Hamiltonian Cycle

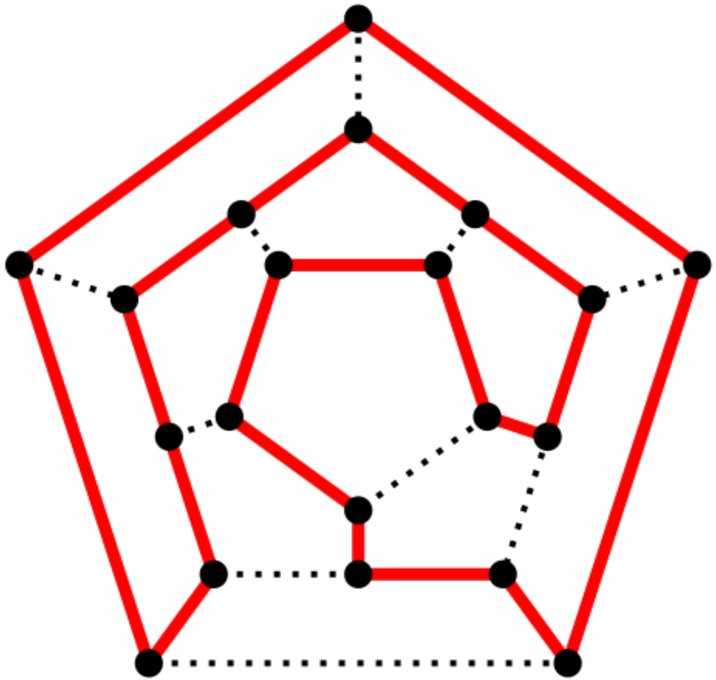
Zero Knowledge Proof

Hamiltonian cycle

Hamiltonian cycle

- A path that visits each vertex exactly once, and ends at the same point it started

