Filippos Vogiatzian

Quantum Key Distribution

Quantum Cryptography

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Quantum Cryptography

Post-quantum Cryptography

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- Perfect Security
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Post-quantum Cryptography

- Quantum Computational Security
- Relies on primitives that are equally hard for classical and quantum computers to solve
  - Lattice-Based Cryptography

QKD 2PC and more

- Quantum Key Distribution (QKD): Two parties (Alice and Bob) communicate with perfect secrecy in the presence of an eavesdropper (Eve) [1,3,4]
- Two-Party Cooperation (2PC): Two parties that don’t trust each other cooperate in a secure way
- How to encrypt or authenticate a quantum state
- Implementations

$$\langle \phi_k | \phi_{k'} \rangle = \langle \phi_k | \int_{-L/2}^{L/2} dx\, |x\rangle\langle x| \phi_{k'} \rangle$$

$$\Rightarrow \left(\frac{2\pi}{L} n + k_0\right)\frac{L}{2} = \frac{\pi}{2}(2l-1),\ l = 1, 2, \ldots \Rightarrow k_0 = -\frac{\pi}{L}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\langle \phi_k | \phi_{k'} \rangle = \int_{-L/2}^{+L/2} dx\, \phi_k^*(x) \cdot \phi_{k'}(x)$$

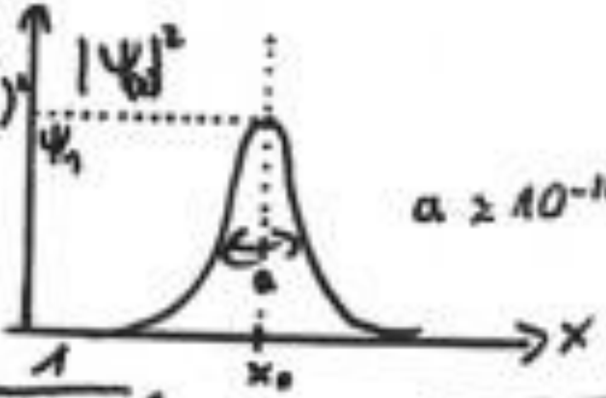
$$\psi_n(x) = \sqrt{\frac{2}{L}} \cos\left[\frac{\pi}{L}(2n-1)x\right];\ \varphi_a - \varphi_b = \pi;\ \psi_n(x) = \sqrt{\frac{2}{L}} \sin\left[\frac{2\pi}{L} n x\right]$$

$$\langle \phi_k | \phi_{k'} \rangle = \frac{1}{L}\int_{-L/2}^{+L/2} dx\, e^{-ikx} e^{ik'x} \stackrel{!}{=} 0;\ k \neq k'$$

$$\hat{H}\psi_{ns}(x) = -\frac{\hbar^2}{2m}\partial_x^2 \psi_{ns}(x) = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}[2n-1]\right)^2 \psi_{ns}(x)$$

$$E_{ns} = \frac{\hbar^2}{2m}\frac{\pi^2}{L^2}(2n-1)^2,\ n = 1, 2, \ldots;\ \hat{H}\psi_{na}(x) = \frac{\hbar^2}{2m}\left(\frac{2\pi}{L}\right.$$

$$|\psi(x)|^2 = |\psi_0|^2 e^{-\frac{(x-x_0)^2}{2a^2}}$$



$$\int_{-\infty}^{+\infty} dx\, e^{-Ax^2} = \sqrt{\frac{\pi}{A}}$$

$$a \approx 10^{-10}\,m$$

$$\hat{H}\psi_a = -\frac{\hbar^2}{2m}\partial_x^2\psi_a(x) = \frac{\hbar^2}{2m}\frac{1}{2a^2}\psi_a(x) - \frac{\hbar^2}{2m}\frac{1}{4a^4}(x-x_0)^2\psi_a(x)$$

$$= -\frac{\hbar^2}{2m}\left(-\frac{1}{2a^2} + \left(\frac{1}{2a^2}(x-x_0)\right)^2\right) e^{-\frac{(x-x_0)^2}{4a^2}}\psi;\ V(x) = \frac{\hbar^2}{2m}\frac{1}{4a^4}(x-x_0)^2$$

$$A = \frac{1}{2a^2} \Rightarrow |\psi_0| = \frac{1}{(2\pi a^2)^{1/4}}$$

$$\hat{H} \to \hat{H} = -\frac{\hbar^2}{2m}\partial_x^2 + V(x);\ \hat{H}\psi_a = \frac{\hbar^2}{2m}\frac{1}{2a^2}\psi_a = E_a\psi_a$$

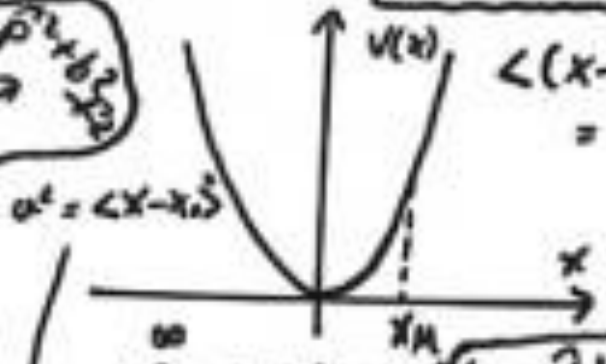
$$V(x) = \frac{1}{2}m\omega^2(x-x_0)^2 \to m\omega^2 = \frac{\hbar^2}{m4a^4} \Rightarrow \boxed{\omega = \frac{\hbar}{2ma}}$$

$$E_a = \frac{\hbar^2}{2m}\frac{1}{2a}$$

$$[\hat{p}, \hat{x}] = \frac{\hbar}{i};\ \hat{p} = \frac{\hbar}{i}\partial_x\ /\ \hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$$

$$a^2 + b^2 = (a+ib)(a-ib);\ a, b \in \mathbb{R};\ 2\ (a\hat{p}\ ib\hat{x})(a\hat{p} - ib\hat{x}),\ a, b \in \mathbb{R}$$

$$= a^2\hat{p}^2 + iba\hat{x}\hat{p} - iab\hat{p}\hat{x} + b^2\hat{x}^2 = a^2\hat{p}^2 + b^2\hat{x}^2 - ba\hbar$$



$$\langle (x-x_0)^2 \rangle = \langle \psi_a | (x-x_0)^2 | \psi_a \rangle$$

$$\hat{H} = (a\hat{p} + ib\hat{x})(a\hat{p} - ib\hat{x}) + ba\hbar;\ a^2 = \frac{1}{2m};\ b^2 = \frac{1}{2}m\omega^2$$

$$\int_{-\infty}^{\infty} dx\, |x\rangle\langle x| = \mathbb{I}$$

$$\text{Def: } c^+ = \frac{1}{\sqrt{\hbar\omega}}(a\hat{p} + ib\hat{x});\ c^- = \frac{1}{\sqrt{\hbar\omega}}(a\hat{p} - ib\hat{x}) \Rightarrow \hat{H} = \hbar\omega c^+ c + \frac{1}{2}\hbar\omega$$

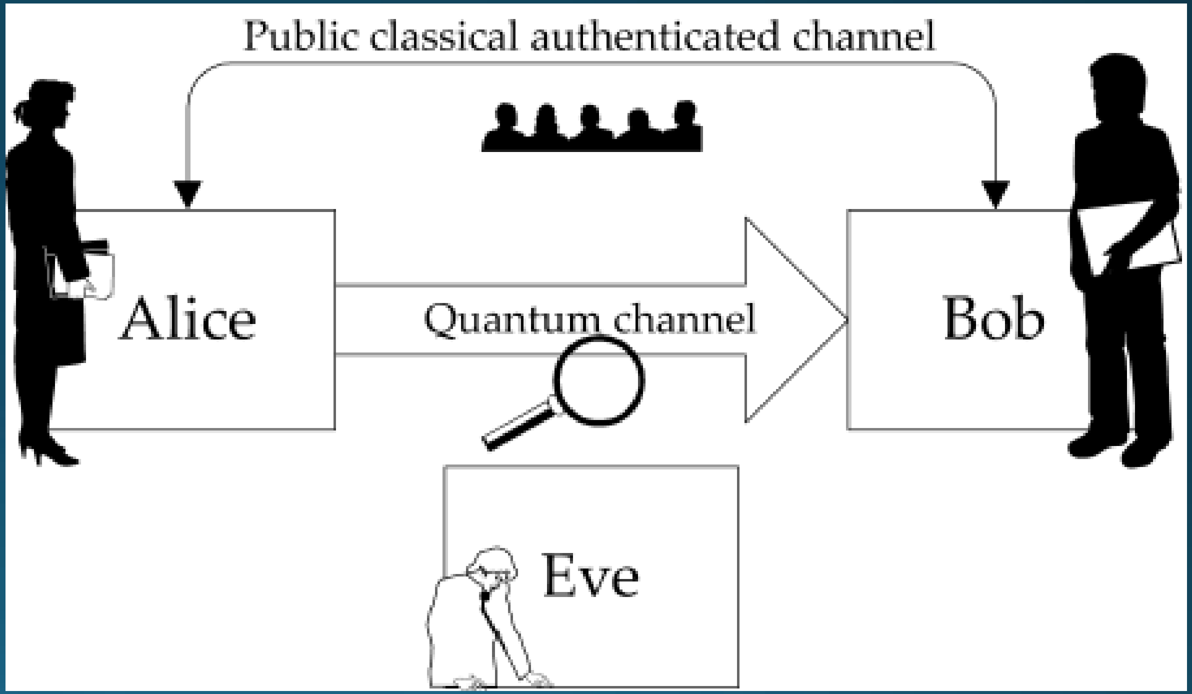
$$\psi_a^*(x)\langle \psi_a |$$

$$\hat{x}|x\rangle = x|x\rangle$$

$$= \int_{-\infty}^{\infty} dx\, \psi_n^*(x)(x-x_0)^2\psi_a(x)$$

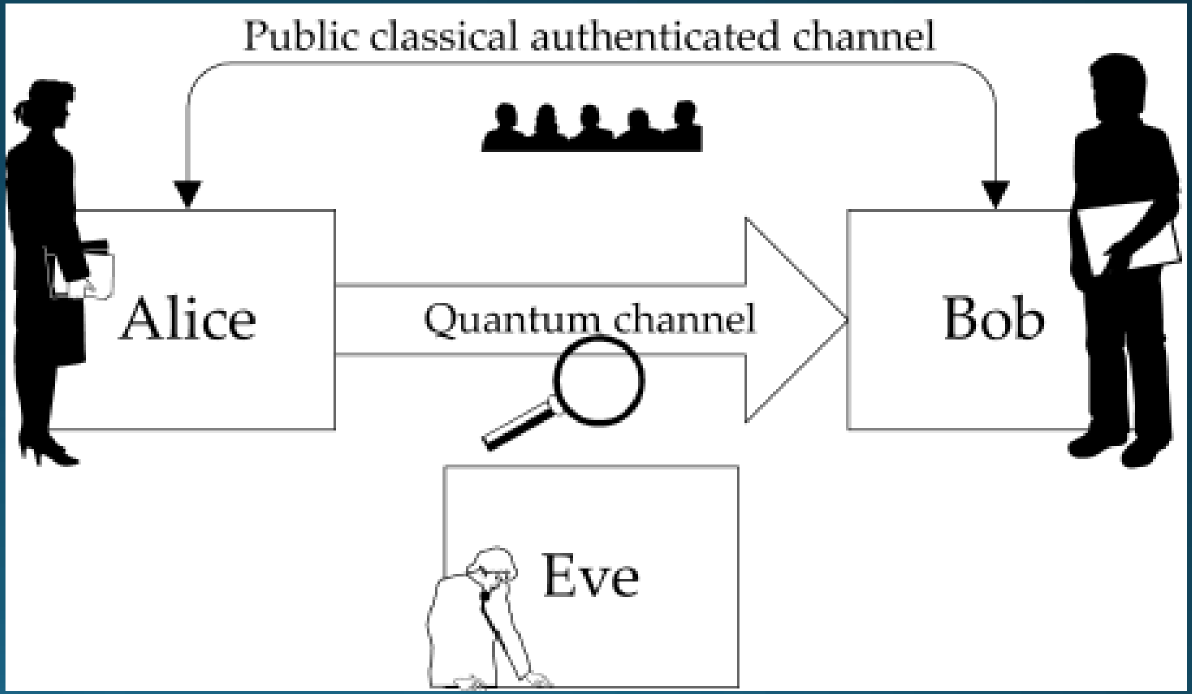
$$\left\{\left(\frac{w}{-\bar{t}}\ \frac{t}{\bar{w}}\right) \middle| w, t \in \mathbb{C}\right\}\ \{\pm 1\};\ SU(2) \cong S^3\quad A \mapsto wAw^{-1}$$

