1st Homework sheet Proof Theory

- Deadline: 7 November.
- Submit your solutions by handing them to the lecturer at the *beginning of the lecture.*
- Good luck!

Exercise 1 (a) *(50 points)* Let (W, R, f) be a Kripke model. Instead of $f(w)$ we will also write $\mathcal{M}_w$ to emphasize that the set of propositional letters $f(w)$ can also be regarded as a classical model. Let $w \in W$ be a world and $p_1, \ldots, p_k$ be a finite set of propositional variables and assume that the truth value of $p_1, \ldots, p_k$ in worlds reachable from w is the same as that in w; more formally,

$$\text{if } wRw' \text{ then } f(w) \cap \{p_1, \ldots, p_k\} = f(w') \cap \{p_1, \ldots, p_k\}.$$

Finally, assume that φ is a formula which only contains propositional variables belonging to $\{p_1, \ldots, p_k\}$. Show that φ is forced at w if and only if it holds in the classical model $\mathcal{M}_w$.

(b) *(50 points)* Let φ be a formula in propositional logic. Use part (a) to show that φ is a classical tautology if and only if $\neg\neg\varphi$ is an intuitionistic tautology.

Hint: What does it mean to force $\neg\neg\varphi$?