Discrete Channels

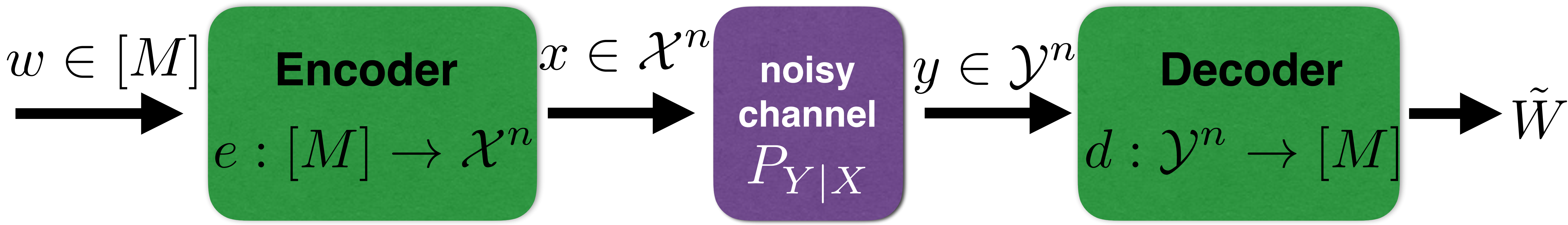
Def: A **discrete channel** is denoted by $(\mathcal{X}, P_{Y|X}, \mathcal{Y})$ where $\mathcal{X}$ is a finite input set, $\mathcal{Y}$ is a finite output set and $P_{Y|X}$ is a conditional probability distribution, d.h.

$$\forall x \in \mathcal{X} \quad \forall y \in \mathcal{Y} : P_{Y|X}(y|x) \geq 0$$

$$\forall x \in \mathcal{X} : \sum_y P_{Y|X}(y|x) = 1$$

$P_{Y|X}(y|x)$ = the probability that the channel outputs y when given x as input

Noisy-Channel Coding

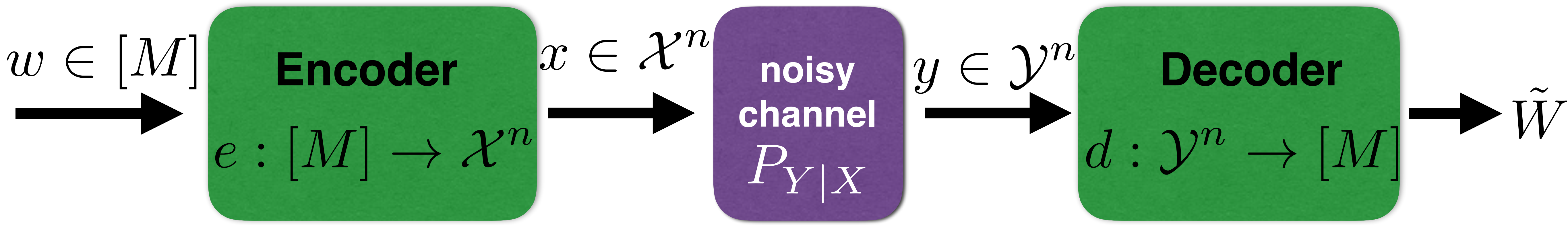


Def: A **(M,n)-code** for the channel $(\mathcal{X}, P_{Y|X}, \mathcal{Y})$ consists of

1. message set: $[M] = \{1, 2, \ldots, M\}$
2. encoding function: $e : [M] \to \mathcal{X}^n$
   codebook: $\{e(1), e(2), \ldots, e(M)\}$
3. deterministic decoding function assigning a guess to each possible received vector $d : \mathcal{Y}^n \to [M]$

The **rate** of a (M,n)-code denotes the transmitted bits per channel use $R := \frac{\log M}{n}$

Rate and Error



The **rate** of a (M,n)-code denotes the transmitted bits per channel use

$$R := \frac{\log M}{n}$$

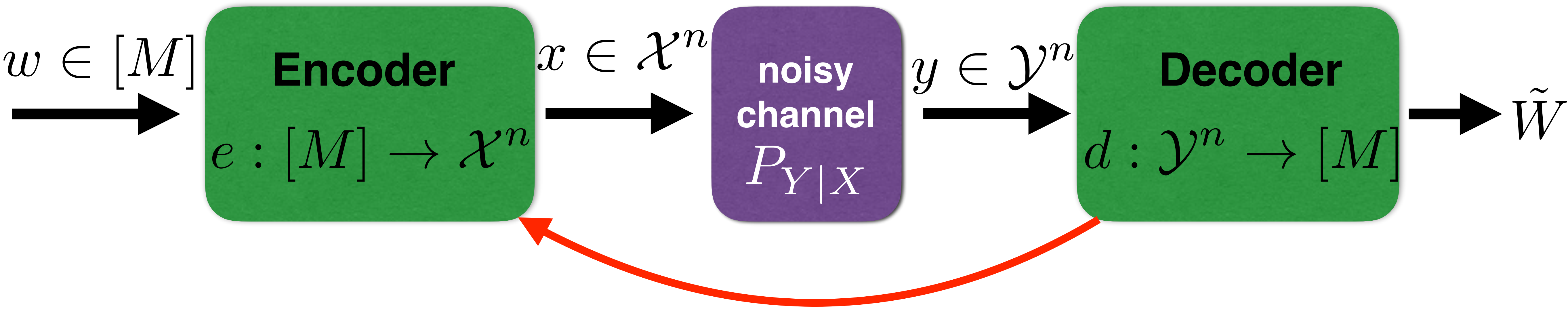
probability of error when sending $w \in [M]$

$$\lambda_w := \Pr[\tilde{W} = d(Y^n) \neq w \mid X^n = e(w)]$$

maximal probability of error: $\lambda^{(n)} := \max_{w \in [M]} \lambda_w$

average probability of error: $p_e^{(n)} := \frac{1}{M} \sum_{w=1}^{M} \lambda_w$

Encoding with Feedback



encoding **with feedback:**
X_{i+1} can depend on w, $X_1,\ldots,X_i$ and $Y_1,\ldots,Y_i$

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