Quantum Channel



Quantum Channel

- Eve has complete control over the channel



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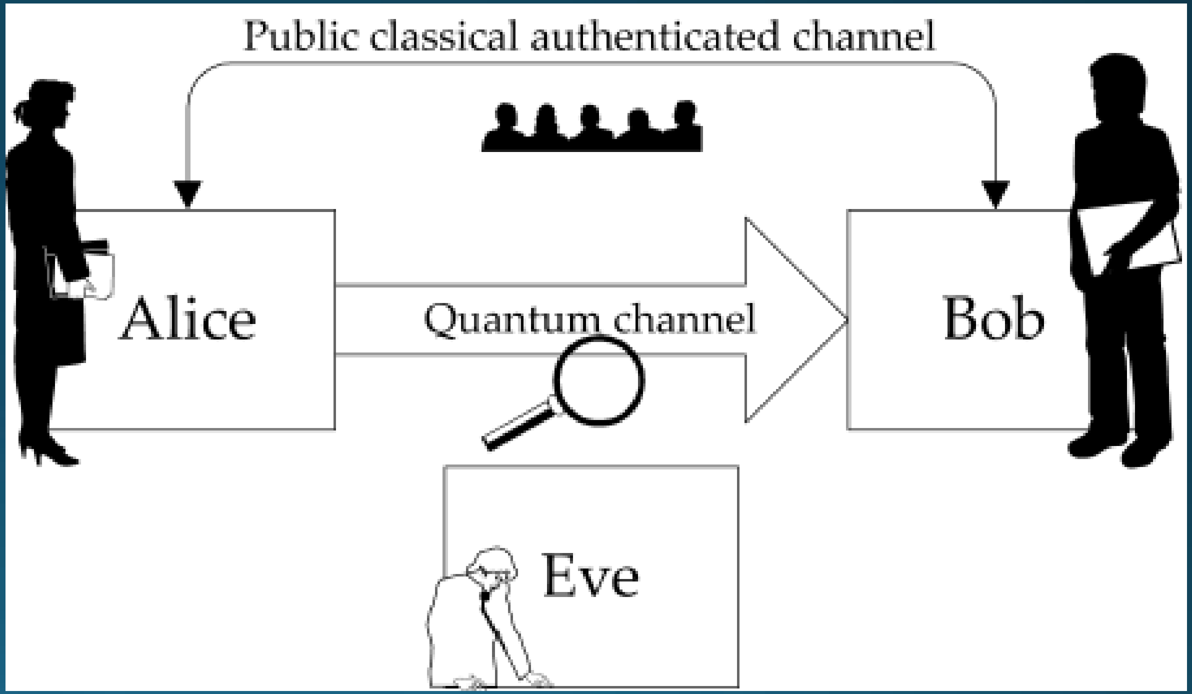

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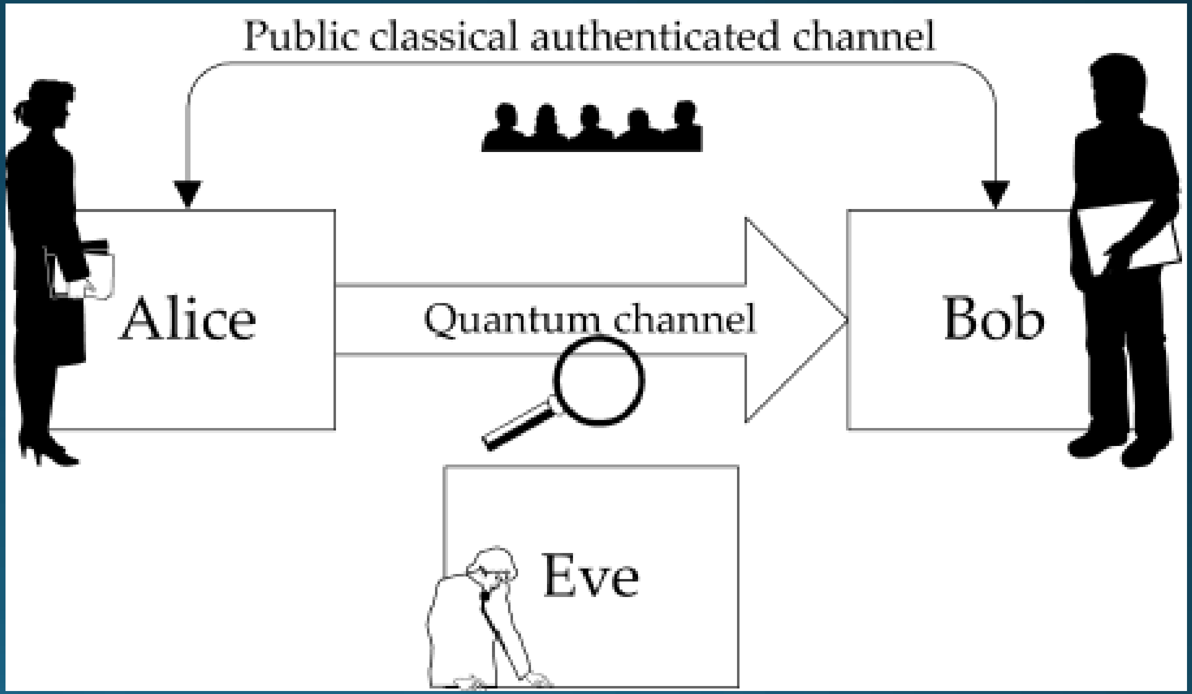
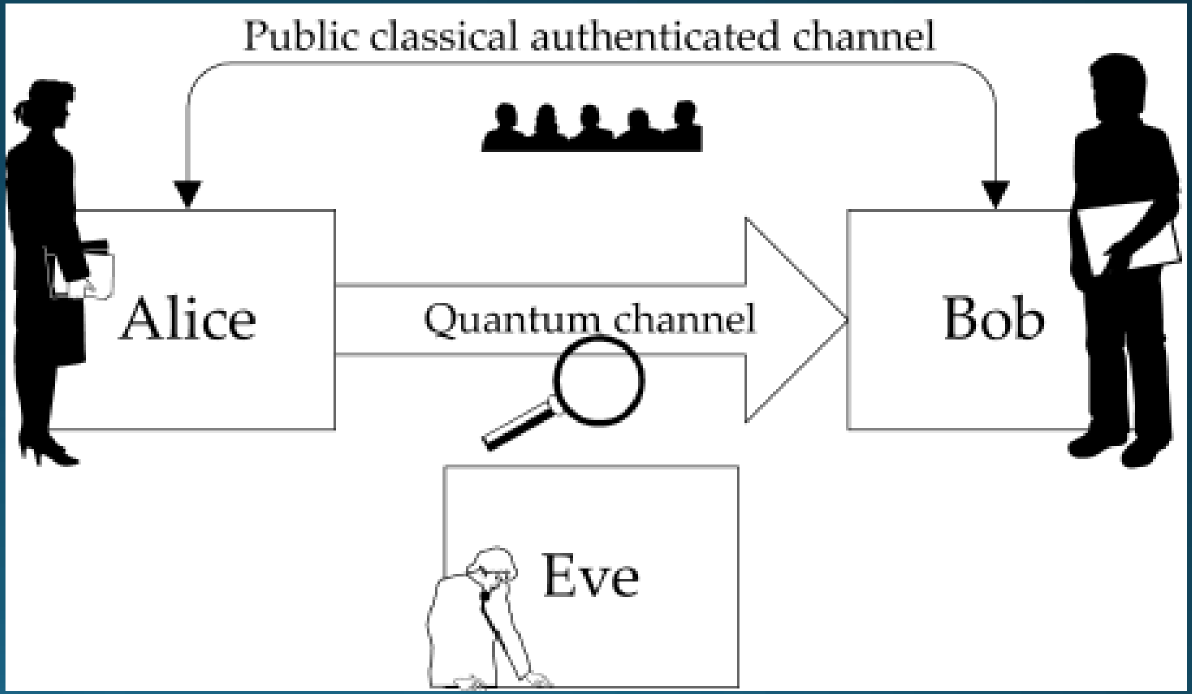


Quantum Channel

- Eve has complete control over the channel
- She can intercept or measure the sent qubits
- No cloning theorem: Forbidden to clone of an unknown quantum state (Wooters and Zurek and Dieks 1982)
- Eve can block the channel by sending random qubits and prevent communication over the channel



Quantum Key Distribution

- Alice and Bob use the public quantum channel to agree on a private secure key
- Eve has no information about the key
- Having a private key they can use any other classical encryption scheme to communicate through the public classical channel
- If they use OTP they can communicate with perfect secrecy!

Quantum Bits (qubits)

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- Two-state quantum-mechanical system

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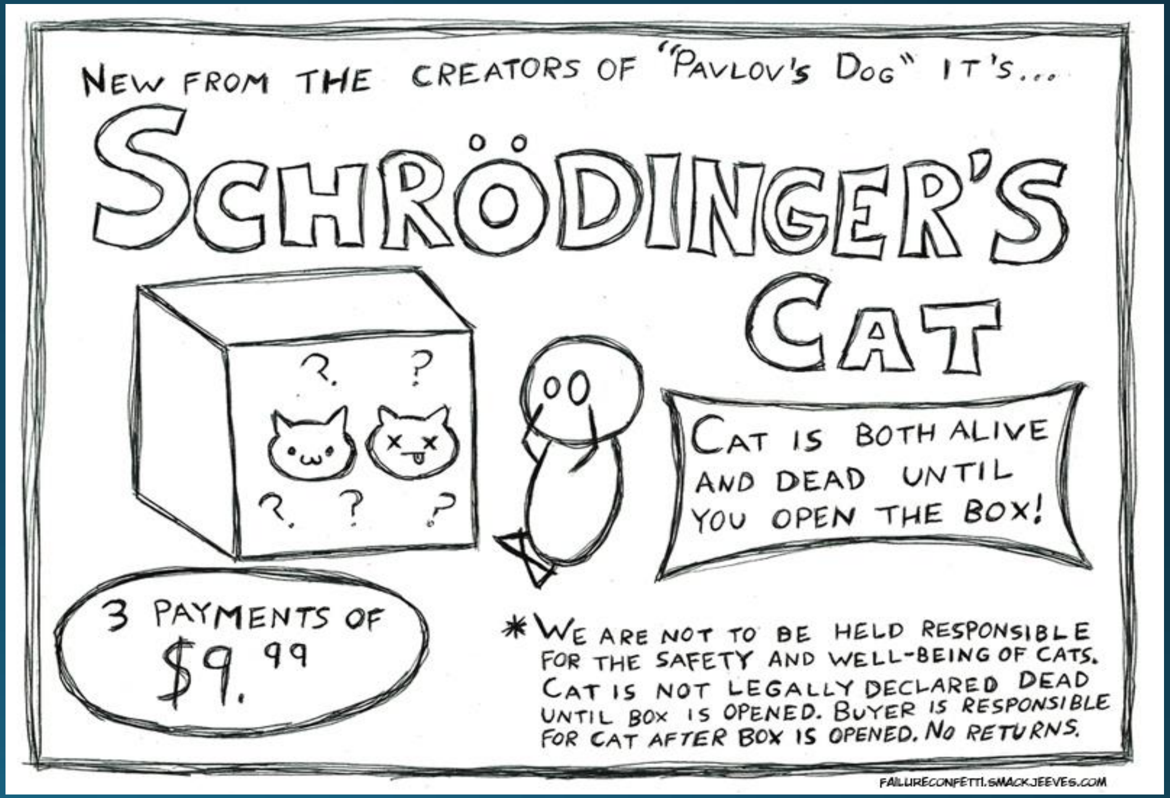
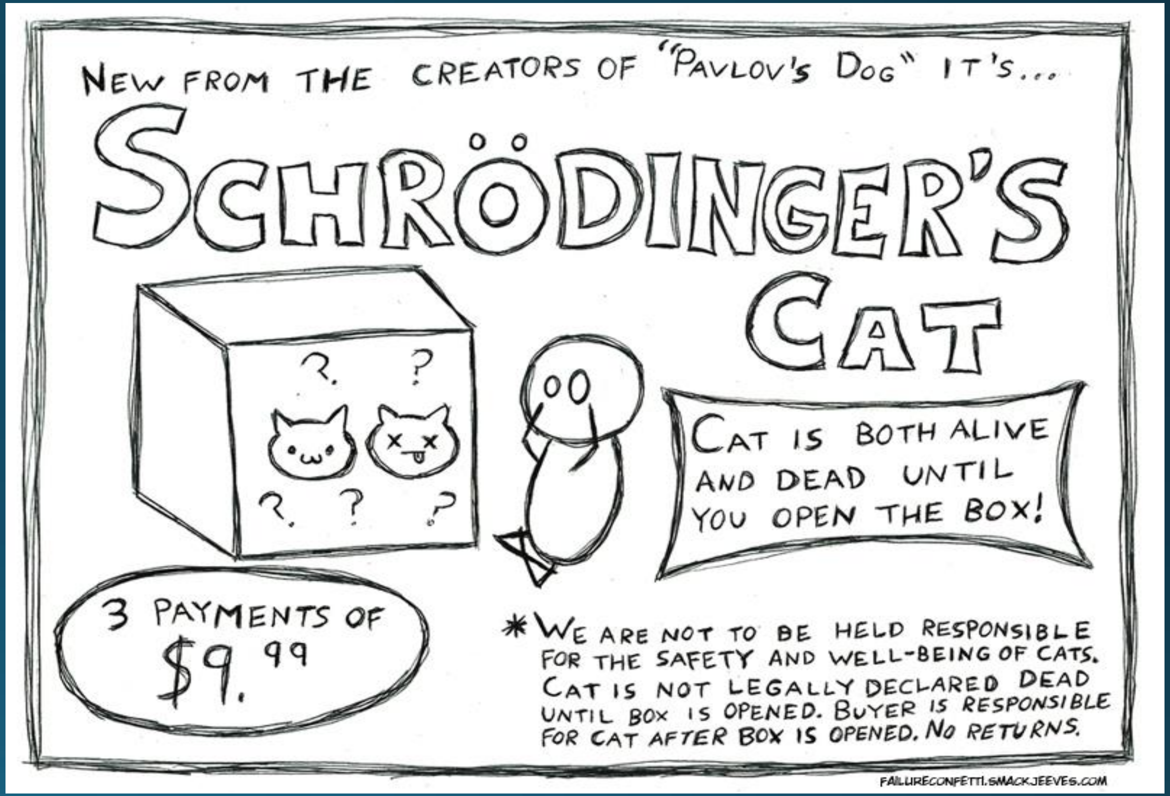


