2nd Exercise sheet Model Theory
12 Feb 2015

Exercise 1 A class of models $\mathcal{K}$ in some fixed signature is called an *elementary class* if there is a first-order theory such that $\mathcal{K}$ consists of precisely those L-structures that are models of T.

Show that if $\mathcal{K}$ is a class of L-structures and both $\mathcal{K}$ and its complement (in the class of all L-structures) are elementary, then there is a sentence φ such that M belongs to $\mathcal{K}$ if and only if $M \models \varphi$.

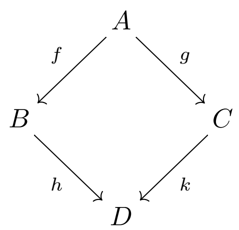
Exercise 2 We work over the empty language L (no constants, function or relations symbols). Show that the class of infinite L-structures is elementary, but the class of finite L-structures is not. Deduce that there is no sentence φ that is true if and only if the L-structure is infinite.

Exercise 3 (Exam question from last year.) A class $\mathcal{K}$ of L-structures is a PC_Δ-*class*, if there is an extension L' of L and an L'-theory T' such that $\mathcal{K}$ consists of all reducts to L of models of T'.

Show that a PC_Δ-class of L-structures is L-elementary if and only if it is closed under L-elementary substructures.

Exercise 4 In the lecture we deduced the Craig Interpolation Theorem from the Robinson Consistency Theorem. Show how one can deduce the Robinson Consistency Theorem from the Craig Interpolation Theorem.

Exercise 5 Use Robinson's Consistency Theorem to prove the following Amalgamation Theorem: Let L_1, L_2 be languages and $L = L_1 \cap L_2$, and suppose A, B and C are structures in the languages L, L_1 and L_2, respectively. Any pair of L-elementary embeddings $f\colon A \to B$ and $g\colon A \to C$ fit into a commuting square



where D is an $L_1 \cup L_2$-structure, h is an L_1-elementary embedding and k is an L_2-elementary embedding.

Exercise 6 (Challenging!) An *existential sentence* is a sentence which consists of a string of existential quantifiers followed by a quantifier-free formula.

Show that a theory T can be axiomatised using existential sentences if and only if its models are closed under extensions.