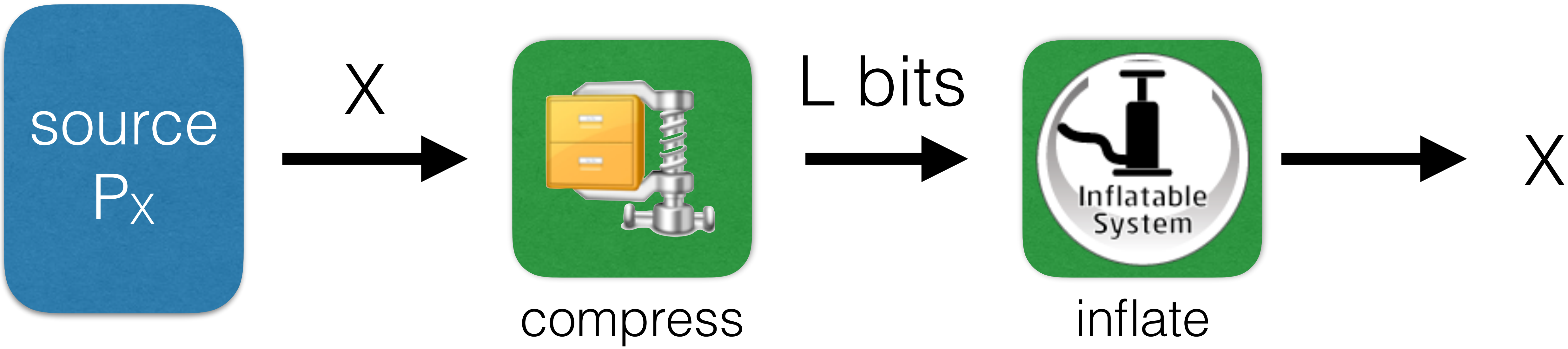
Data Compression / Source Coding



How much "information" is contained in X?

- compress it into minimal number of L bits per source symbol
- decompress reliably

⇒ average information content is L bits per symbol

Shannon's source-coding theorem: L ≈ H(X)

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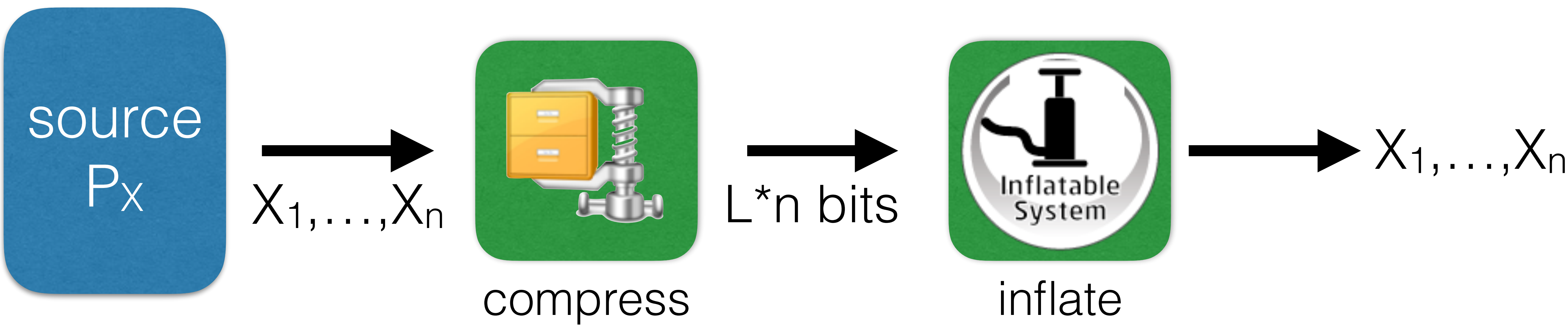

How much "information" is contained in X?

- compress it into minimal number of L bits per source symbol
- decompress reliably

$\Rightarrow$ average information content is L bits per symbol

Shannon's source-coding theorem: $L \approx H(X)$

Examples of Symbol Codes

x	P_X	C(x)	D(x)	E(x)
a	1/2	0	0	0
b	1/4	10	010	01
c	1/8	110	01	011
d	1/8	111	10	111

expected code word length:

