

3rd Exercise sheet Proof Theory

15 Nov 2016

Exercise 1 Use backwards proof search to either find classical countermodels or cut free derivations in the classical sequent calculus for the following sequents:

- (a) $p \wedge q, p \rightarrow s \Rightarrow s$
- (b) $p \vee q, p \rightarrow s \Rightarrow s$
- (c) $q \rightarrow p, \neg p, r \vee q \Rightarrow r$
- (d) $\neg(p \wedge q) \Rightarrow \neg p \vee \neg q$
- (e) $p \vee q, q \vee r, r \vee s, \neg r \Rightarrow (q \wedge s) \rightarrow p$

Exercise 2 Construct for any formula φ a cut free derivation of $\Gamma, \varphi \Rightarrow \varphi$ in the intuitionistic sequent calculus.

Exercise 3 Find effective procedures for converting proofs of $\Gamma \vdash \varphi$ in intuitionistic natural deduction into proofs of $\Gamma \Rightarrow \varphi$ in the intuitionistic sequent calculus (with cut rule), and *vice versa*.

Exercise 4 Show that the rules introducing disjunction on the right and implications on the left are not invertible in the intuitionistic sequent calculus.