Example



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Eulerian path/cycle

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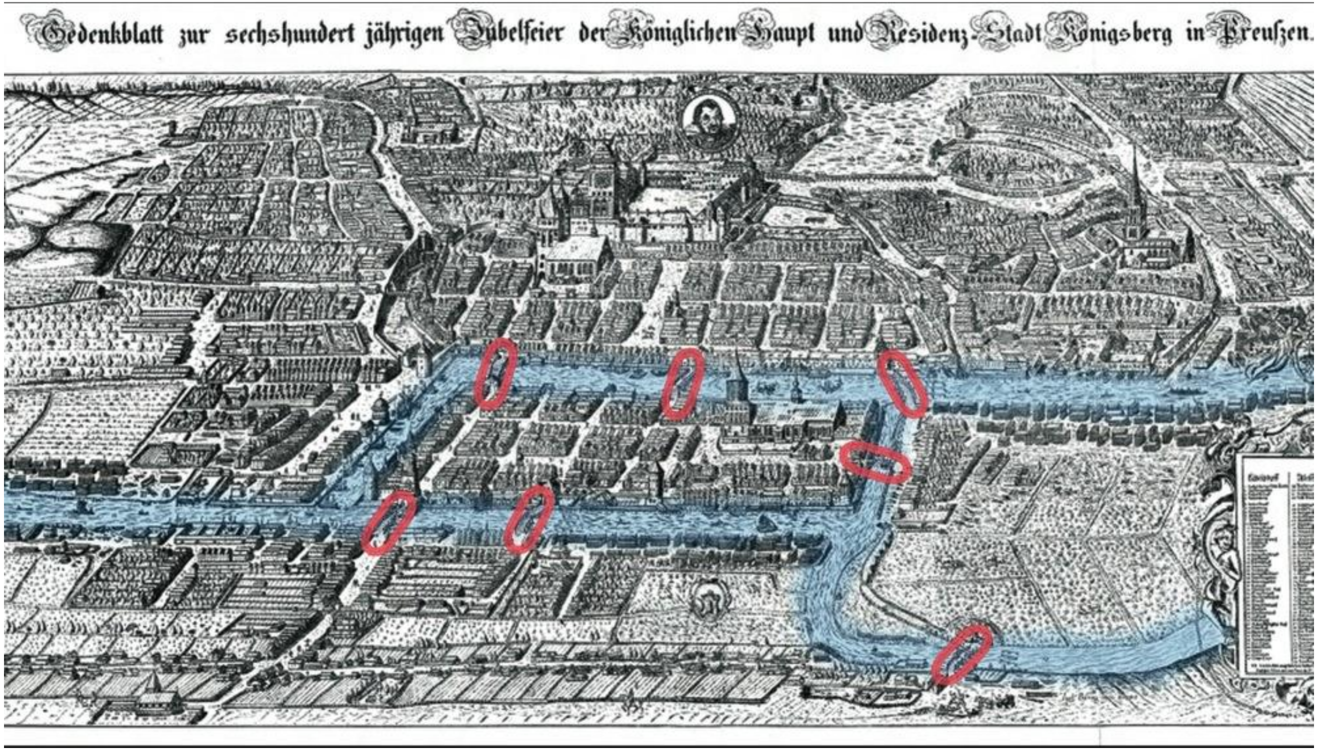
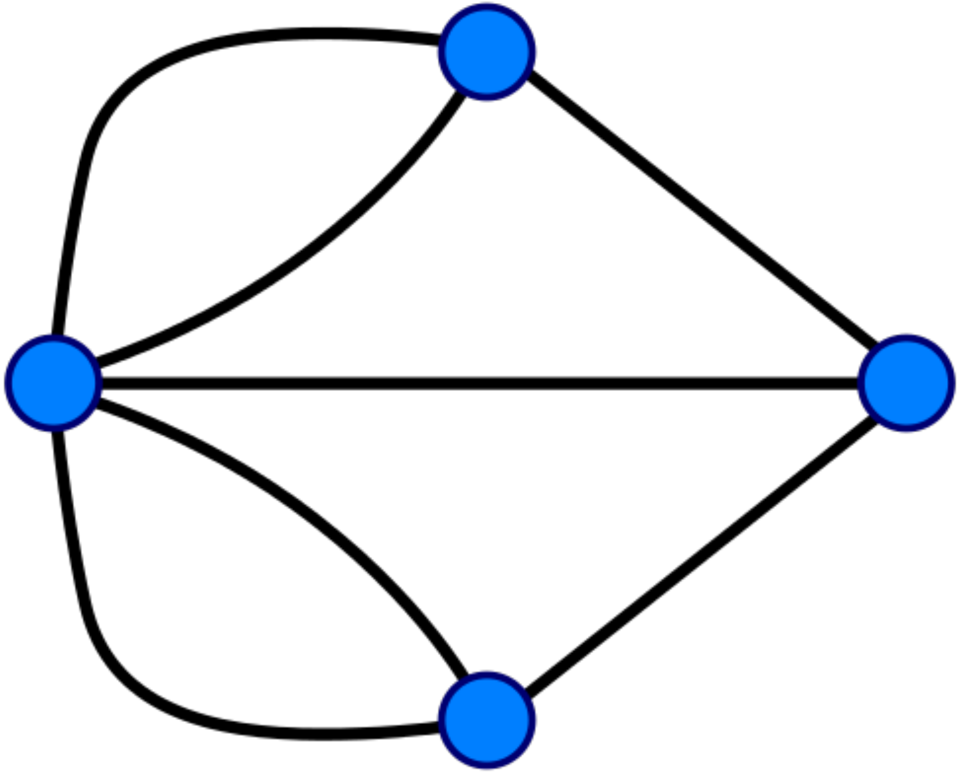
Seven Bridges



Seven Bridges



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- Eulerian path: visits each edge exactly once
- Eulerian cycle: starts and ends at the same point
- Graph has Eulerian circuit iff (1) connected and (2) all vertices have even degree.

Complexity

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Complexity

- Euler vs. Hamilton
- Edges vs. Vertices
- P vs. NP
- No necessary and sufficient conditions for a Hamiltonian cycle
- No good algorithm for finding one (there are known algorithms with running time $O(n^2 2^n)$ and $O(1.657^n)$), so exponential

Zero Knowledge (1)

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- Difference cycle and Hamiltonian cycle
- Permuting: Create a graph F that is isomorphic to G

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- Step 2: P commits to F (how?)
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- Step 4: P reveals (1) F completely plus the isomorphism or (2) the Hamiltonian cycle

