6th Homework sheet Proof Theory

- Deadline: 7 December.
- Submit your solutions by handing them to the TA *at 9:00 sharp*.
- Good luck!

In both exercises you have to justify your answer by providing a proof showing that it is correct.

Exercise 1 *(50 points)* Construct a closed term **minus** of type $0 \to (0 \to 0)$ such that HA^ω proves

$$\begin{array}{rcl} \mathbf{minus}\, 0\, n & = & 0 \\ \mathbf{minus}\, Sm\, 0 & = & Sm \\ \mathbf{minus}\, Sm\, Sn & = & \mathbf{minus}\, m\, n \end{array}$$

Hint: Use the recursor twice: one time to define a sequence of objects $f(x^0)$ of type $0 \to 0$ such that $f(x)(y) = \mathbf{minus}\, x\, y$ and another time in the definition of $f(Sx)$ in terms of $f(x)$.

Exercise 2 *(50 points)* Let $\varphi(x)$ and $\psi(x)$ be two formulas in the language of HA^ω. Suppose that their types are σ and τ, respectively, and that both these formulas only have the variable x^ρ as a free variable. Construct a closed term t in the language of HA^ω such that HA^ω proves that

$$t \;\mathsf{mr}\; \big[\forall x^\rho \,(\, \varphi(x) \to \psi(x)\,) \to (\, \exists x^\rho\, \varphi(x) \to \exists x^\rho\, \psi(x)\,)\big].$$