

## 5th Homework sheet Model Theory

- Deadline: 11 March.
- Submit your solutions by handing them to the lecturer at the *beginning of the lecture at 15:00*.
- Good luck!

**Exercise 1** Let  $M$  be an  $L$ -structure and  $A$  be a subset of  $M$ . We say that  $b$  is *algebraic over  $A$*  if there is an  $L$ -formula  $\varphi(x, \bar{y})$  and a tuple  $\bar{a}$  from  $A$  such that

$$M \models \varphi(b, \bar{a})$$

and the set

$$\{x \in M : M \models \varphi(x, \bar{a})\}$$

is finite. We write  $\text{acl}(A)$  for the set of elements in  $M$  that are algebraic over  $A$ .

- Show that  $A \subseteq \text{acl}(A)$ .
- Show that  $\text{acl}(\text{acl}(A)) = \text{acl}(A)$ .
- Write  $T = \text{Th}_{L_A}((M, a)_{a \in A})$ , the set of all  $L_A$ -sentences true in  $M$ . Show that if  $b$  is algebraic over  $A$ , then  $\text{tp}_M^{L_A}(b)$  is isolated in  $T$ .
- Suppose that  $T$  is a nice  $\omega$ -categorical theory. Show that there is a function  $f: \mathbb{N} \rightarrow \mathbb{N}$  such that for any model  $M$  of  $T$  and any subset  $A \subseteq M$  with  $|A| \leq n$ , we have  $|\text{acl}(A)| \leq f(n)$ .