3rd Homework sheet Model Theory

- Deadline: 27 February, 13:00 sharp.
- Submit your solutions by handing them to the lecturer or the teaching assistant at the *beginning of the lecture*.
- Good luck!

Exercise 1 Let $L_3 = \{<, c_0, c_1, c_2, \ldots\}$, where $c_0, c_1, \ldots$ are constant symbols. Let T_3 be the theory of dense linear orders without end points with sentences added asserting $c_0 < c_1 < \ldots$.

(a) *(25 points)* Show that T_3 has exactly three countable models up to isomorphism.

Hint: Consider the questions: Does $c_0, c_1, c_2, \ldots$ have an upper bound? A least upper bound?

(b) *(25 points)* Prove the following two general results and use them to prove that T_3 is complete.

(i) For any language L, two L-structures M and N are elementarily equivalent if and only if their reducts $M \restriction L'$ and $N \restriction L'$ are elementarily equivalent for any finite sublanguage L' of L.

(ii) If L is countable, T is an L-theory with no finite models, and any two countable models of T are elementarily equivalent, then T is complete.

(c) *(25 points)* Let $L_4 = L_3 \cup \{P\}$, where P is a unary predicate. Let T_4 be T_3 with the added sentences $P(c_i)$ and

$$\forall x \, \forall y \, \big(x < y \rightarrow \exists z \, \exists w \, (x < z < y \wedge x < w < y \wedge P(z) \wedge \neg P(w)) \big).$$

In other words, P is a dense-codense subset. Show that T_4 is a complete theory with exactly four countable models.

(d) *(25 points)* Generalise (c) to give examples of complete theories which have exactly n countable models for $n = 5, 6, \ldots$