Quantum Bits (qubits)

- Two-state quantum-mechanical system
- Polarisation of photon
  - rectilinear / diagonal polarization



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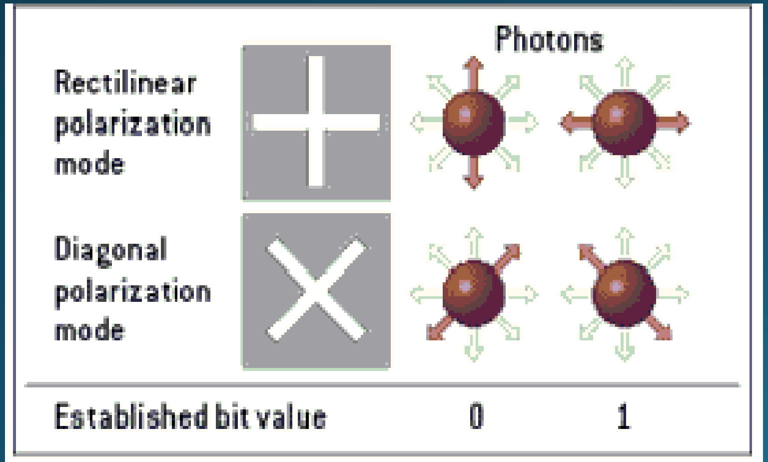
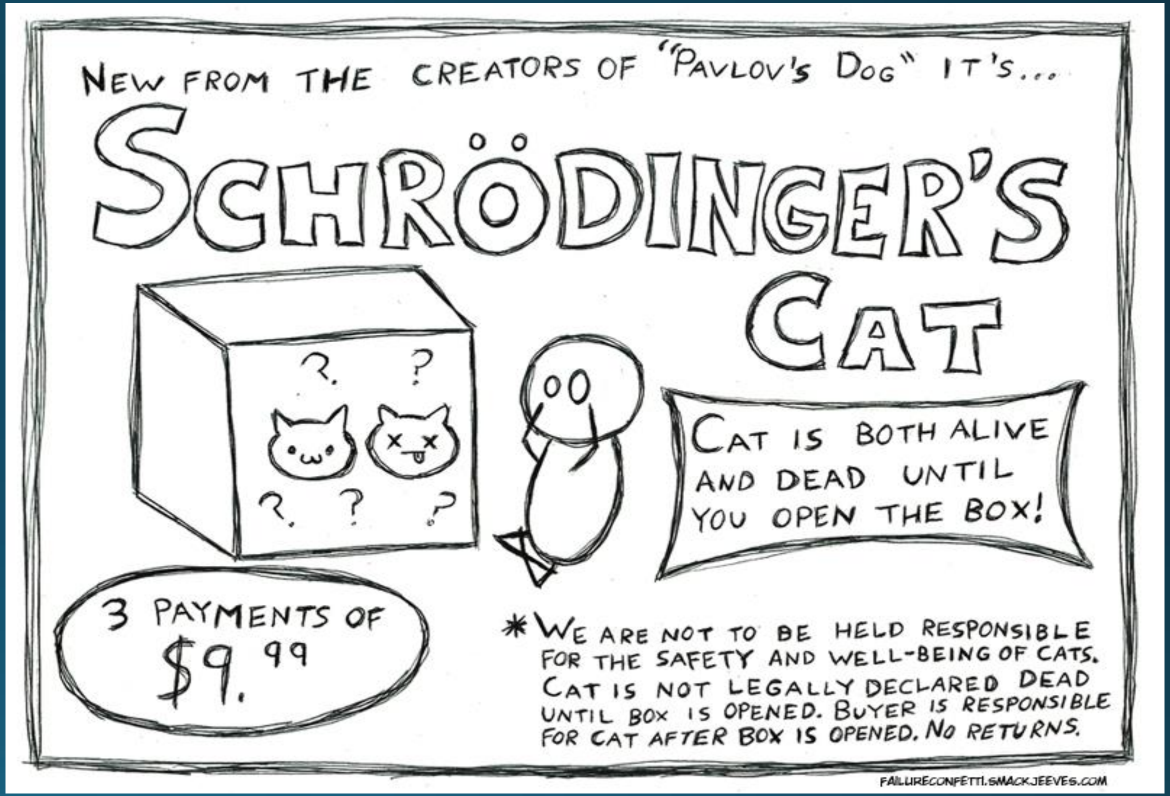
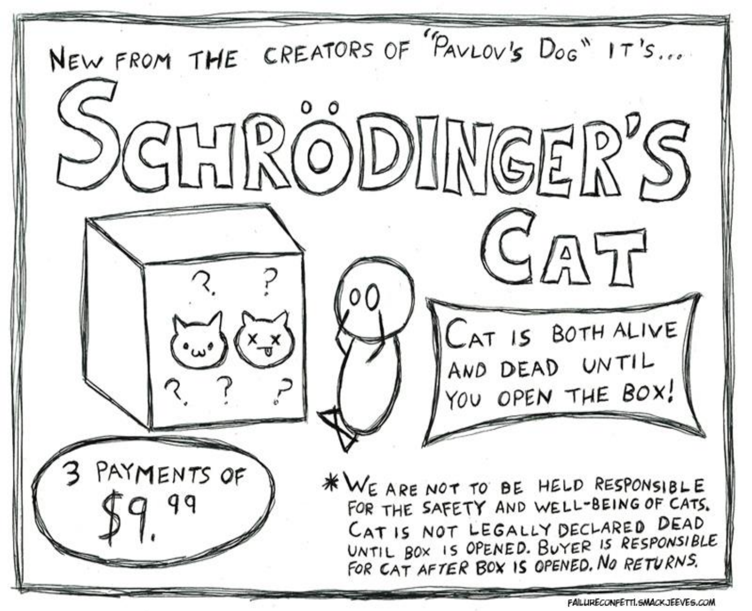


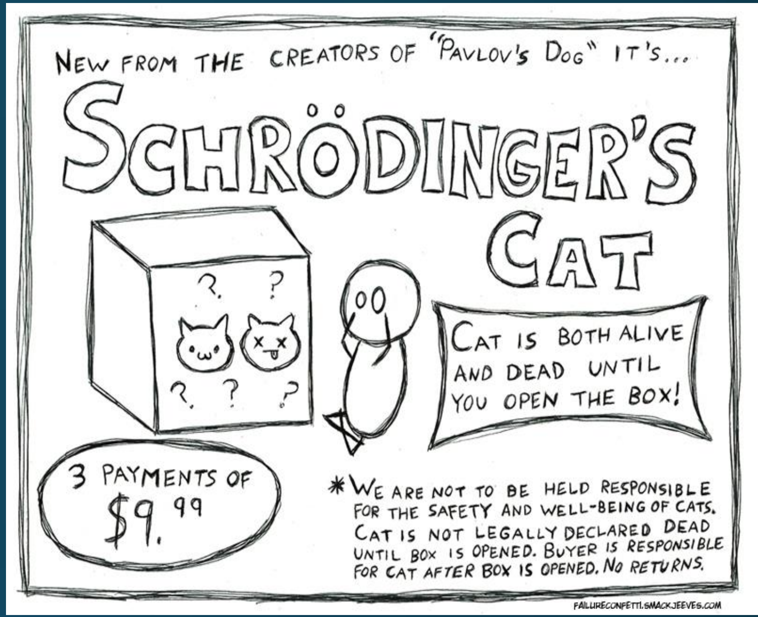


Measuring Qubits



Measuring Qubits

- Measuring a qubit:
  - opening the box



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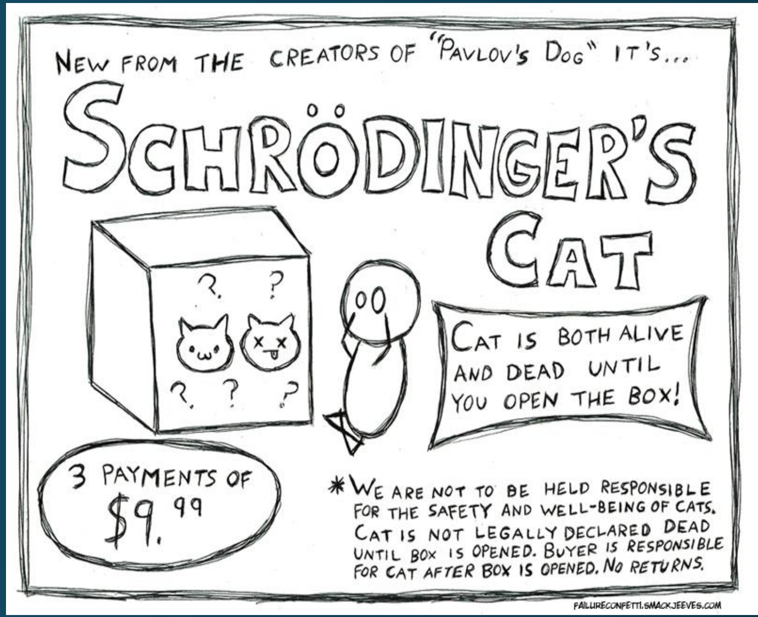

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  - The cat is DEAD or ALIVE

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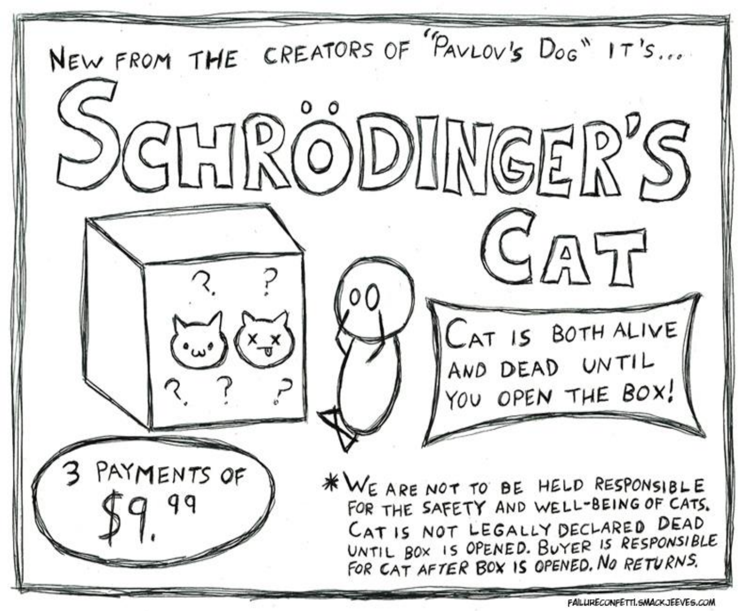

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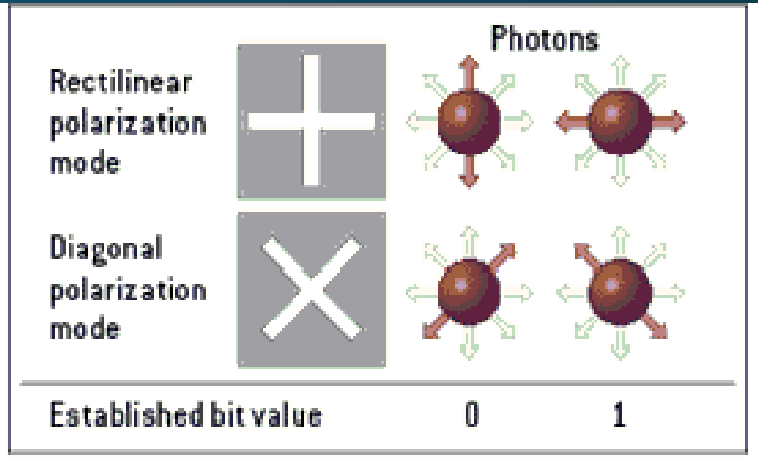

Measuring Qubits

- Measuring a qubit:
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  - filter the photon through one of the modes
- Collapses the wavefunction to one of the two states:
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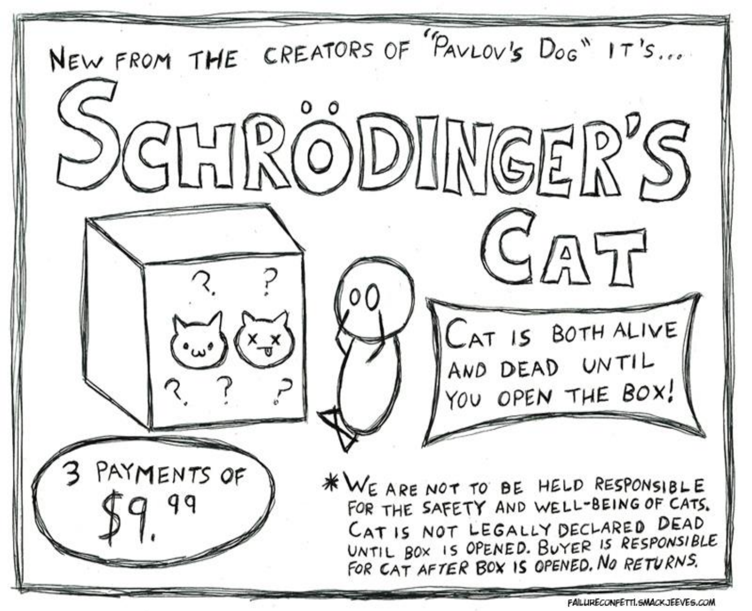