$$\ell_C(X) = \mathbb{E}[\ell(C(X))] = \sum_x P_X(x)\ell(C(x))$$

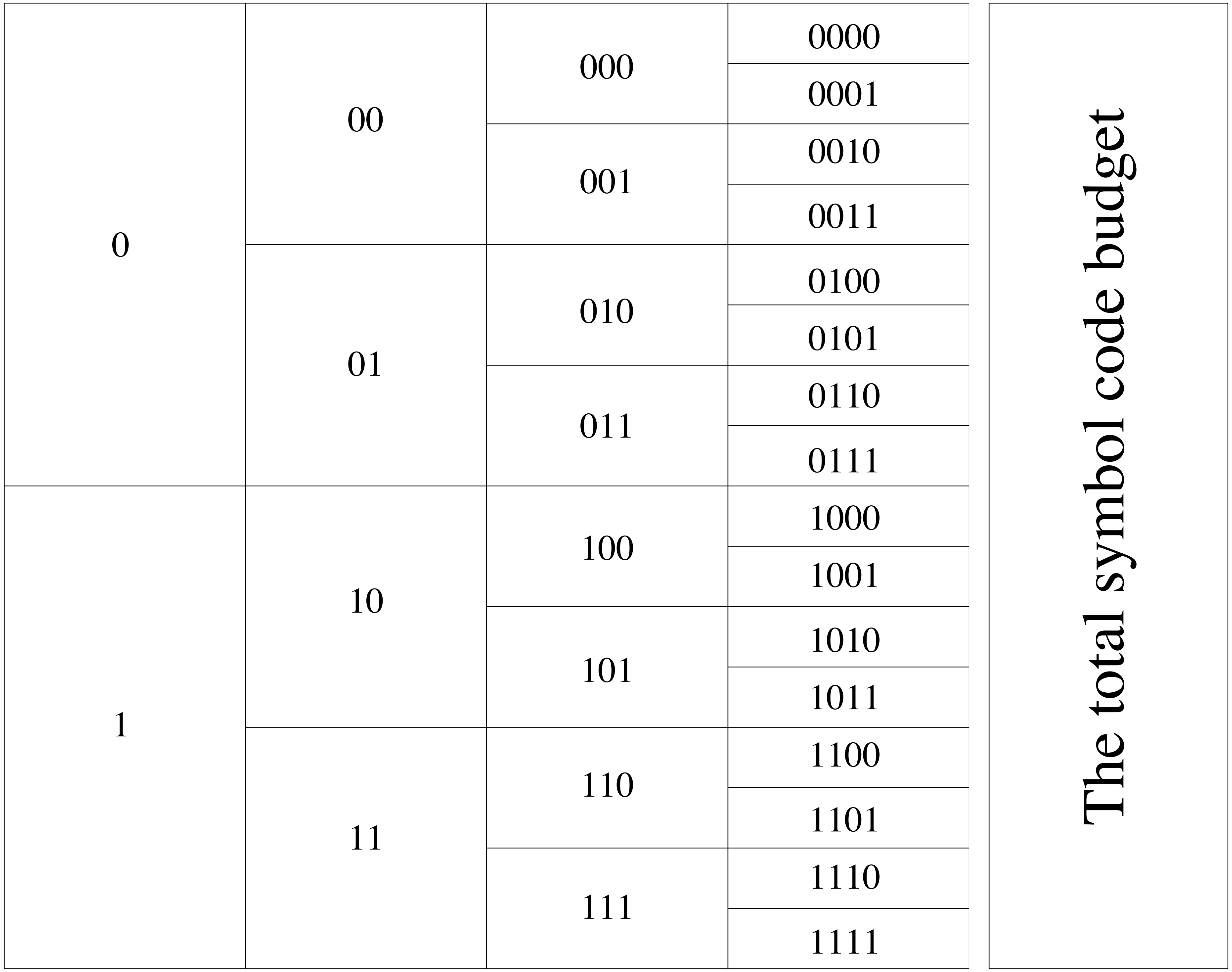

Figure 5.1. The symbol coding budget. The ‘cost’ 2^{-l} of each codeword (with length l) is indicated by the size of the box it is written in. The total budget available when making a uniquely decodeable code is 1.

You can think of this diagram as showing a *codeword supermarket*, with the codewords arranged in aisles by their length, and the cost of each codeword indicated by the size of its box on the shelf. If the cost of the codewords that you take exceeds the budget then your code will not be uniquely decodeable.

