6th Exercise sheet Model Theory
11 Mar 2015

Exercise 1 Let $L = \{E\}$ where E is a binary relation symbol. For each of the following theories either prove that they have quantifier elimination or give an example showing that they do not have quantifier elimination and a natural $L' \supseteq L$ in which they do have quantifier elimination.

(a) E is an equivalence relation and has infinitely many equivalence classes and each equivalence class has size 2.

(b) E is an equivalence relation and has infinitely many equivalence classes and each equivalence class is infinite.

(c) E is an equivalence relation and has infinitely many equivalence classes of size 2 and infinitely many equivalence classes of size 3, and every equivalence class has size 2 or 3.

(d) E is an equivalence relation and has precisely one equivalence class of size n for each finite n, but no infinite equivalence classes.

Exercise 2 Let $M = (\mathbb{Z}, s)$, where $s(x) = x + 1$, and let $T = \mathrm{Th}(M)$.

(a) Show that T has quantifier elimination.

(b) Give a concrete description of a countable ω-saturated model of T.

(c) Describe the type spaces of T.

(d) Show that $\mathrm{Th}(\mathbb{N}, s)$ does not have quantifier elimination.

Exercise 3 (a) Show that the theory of $(\mathbb{N}, <)$ has quantifier elimination in the language where we add a function symbol s for the function $s(x) = x + 1$ and a constant symbol for 0.

(b) Give a concrete description of a countable ω-saturated model of $\mathrm{Th}(\mathbb{N}, <)$.

(c) Describe the type spaces of $\mathrm{Th}(\mathbb{N}, <)$