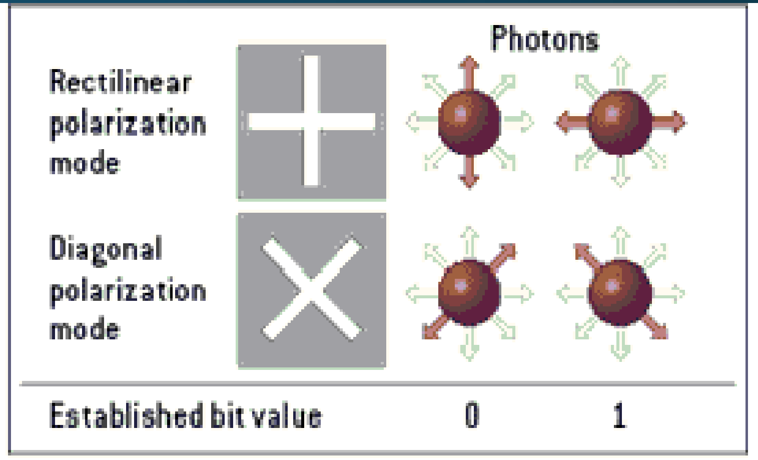

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- Measuring a qubit:
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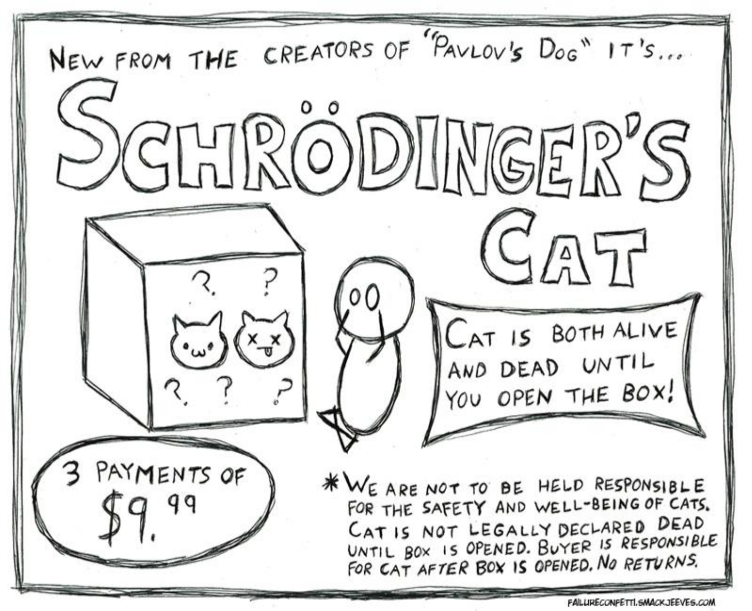
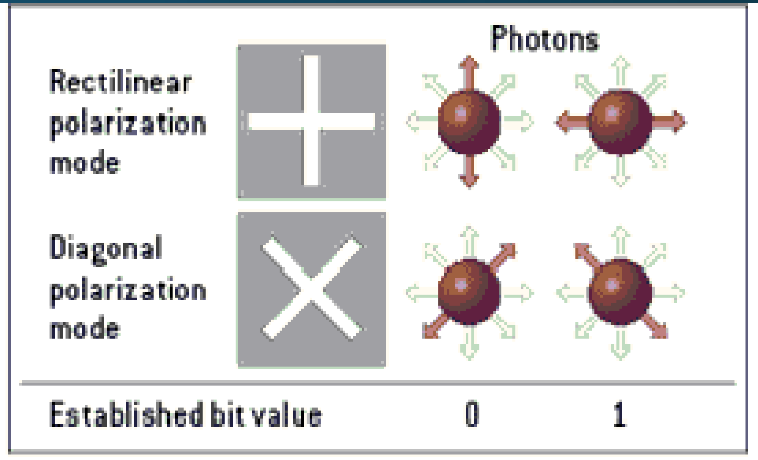
  - filter the photon through one of the modes
- Collapses the wavefunction to one of the two states:
  - The cat is DEAD or ALIVE
  - The polarization is VERTICAL or HORIZONTAL
  - The bit is 0 or 1





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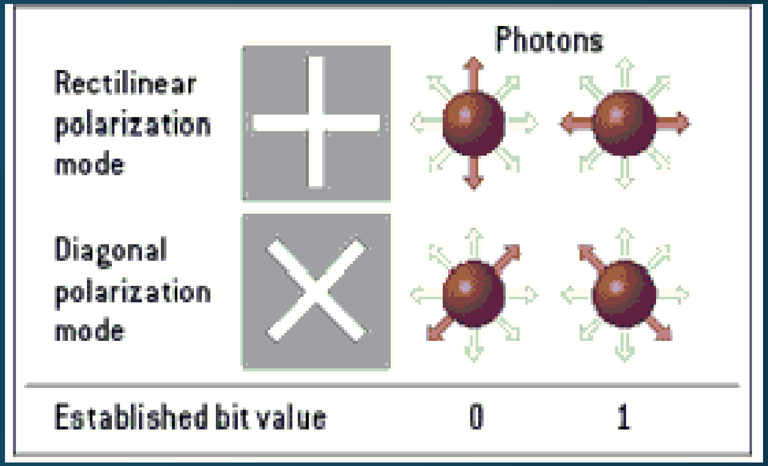

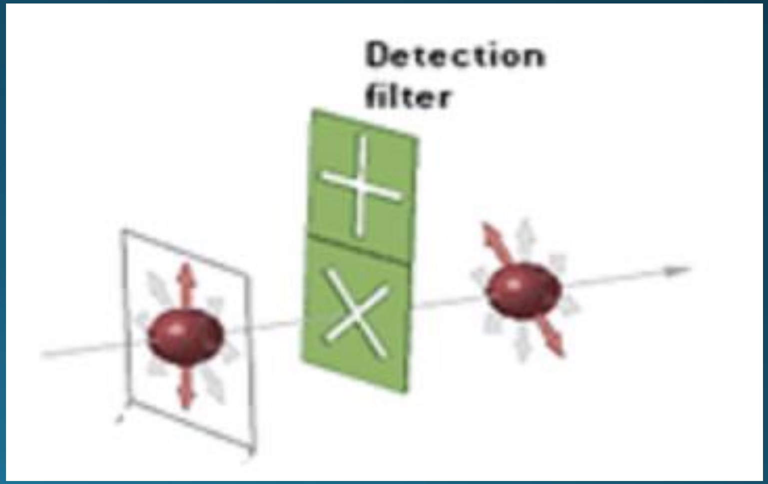

Measuring Qubits

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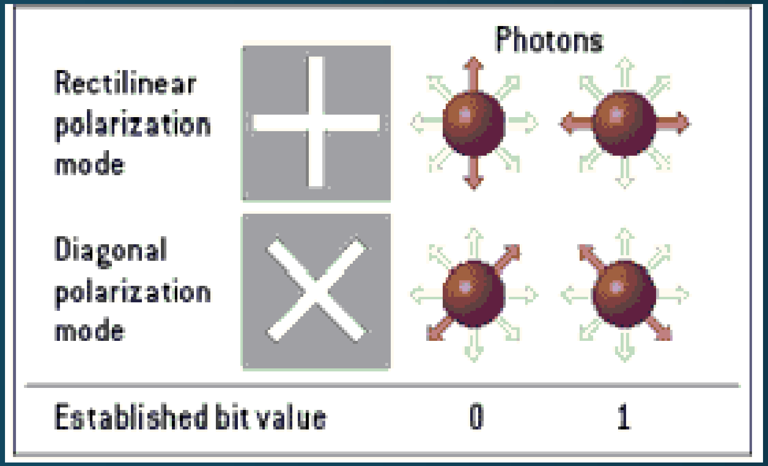

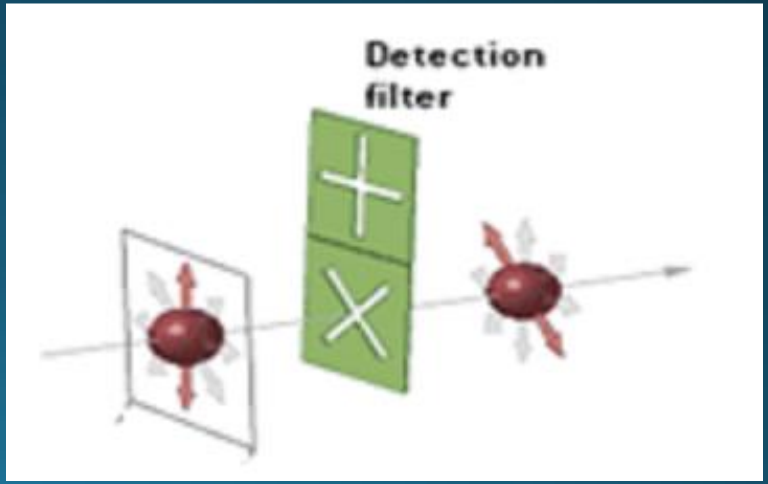

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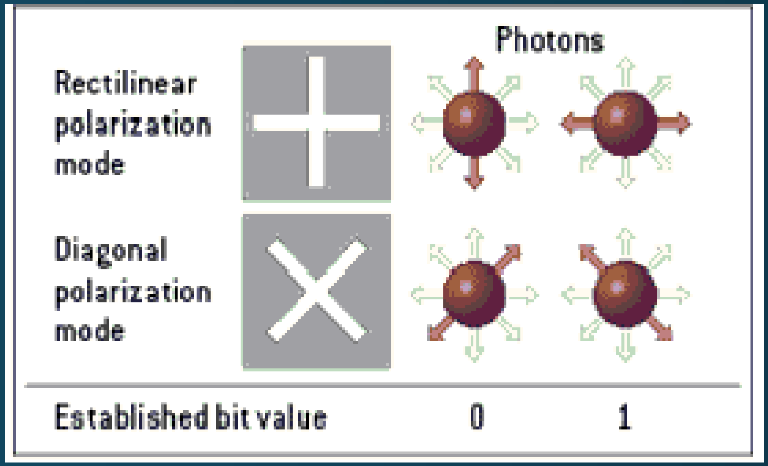
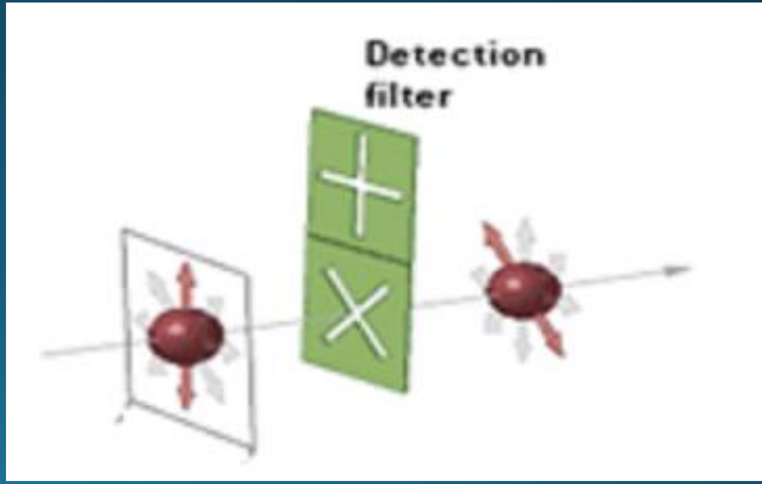
2. No information gain! (we get 0 or 1 with P=0.5)





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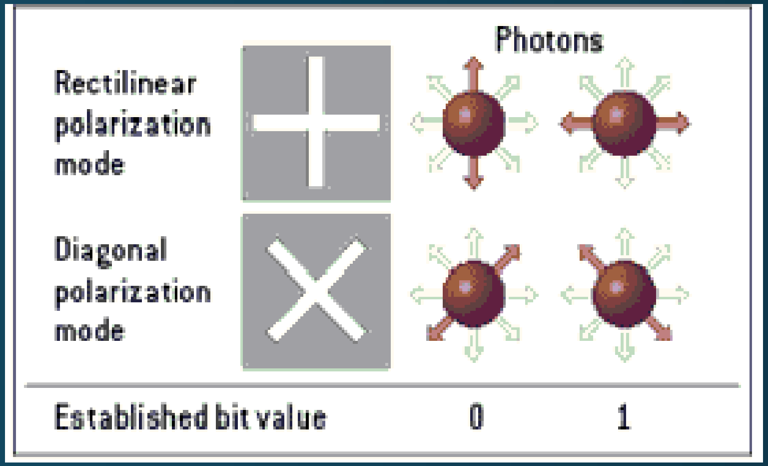
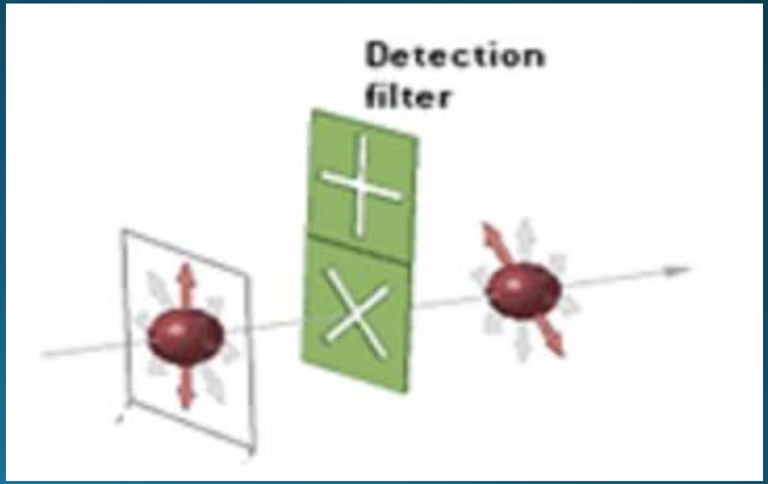
3. It changes the state to one of the states corresponding to the new basis