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a_i	p_i	$\log_2 \frac{1}{p_i}$	l_i	$c(a_i)$
a	0.0575	4.1	4	0000
b	0.0128	6.3	6	001000
c	0.0263	5.2	5	00101
d	0.0285	5.1	5	10000
e	0.0913	3.5	4	1100
f	0.0173	5.9	6	111000
g	0.0133	6.2	6	001001
h	0.0313	5.0	5	10001
i	0.0599	4.1	4	1001
j	0.0006	10.7	10	1101000000
k	0.0084	6.9	7	1010000
l	0.0335	4.9	5	11101
m	0.0235	5.4	6	110101
n	0.0596	4.1	4	0001
o	0.0689	3.9	4	1011
p	0.0192	5.7	6	111001
q	0.0008	10.3	9	110100001
r	0.0508	4.3	5	11011
s	0.0567	4.1	4	0011
t	0.0706	3.8	4	1111
u	0.0334	4.9	5	10101
v	0.0069	7.2	8	11010001
w	0.0119	6.4	7	1101001
x	0.0073	7.1	7	1010001
y	0.0164	5.9	6	101001
z	0.0007	10.4	10	1101000001
–	0.1928	2.4	2	01

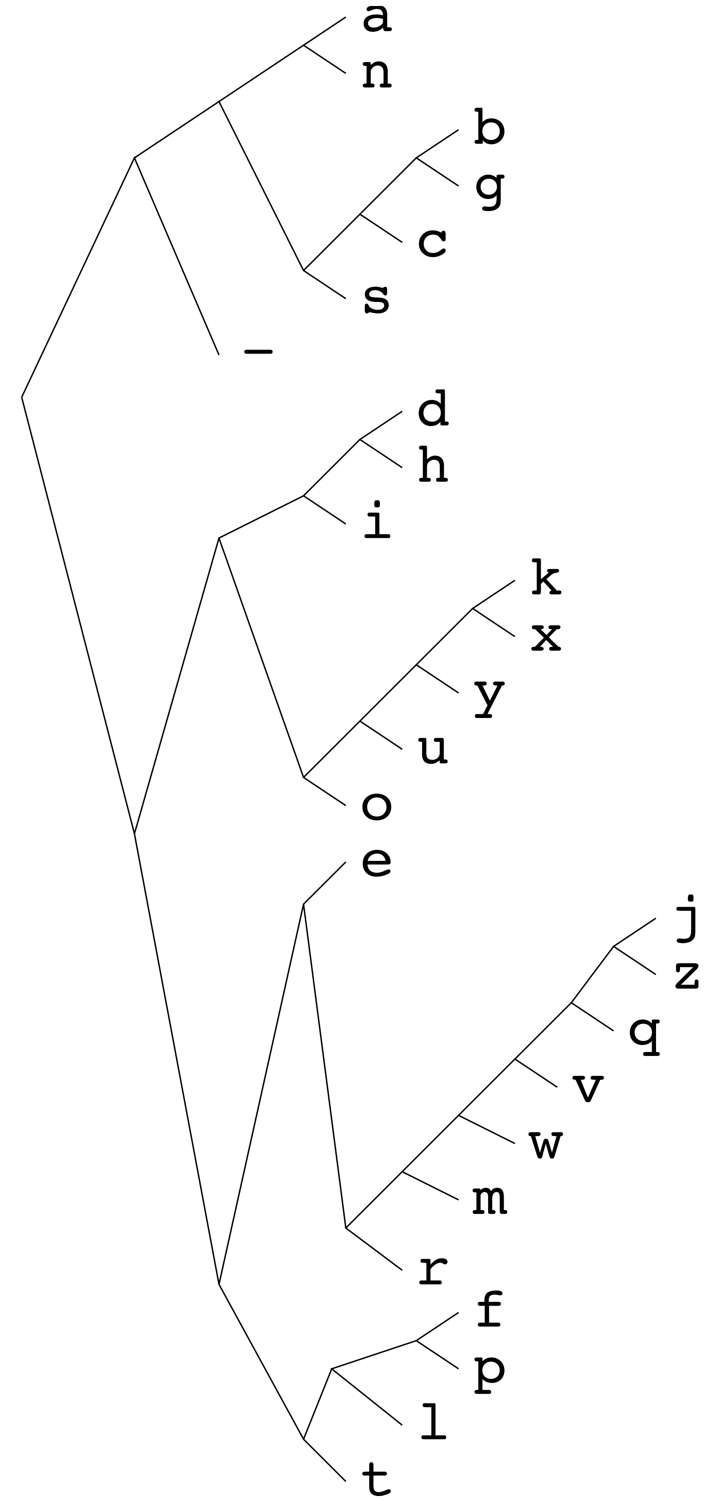

Figure 5.6. Huffman code for the English language ensemble (monogram statistics).

ure 5.6. This code has an expected length of 4.15 bits; the entropy of the ensemble is 4.11 bits. Observe the disparities between the assigned codelengths and the ideal codelengths $\log_2 1/p_i$.

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