

3rd Homework sheet Proof Theory

- Deadline: 2 March 2018.
- Submit your solutions by handing them to the TA at the *beginning of the exercise class*.
- Good luck!

In this exercise we work in intuitionistic propositional logic. The aim of this exercise is to give two proofs of the following fact:

Let $\Gamma = \{A_1 \rightarrow B_1, \dots, A_n \rightarrow B_n\}$ and C be a formula not containing \rightarrow . If $\Gamma \vdash C$, then $\Gamma \vdash A_i$ for some $i \leq n$.

- (a) (50 points) Give an effective argument using the intuitionistic sequent calculus: that is, show that one can effectively compute from a derivation π of $\Gamma \Rightarrow C$ in the intuitionistic sequent calculus a natural number $i \leq n$ as well as a derivation π' of $\Gamma \Rightarrow A_i$ in the same calculus.
- (b) (50 points) Also give a purely semantic proof of this fact using Kripke models (that is, without using completeness and part (a)).