Commitment and example

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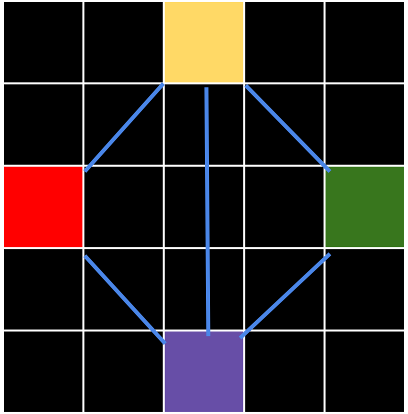
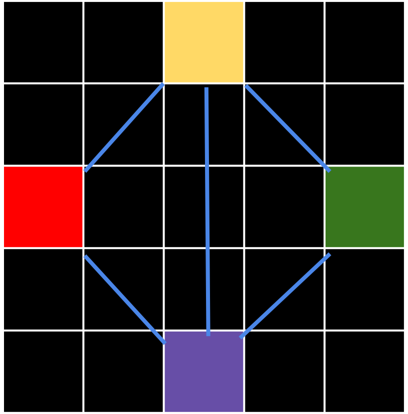


G

Commitment and example



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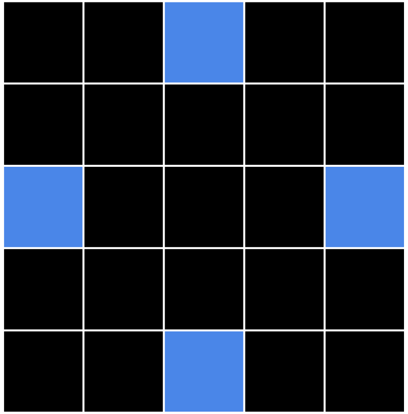
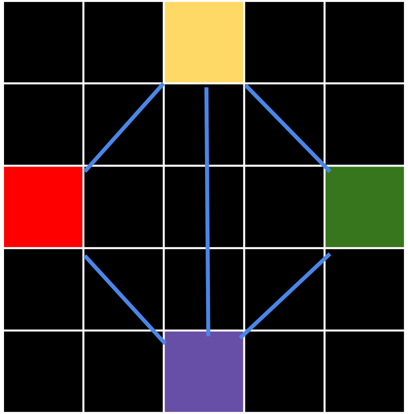


F

Commitment and example



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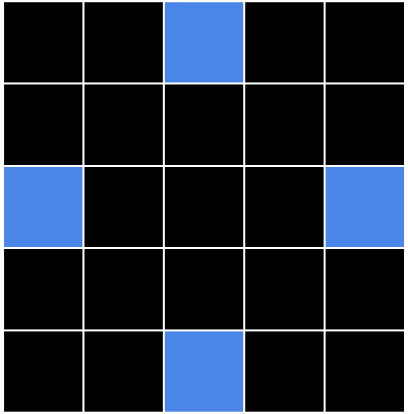

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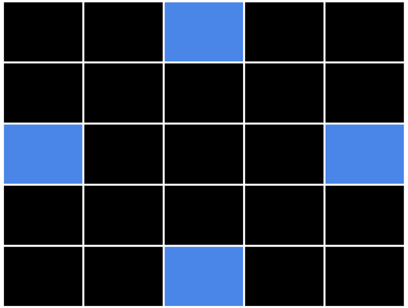

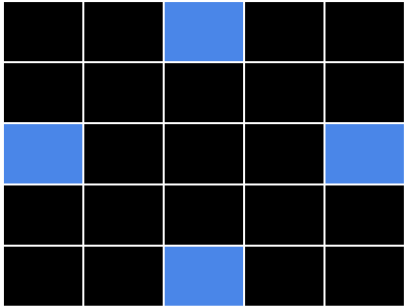

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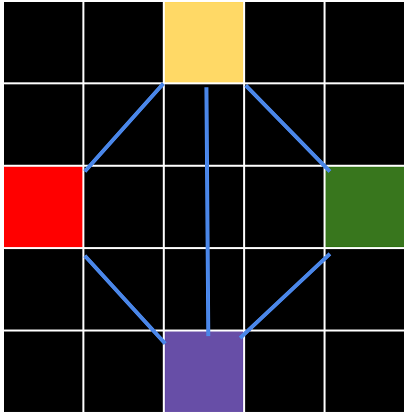