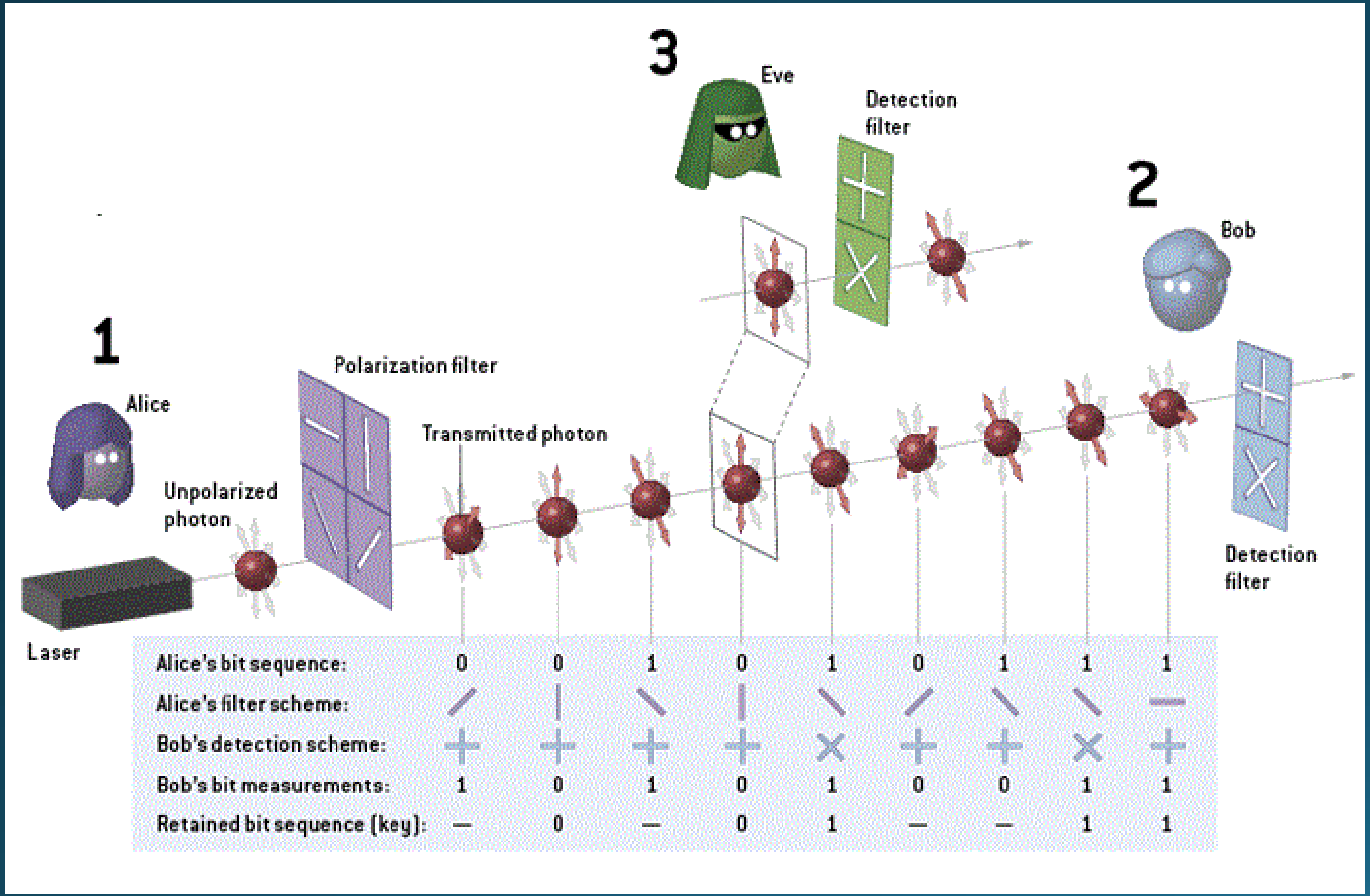




BB84 QKD Scheme

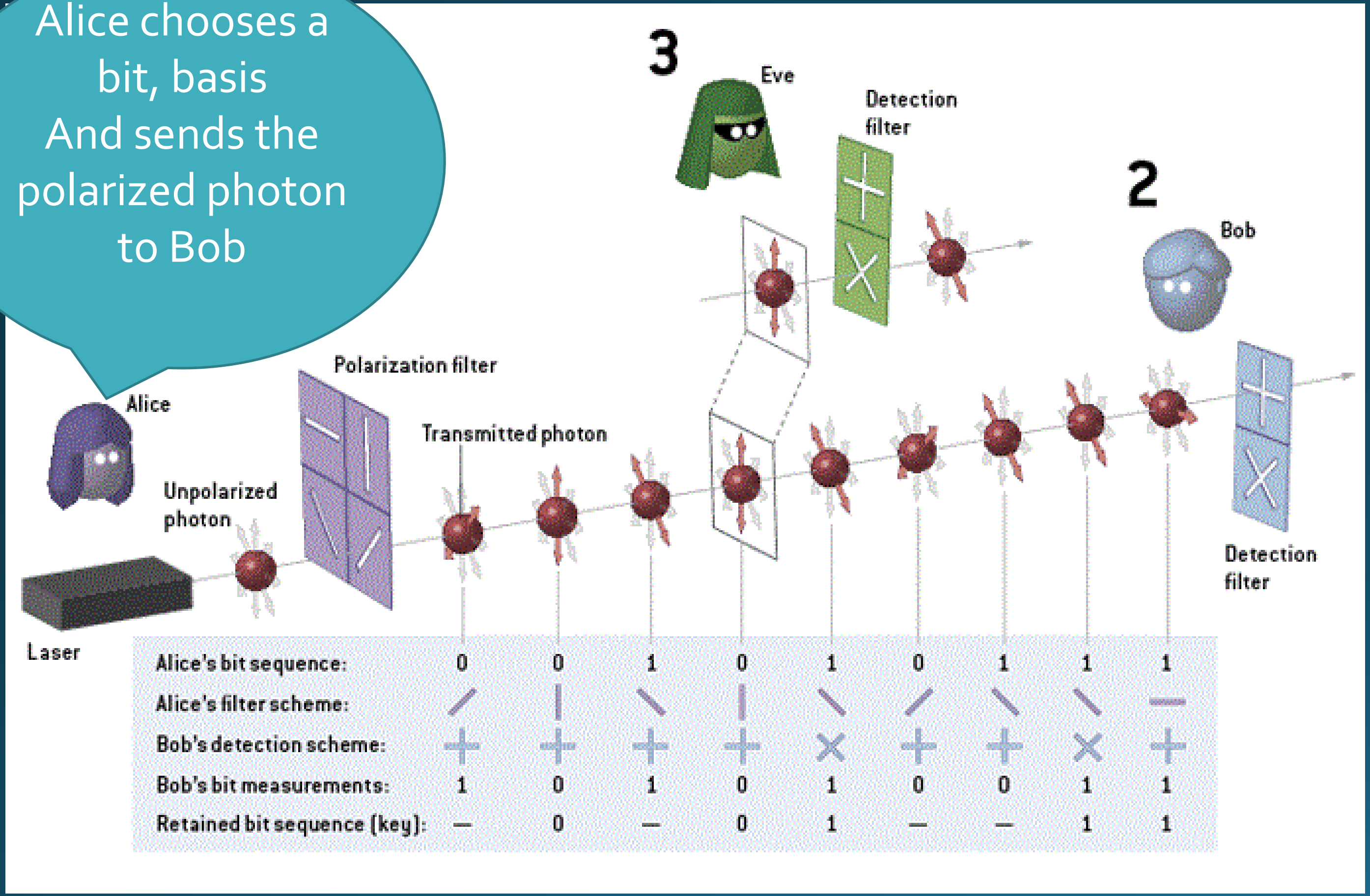
1. Alice chooses a random bit (0,1) and basis (+ or X)
2. She sends the qubit to Bob with the appropriate polarization
3. Bob measures the qubit with a random basis
4. Alice and Bob compare the string of bases they used and only keep those bits where they used the same basis
5. Error estimation and correction
6. Privacy amplification

BB84 QKD in Action



BB84 QKD in Action

Alice chooses a bit, basis
And sends the polarized photon to Bob



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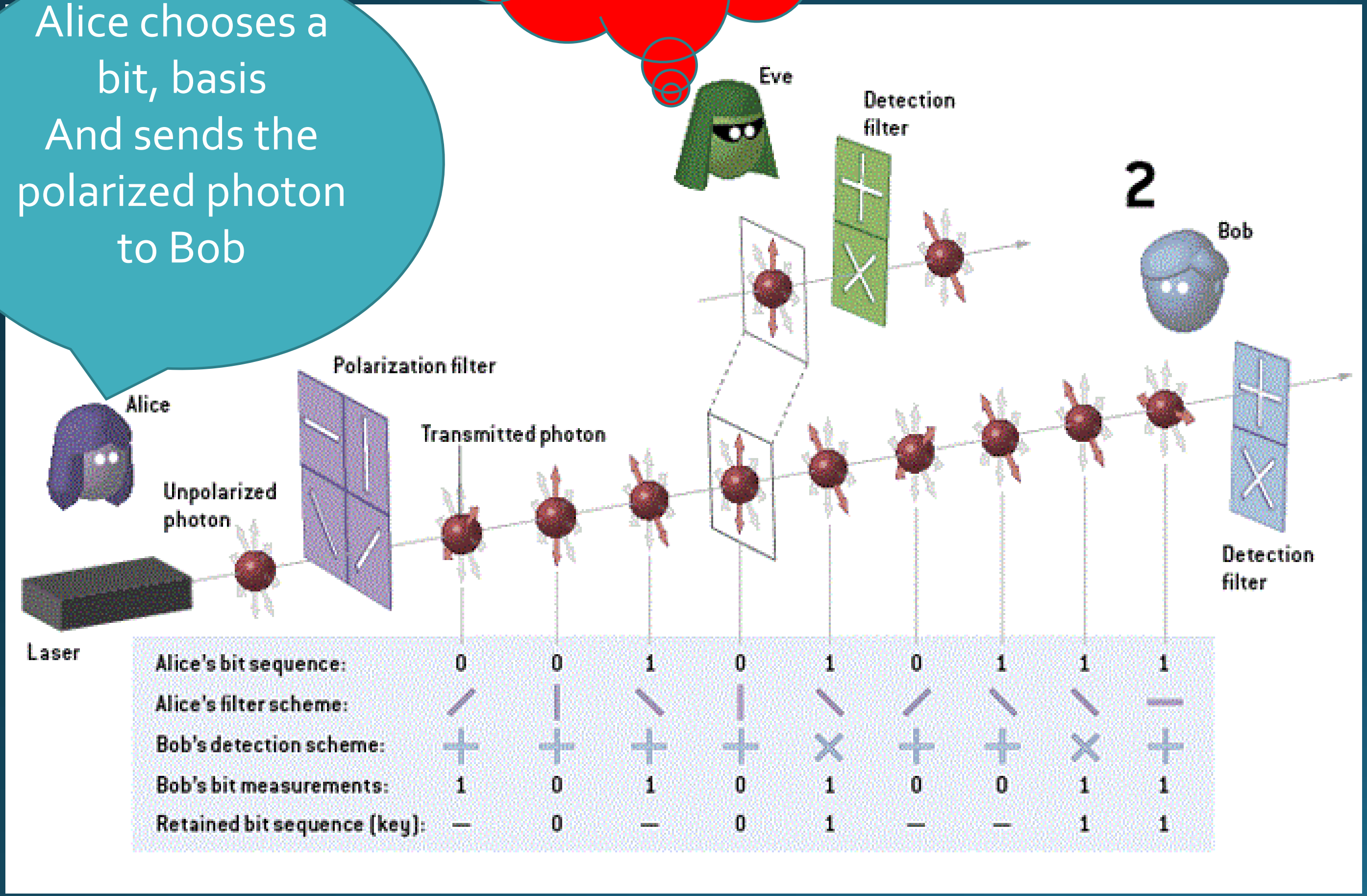

