4th Exercise sheet Proof Theory
21 Feb 2018

Exercise 1 Use backwards proof search to either find classical countermodels or cut free derivations in the classical sequent calculus for the following sequents:

(a) $p \wedge q, p \to s \Rightarrow s$

(b) $p \vee q, p \to s \Rightarrow s$

(c) $q \to p, \neg p, r \vee q \Rightarrow r$

(d) $\neg(p \wedge q) \Rightarrow \neg p \vee \neg q$

(e) $p \vee q, q \vee r, r \vee s, \neg r \Rightarrow (q \wedge s) \to p$

Exercise 2 (a) Construct for any formula φ a cut free derivation of

$$\Gamma, \varphi \Rightarrow \varphi, \Delta$$

in the classical sequent calculus.

(b) Show that for any formula φ there is a cut free derivation of $\Rightarrow \varphi \vee \neg\varphi$.

Exercise 3 This exercise will devoted to show interpolation properties using the cut free sequent calculus for classical propositional logic.

Suppose π is a cut free derivation with endsequent $\Gamma, \Gamma' \Rightarrow \Delta, \Delta'$. Construct by induction on the derivation π a formula φ (an *interpolant*) such that:

(1) $\Gamma \Rightarrow \Delta, \varphi$ is derivable

(2) $\Gamma', \varphi \Rightarrow \Delta'$ is derivable

(3) Every propositional variable occurring in φ occurs both in $\Gamma \cup \Delta$ and $\Gamma' \cup \Delta'$.

Deduce that if $\Gamma \Rightarrow \Delta$ is derivable then there is a formula φ such that both $\Gamma \Rightarrow \varphi$ and $\varphi \Rightarrow \Delta$ are derivable, while every propositional variable occurring in φ occurs both in Γ and Δ.