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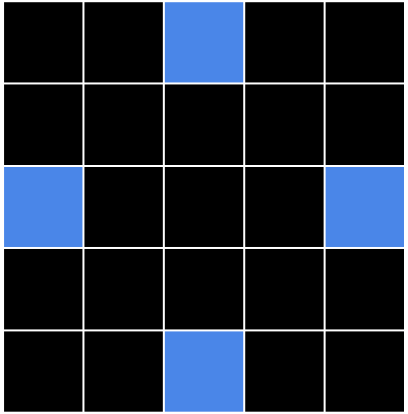

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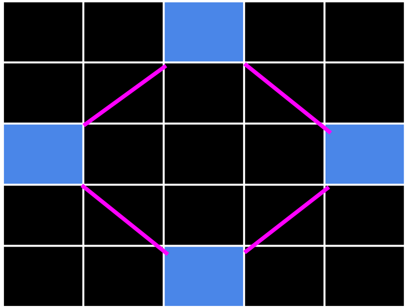
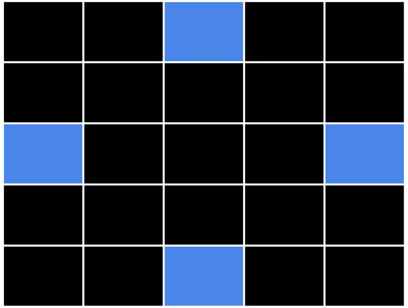
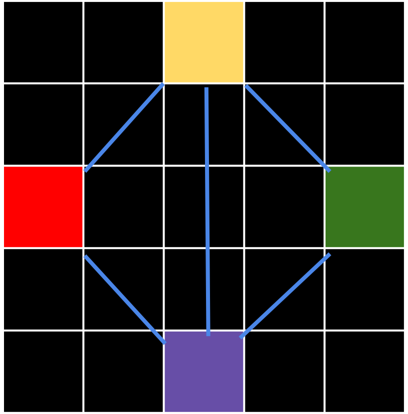


Cycle



Commitment and example



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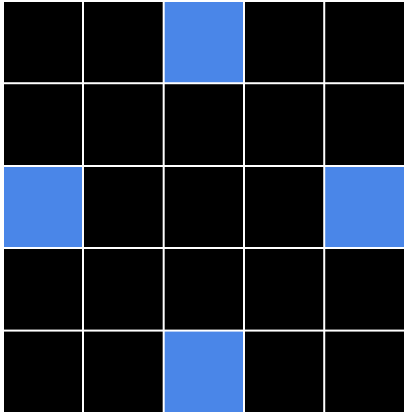

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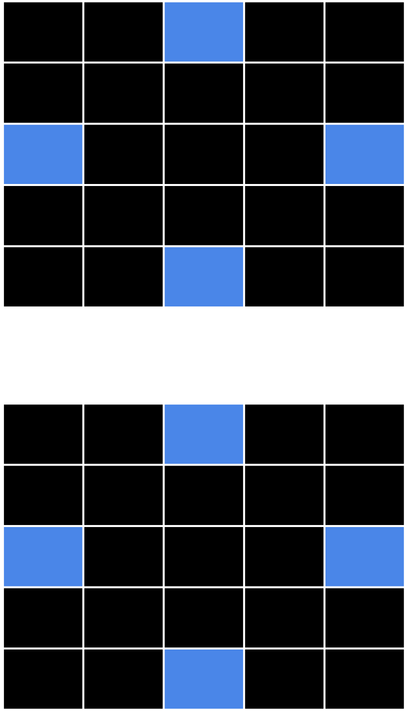

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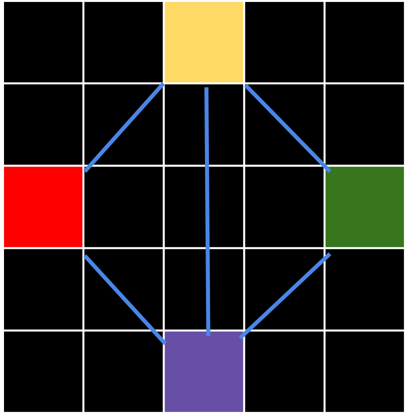

