

5th Homework sheet Proof Theory

- Deadline: 8 December, 9:00 sharp.
- Submit your solutions by handing them to the lecturer or the teaching assistant at the *beginning of the lecture*.
- Good luck!

Exercise 1 In this exercise we work with sequent calculi for predicate logic. Its aim is to show that cut elimination can be used to establish interpolation results.

- (a) In this part of the exercise we work in the intuitionistic setting. Show that if φ is a formula and Γ and Γ' are two finite sets of formulas such that $\Gamma, \Gamma' \Rightarrow \varphi$ is derivable, then there is a formula ψ such that:
- (1) $\Gamma \Rightarrow \psi$ is derivable
 - (2) $\Gamma', \psi \Rightarrow \varphi$ is derivable
 - (3) Every relation symbol occurring in ψ occurs both in Γ and in $\Gamma' \cup \{\varphi\}$.

Deduce that if $\Gamma \Rightarrow \varphi$ is derivable then there is a formula ψ such that both $\Gamma \Rightarrow \psi$ and $\psi \Rightarrow \varphi$ are derivable, while every relation symbol occurring in ψ occurs both in Γ and φ .

- (b) Sketch an argument for the interpolation property for the classical sequent calculus: if $\Gamma \Rightarrow \Delta$ is derivable in the classical sequent calculus, then there is a formula φ such that both $\Gamma \Rightarrow \varphi$ and $\varphi \Rightarrow \Delta$ are derivable, while every relation symbol occurring in φ occurs both in Γ and in Δ .

Please be kind on both your wrists and the TA and only discuss a few pertinent or illustrative cases in your proofs.