still unsolved how to construct and prove secure a 1- m OT protocol, more efficient than via the general reduction.

Finally, finding protocols secure against adversaries in more general noisy-memory models, quickly discussed in Section 5, would certainly be a natural extension of this work to more practical settings.

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