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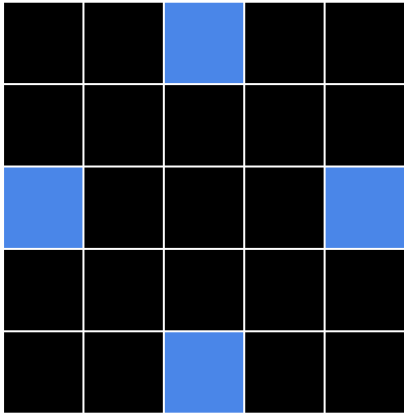

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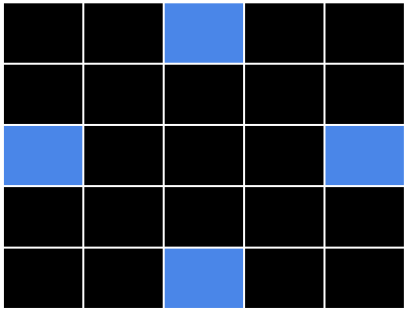

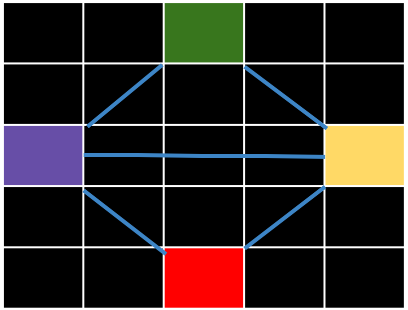

Isomorphism

Completeness, Soundness

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- Soundness: if P does not know, the best he can do is either create an isomorphic F, or create a Hamiltonian cycle. V will accept 50% of the times -> repeat to pass soundness

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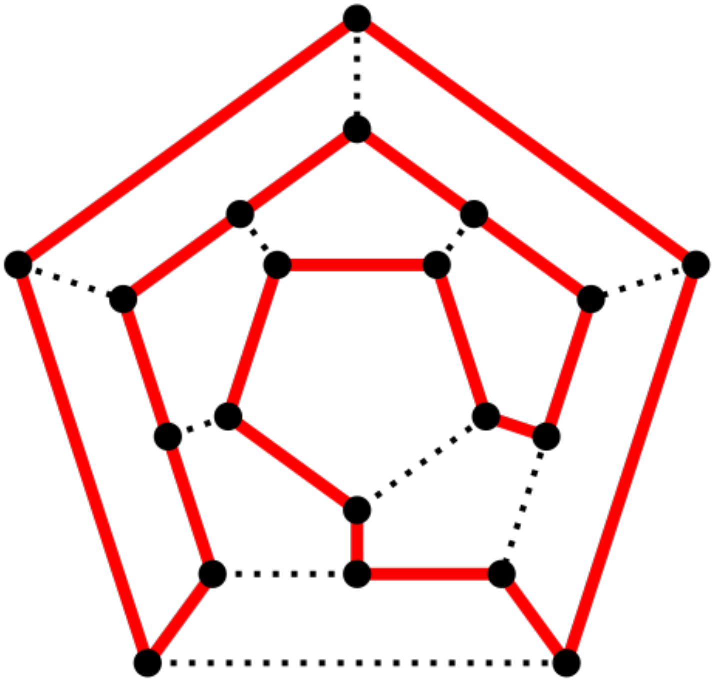
Distribution example



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- Suppose V choses ‘isomorphism’. Then all she sees is a ‘scrambled’ version of G. A simulator does not need P to create a random permutation of G
- Suppose V choses ‘cycle’. Then all she sees is a cycle between some n vertices. Since the permutation was random, a simulator that generates random cycles for n vertices would have the same output distribution
- V does not learn anything!

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- P knows a Hamiltonian cycle for G. Uses random tape to create F, isomorphic to G
- P commits F using some fancy encryption stuff
- V randomly selects 1 or 0

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- If 1, P shows the cycle (in the case of a matrix, this means that every row and every column contains two 1s)
- Verifier checks whether prover is correct

Travelling Salesman

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- Famous variant of Hamilton cycle: given a weighted graph (i.e. edges have a certain value), find the *shortest* Hamiltonian cycle
- NP Hard -> Not only check whether the path is a Hamiltonian cycle, but also whether it is the shortest

Holiday

Shortest path through all US towns/cities with more than 500 citizens