

# 1st Homework sheet Proof Theory

- Deadline: 2 November.
- Submit your solutions by handing them to the teaching assistant at the *beginning of the lecture*.
- Good luck!

**Exercise 1** (a) (50 points) Let  $(W, R, f)$  be a Kripke model. Instead of  $f(w)$  we will also write  $\mathcal{M}_w$  to emphasize that the set of propositional letters  $f(w)$  can also be regarded as a classical model. Let  $w \in W$  be a world and  $p_1, \dots, p_k$  be a finite set of propositional variables and assume that the truth value of  $p_1, \dots, p_k$  in worlds reachable from  $w$  is the same as that in  $w$ ; more formally,

$$\text{if } wRw' \text{ then } f(w) \cap \{p_1, \dots, p_k\} = f(w') \cap \{p_1, \dots, p_k\}.$$

Finally, assume that  $\varphi$  is a formula which only contains propositional variables belonging to  $\{p_1, \dots, p_k\}$ . Show that  $\varphi$  is forced at  $w$  if and only if it holds in the classical model  $\mathcal{M}_w$ .

(b) (50 points) Let  $\varphi$  be a formula in propositional logic. Use part (a) to show that  $\varphi$  is a classical tautology if and only if  $\neg\neg\varphi$  is an intuitionistic tautology.

*Hint:* What does it mean to force  $\neg\neg\varphi$ ?