Zero-Knowledge Proofs

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Introduction

- Zero-knowledge proofs are proofs that yield nothing beyond the validity of the assertion

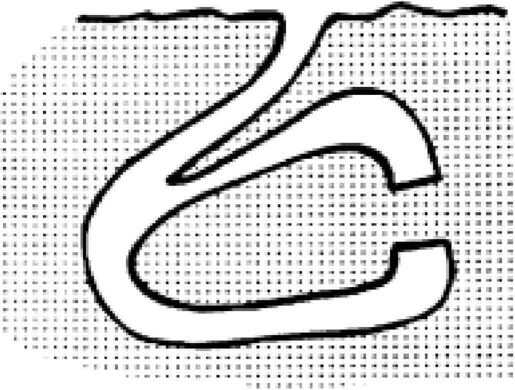

Figure: The special cave [4]

Interactive proofs

- Zero-knowledge proofs are a special case of interactive proofs
- Interactive proofs have two parties: the Prover (P) and the Verifier (V)
- Verifier is a PPT machine, Prover is unbounded and both are able to communicate
- The prover claims a certain statement is true
- If (P,V) accept this statement (completeness) and rejects false statements (soundness), then it is an interactive proof system

Formal Definition Zero-Knowledge Proofs

Fix an interactive machine (the Prover) look at what can be computed by an arbitrary adversary (the Verifier). Now an interactive proof A is zero-knowledge on the set S, if for every feasible strategy B^*, there exists a feasible computation C^*, s.t. the following two probability ensembles are computationally indistinguishable:

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The first ensemble is the execution of an interactive protocol, the second represents a stand-alone procedure. This means that anything that could be extracted from A, was also already in C. So nothing was gained from the interaction. [2]

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- Whats could go wrong in this scheme?

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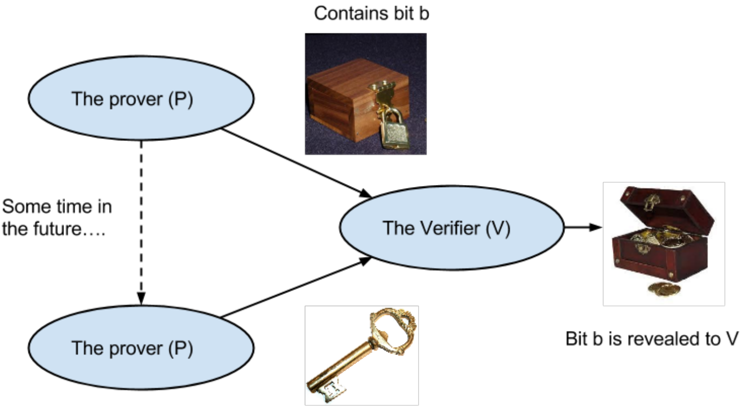
- Whenever P decides to he passes the key to V to open the padlock. In this way P is bound to his original choice and his choice is hidden until he decided to give the key [1]

Commitment Scheme visualized



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- Now the verifier accepts the identity of the prover iff the commitment can be correctly opened and M' = M.

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- It has been shown that using zero-knowledge protocols as sub-protocols it is possible to transform any protocol that assumes players follow the rules into one that is secure even if players deviate from the protocol [3]

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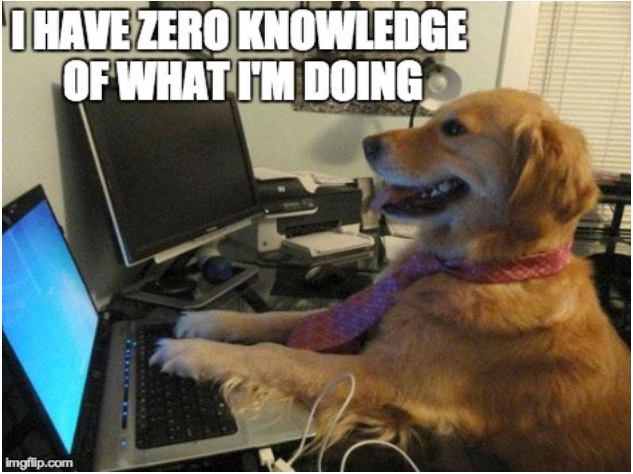
- This works on smartcards (OV-Chipkaart) [5], where computation is very limited



Thank you for your attention!

Time left for a game?
Sudoku Zero-Knowledge

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