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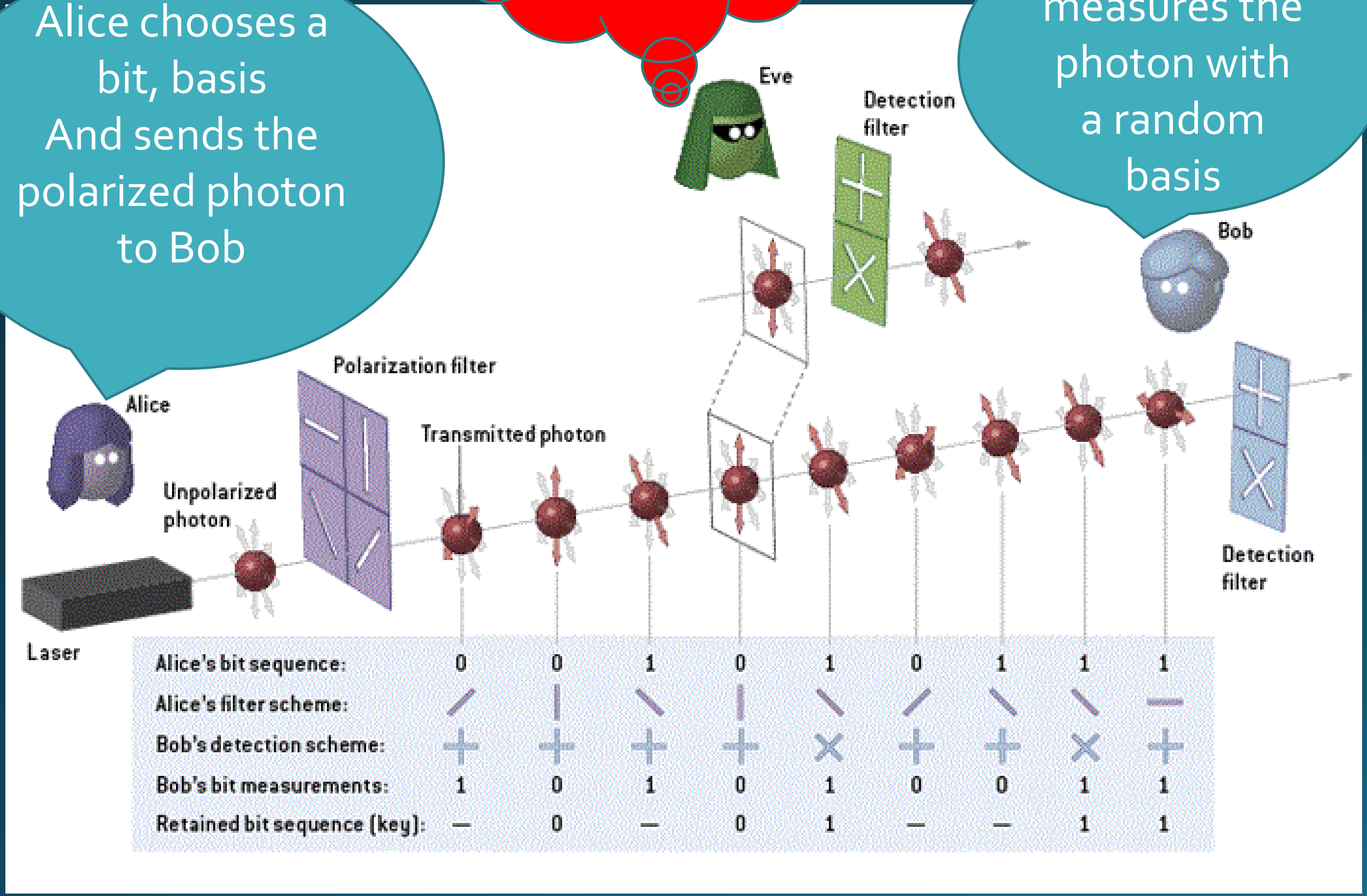
Bob measures the photon with a random basis



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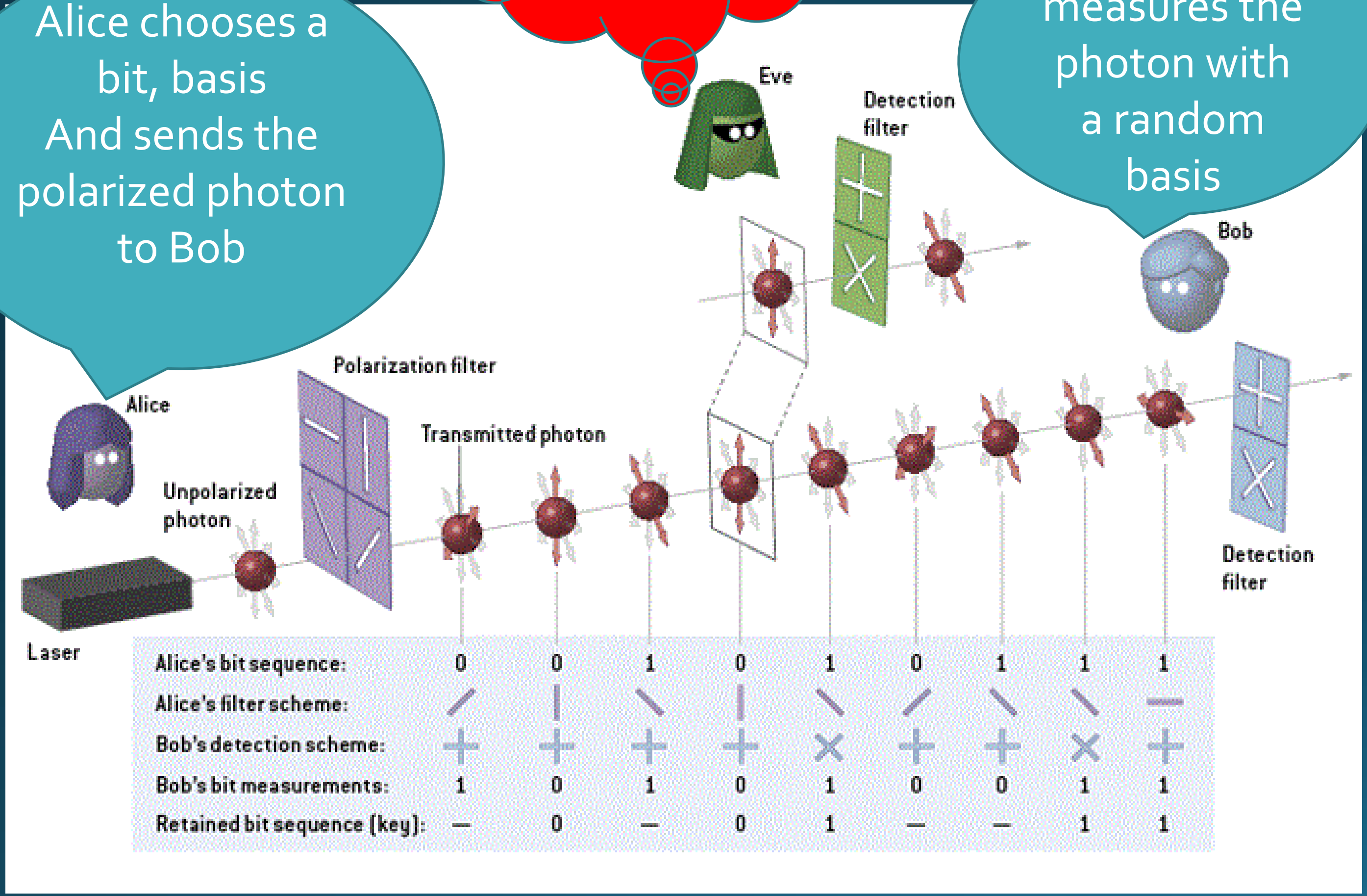
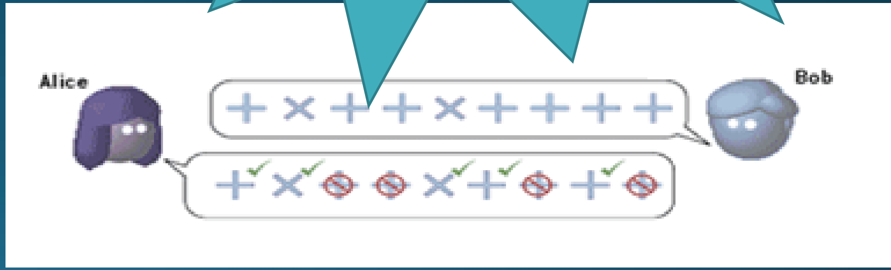


Alice's bit sequence:	0	0	1	0	1	0	1	1	1
Alice's filter scheme:	/	|	\	|	\	/	\	\	—
Bob's detection scheme:	+	+	+	+	×	+	+	×	+
Bob's bit measurements:	1	0	1	0	1	0	0	1	1
Retained bit sequence (key):	—	0	—	0	1	—	—	1	1

-Confirm bases used
-Error estimation
-Privacy Amplification



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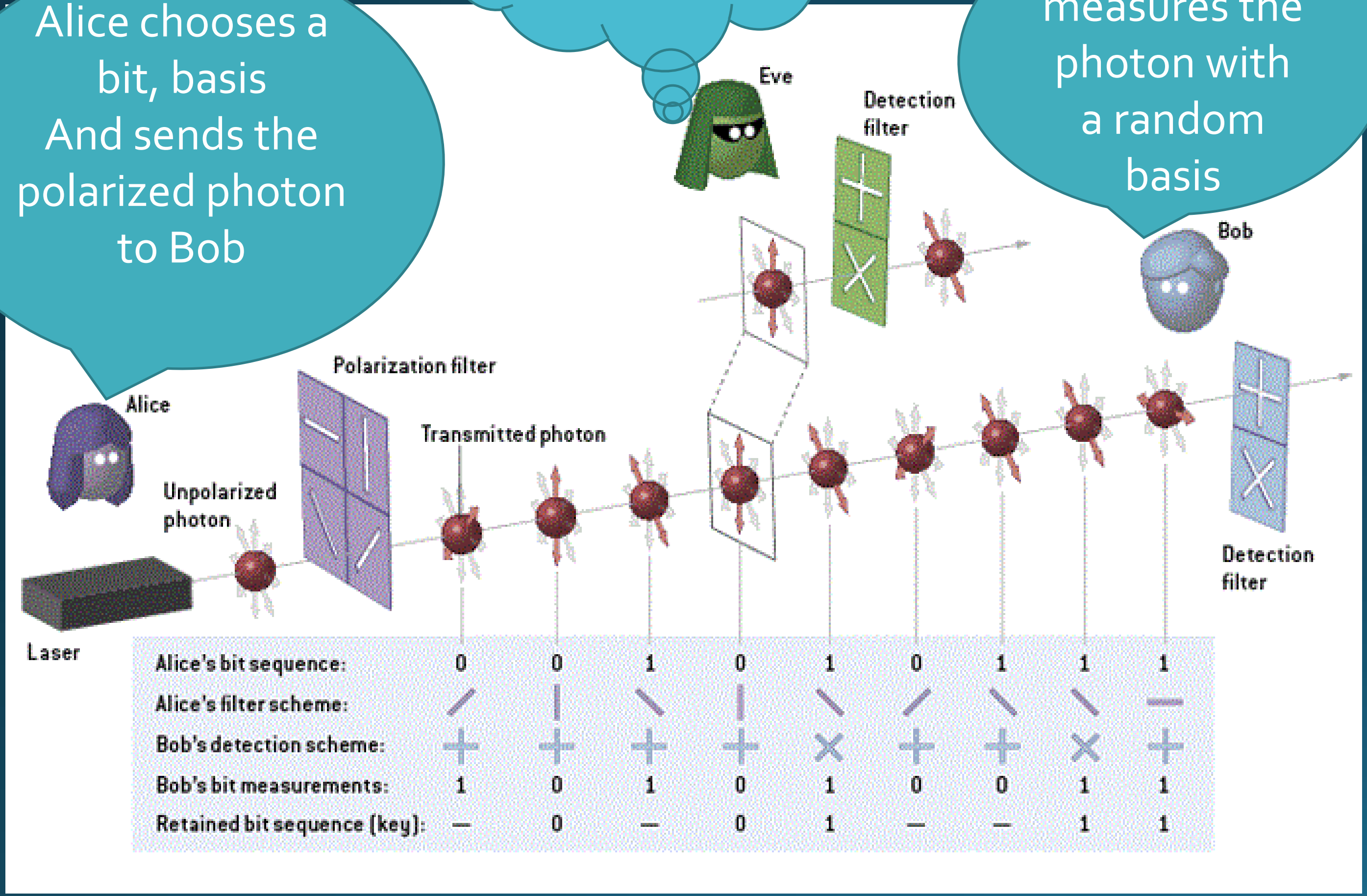

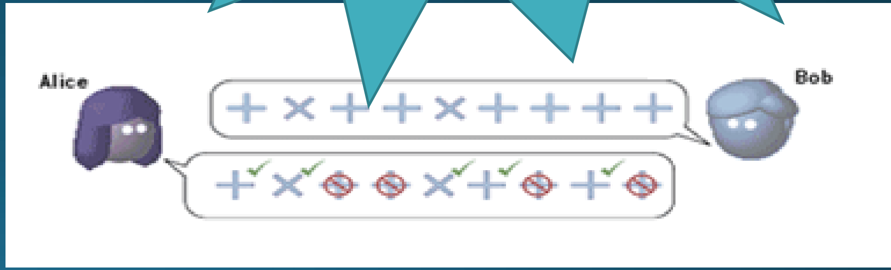

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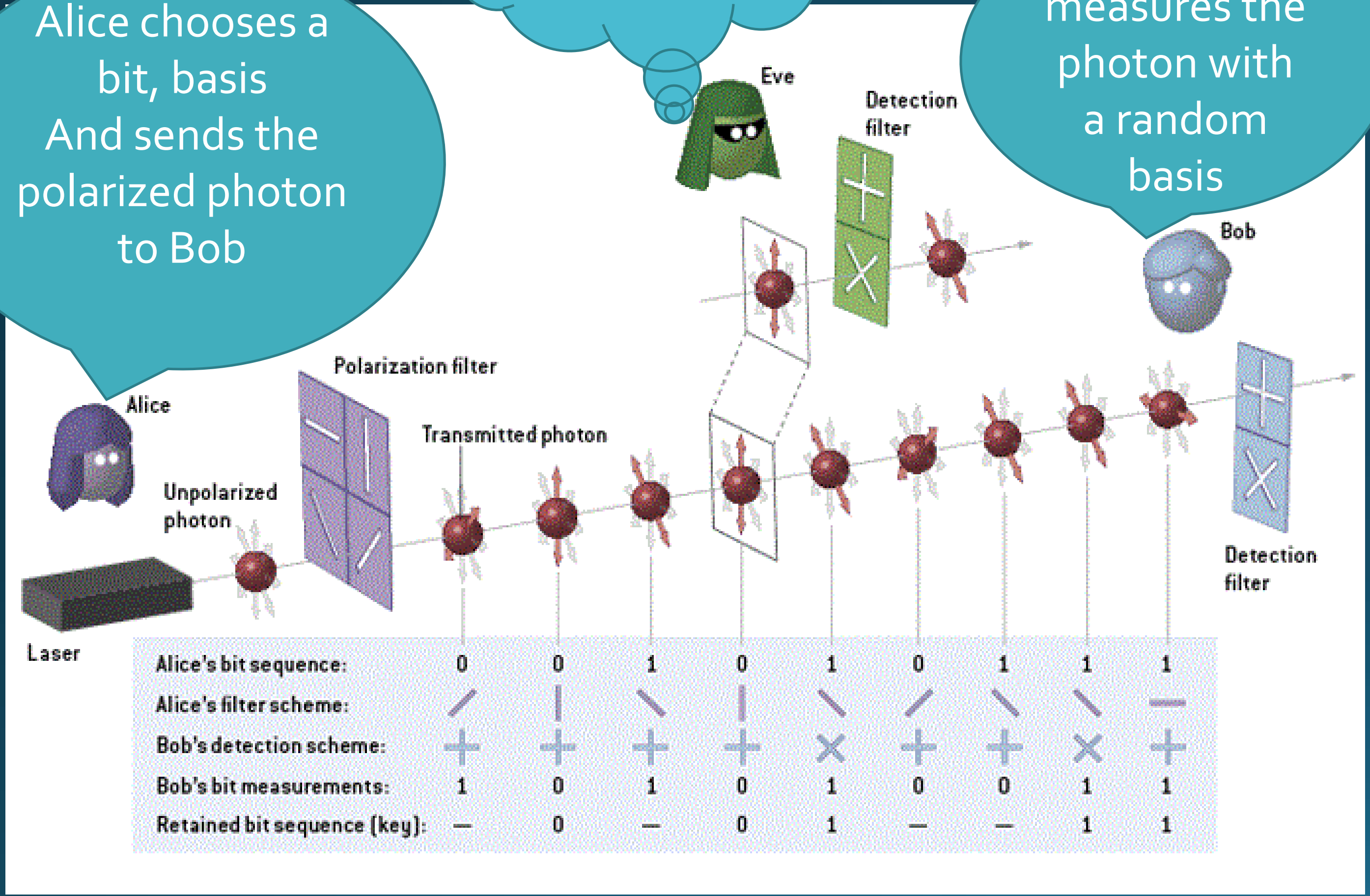

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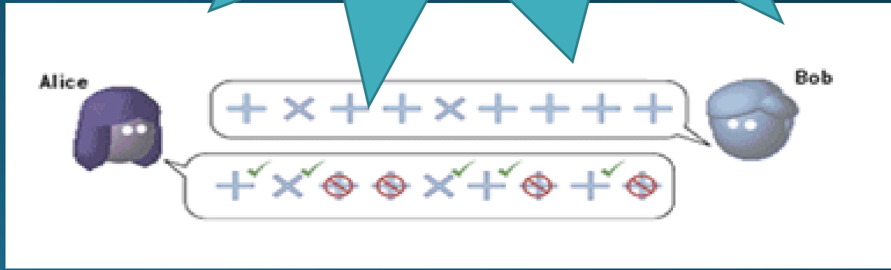
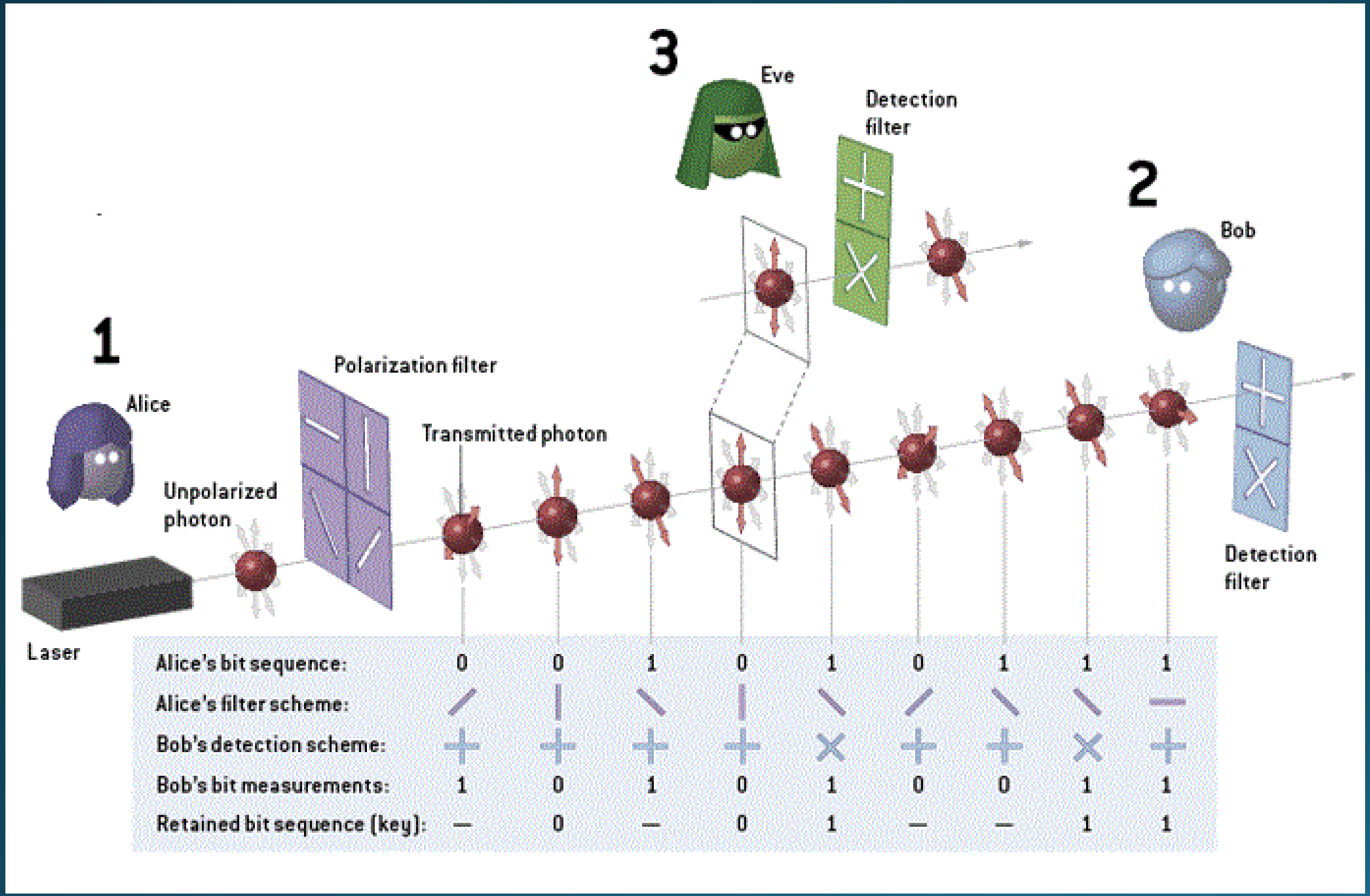
This scheme can be proven to be perfectly secure!

-Confirm bases used
-Error estimation
-Privacy Amplification



Detecting an eavesdropper



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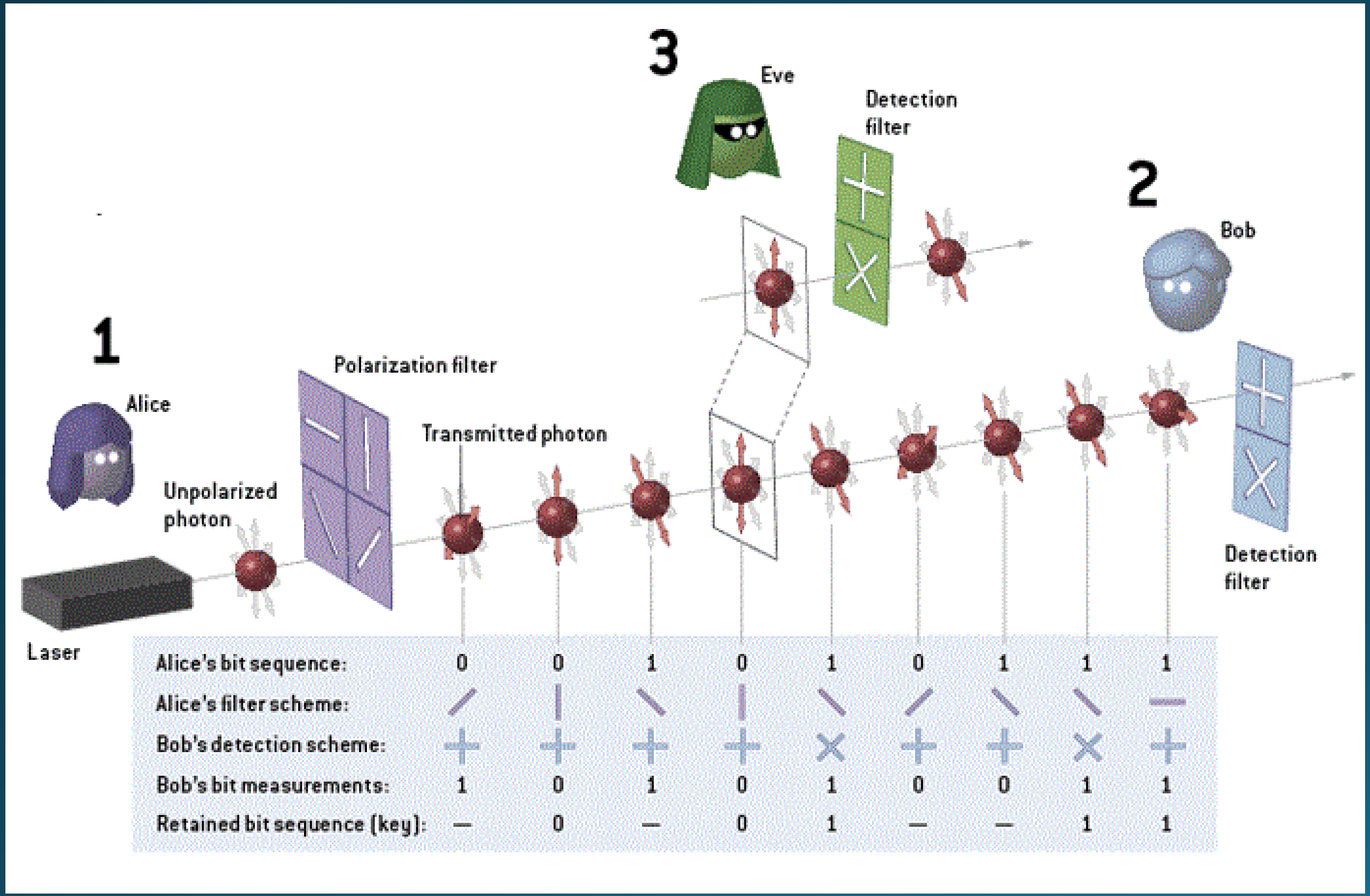
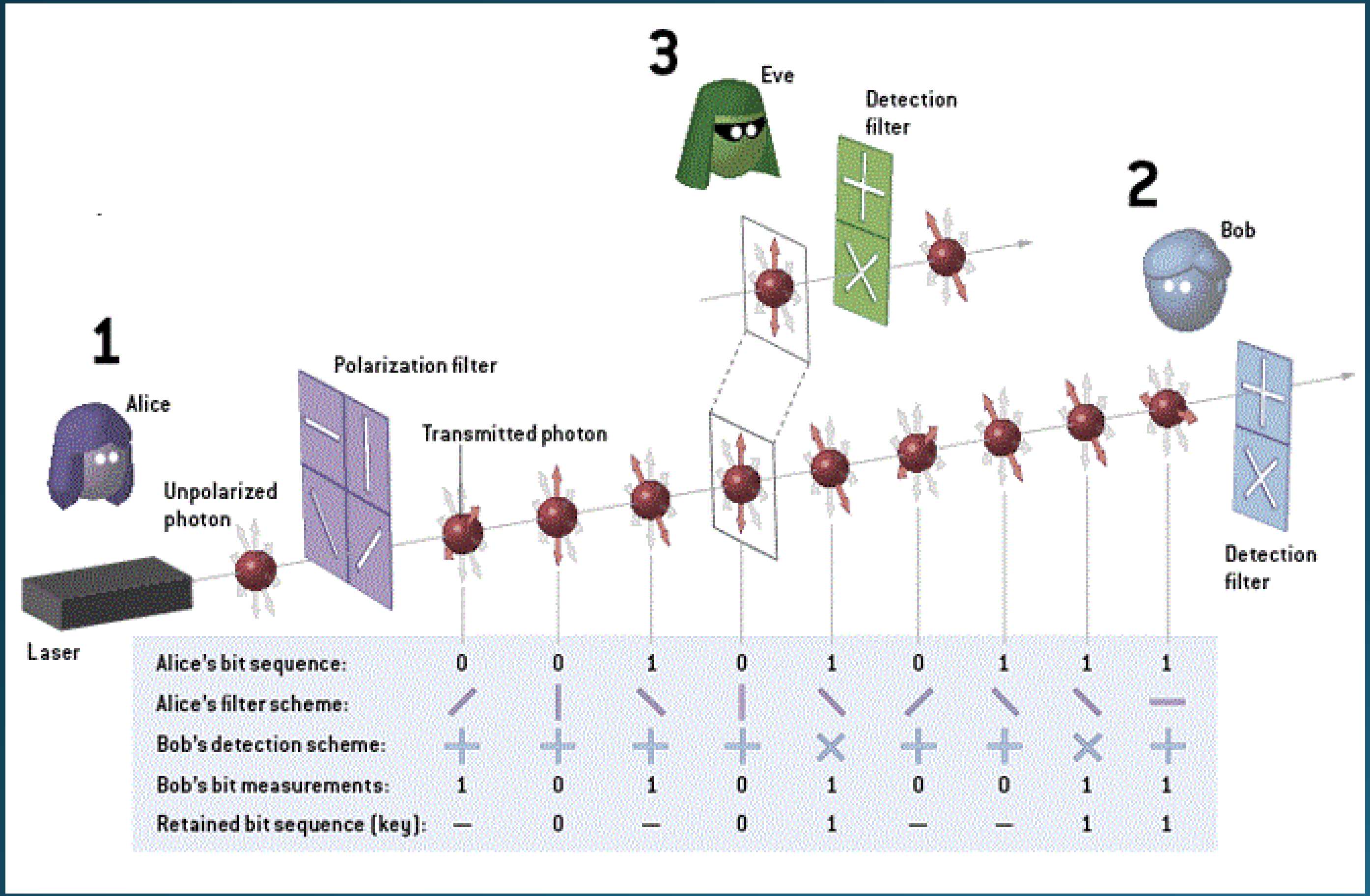


- If Eve measures a state in the wrong basis she will change the state of the photon

Detecting an eavesdropper



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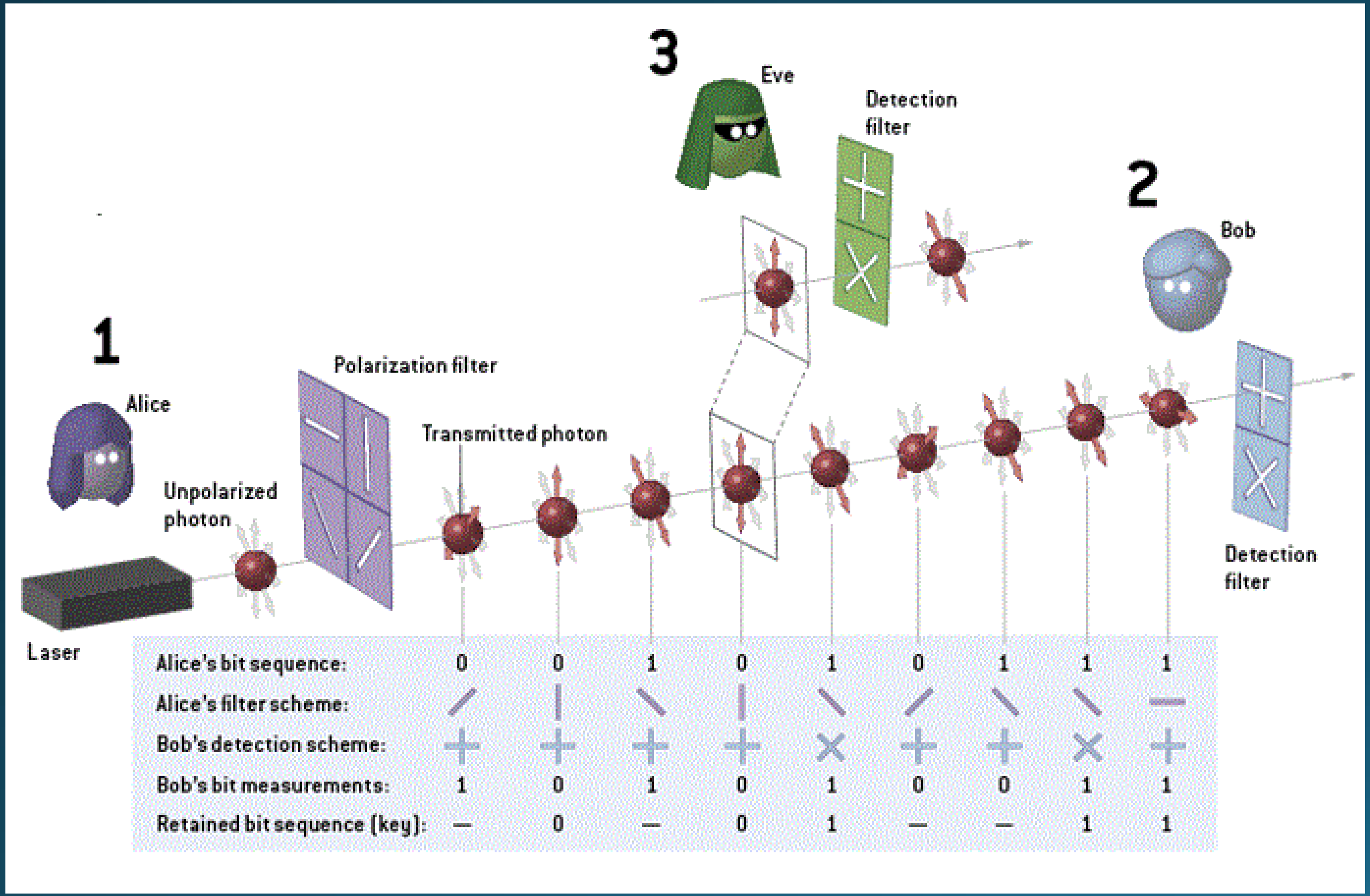
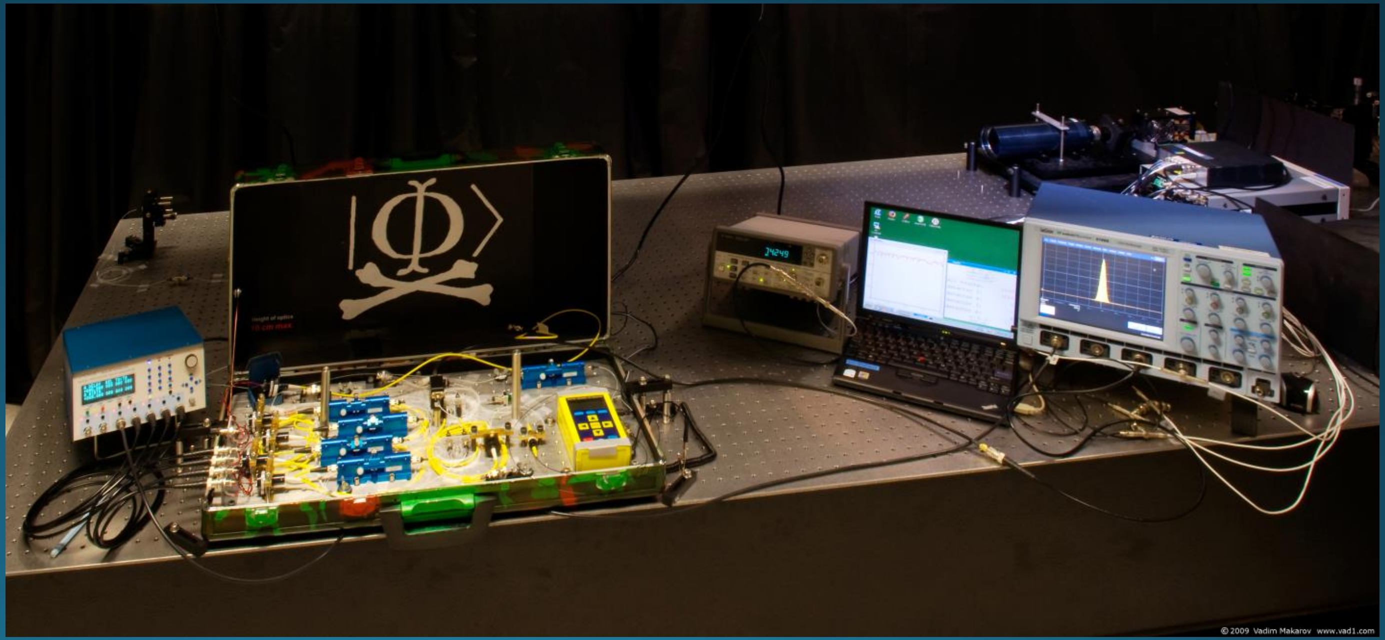


- If Eve measures a state in the wrong basis she will change the state of the photon
- This might introduce errors that can be detected by Alice and Bob
- If too many errors are detected they know that there was an eavesdropper and abort

Applications

- Commercial QKD systems already exist
- 2007 Voting in Geneva [5]
- Approximately 4 commercial companies
- and 5 Quantum Key Distribution Networks



Too good to be true?

- Distances: ~200km using optic fiber
  and much less through free space (air)
- Expensive equipment
- Imperfect implementations, at least two successful attacks

Conclusions

- Quantum Cryptography only relies on laws of nature
- Post-quantum cryptography relies on primitive that are difficult for quantum and classical computers
- Quantum Key Distribution allows two parties to share a key using a public quantum channel
- QKD schemes are perfectly secure, possible and work in practice

  although the implementation of them so far is not perfect

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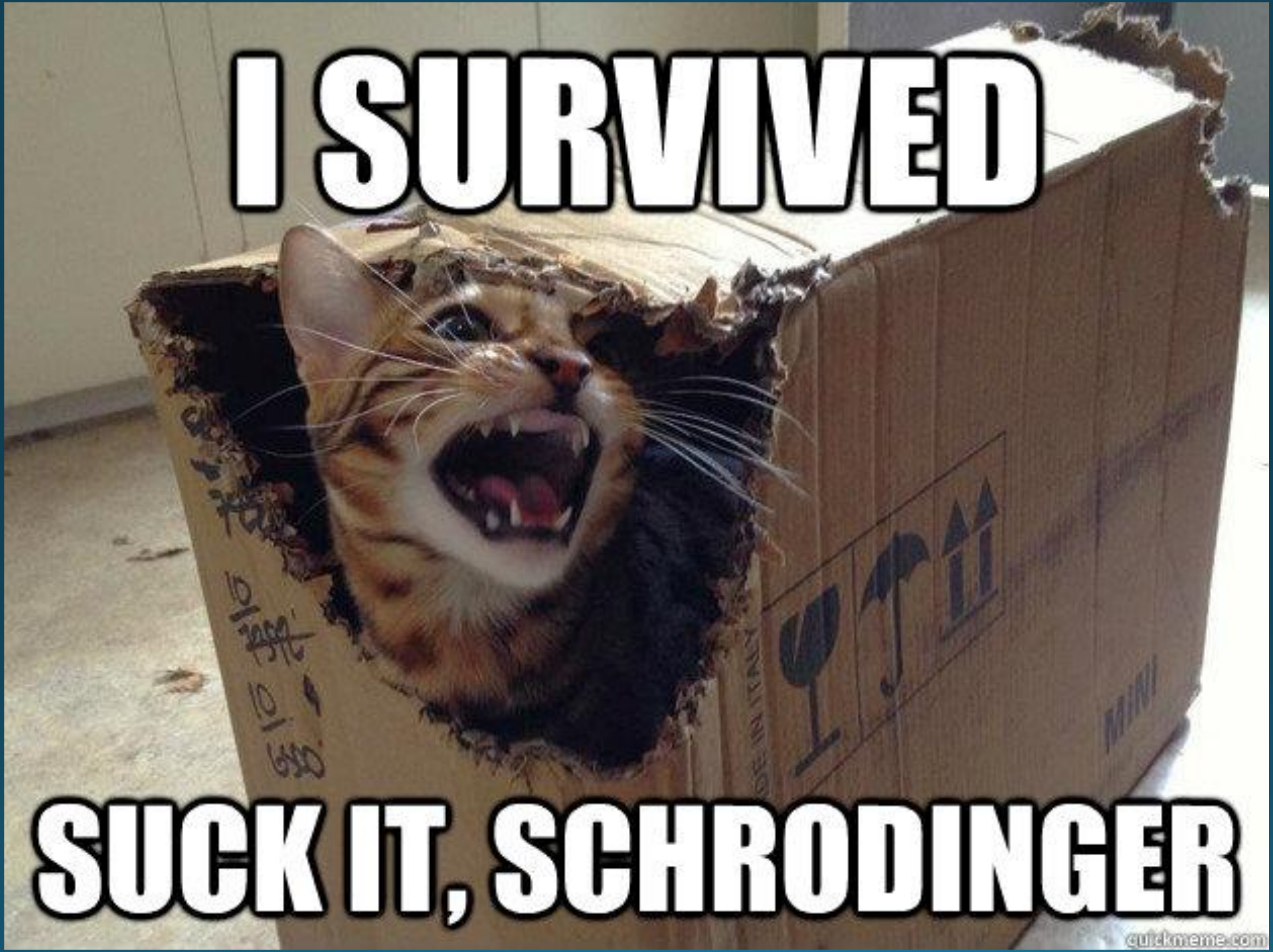
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